

# Grid Shaping Control for High-IBR Power Systems

Stability Analysis and Control Design

**Enrique Mallada**



**Tsinghua University**

Electrical Engineering Department

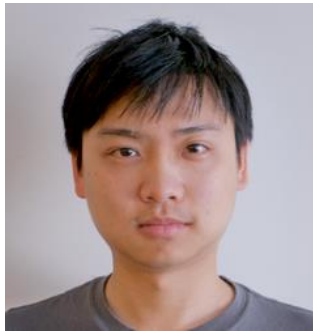
July 15<sup>th</sup>, 2025

# Acknowledgements

## Students



**Yan Jiang**



**Hancheng Min**



**Eliza Cohn**



## Collaborators



**Petr Vorobev**



**Richard Pates**



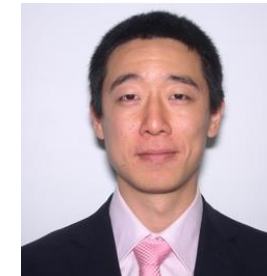
**Fernando Paganini**



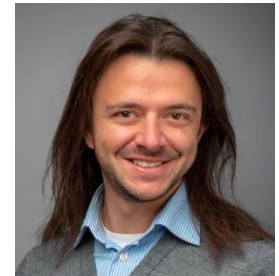
**Dominic Groß**



**Bala K. Poolla**



**Yashen Lin**



**Andrey Bernstein**

# Decarbonization of electricity is key to mitigate climate change

## California lifts renewable energy target to 73% by 2032

The California Public Utilities Commission raised renewable energy procurement targets, plans for a more aggressive decarbonization plan, and includes increased reliability provisions.

FEBRUARY 14, 2022 **RYAN KENNEDY**

## Vermont House passes 75% by 2032 renewable energy mandate

Published March 11, 2015

### ENVIRONMENT

## Maryland bill mandating 50% renewable energy by 2030 to become law, but without Gov. Larry Hogan's signature

By Scott Dance  
Baltimore Sun • May 22, 2019 at 6:40 pm

## New York mandates 70% renewable energy by 2030

By Kelsey Misbrener | October 15, 2020

## Oregon bill targets 100% clean power by 2040, with labor and environmental justice on board

After Democratic cap-and-trade bills faltered in the face of GOP revolts, an electricity-focused, consensus-driven bill gains ground in Oregon.

23 June 2021

## Virginia becomes the first state in the South to target 100% clean power

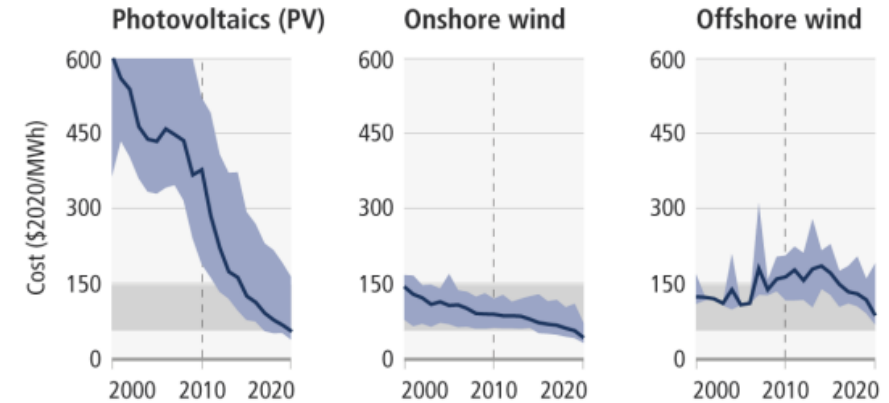
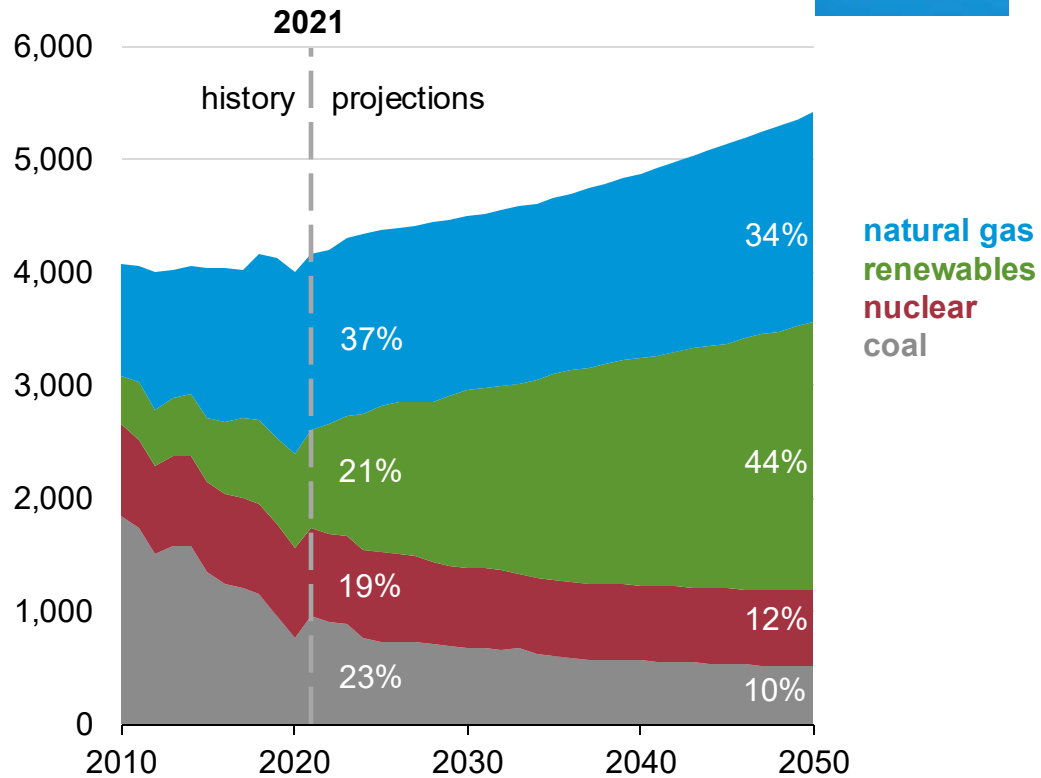
The state's Democratic majority is doing what Democratic majorities do.

By David Roberts | @drvltz | Updated Apr 13, 2020, 2:56pm EDT

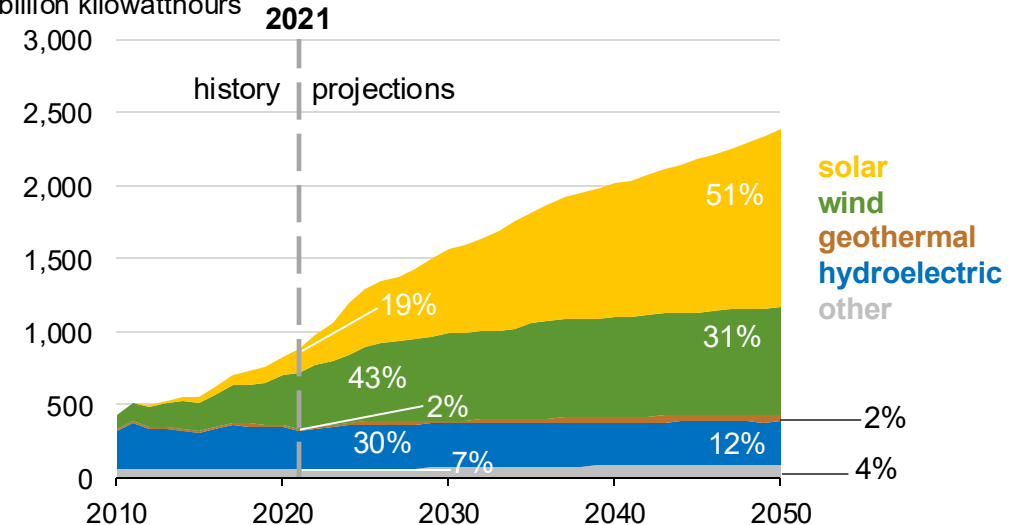


# Decarbonization of electricity is key to mitigate climate change

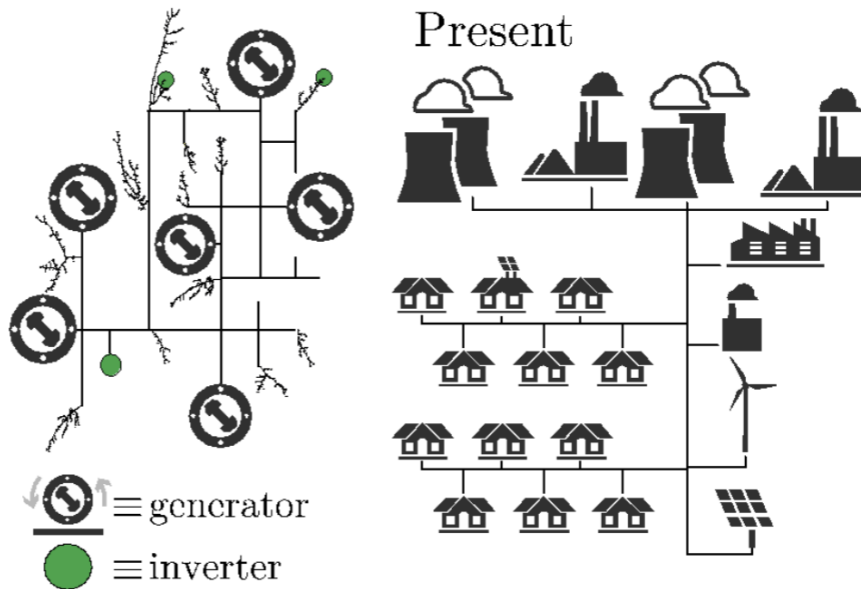
**U.S. electricity generation from selected fuels**  
**AEO2022 Reference case**  
 billion kilowatthours



**U.S. renewable electricity generation, including end use**  
**AEO2022 Reference case**  
 billion kilowatthours

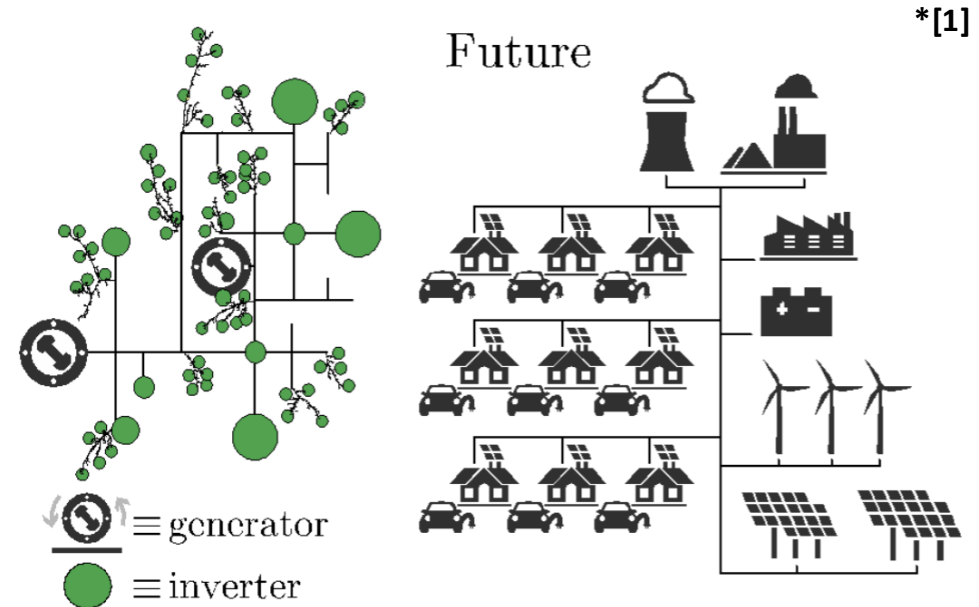


# The Future Grid



## Present grid

- dispatchable generation
- high inertial response
- strong voltage support
- well known physics



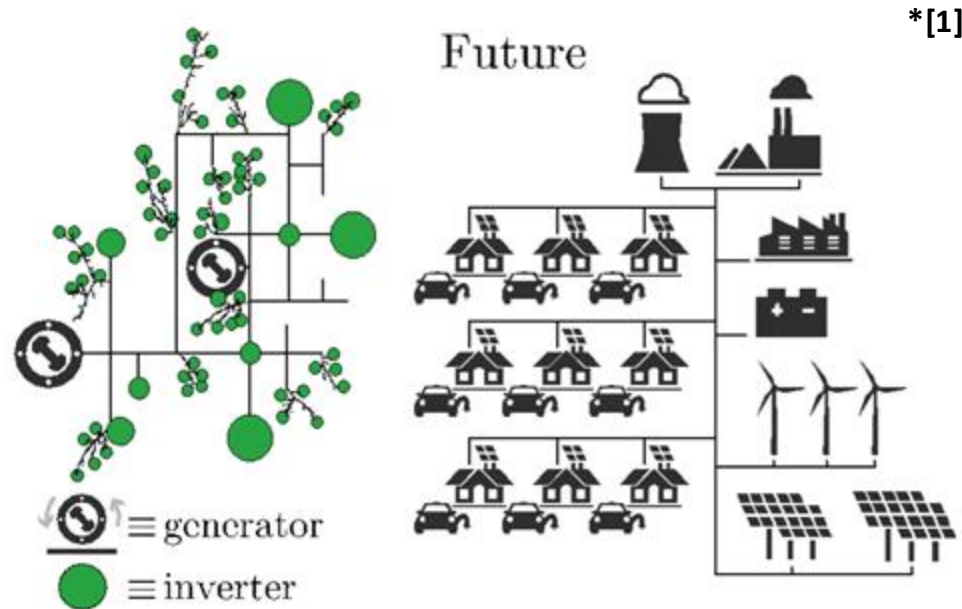
## Future

- variable and distributed generation
- limited inertia levels
- weak voltage support
- proprietary control laws (black box)

[1] Lin et al. Research roadmap on grid-forming inverters. Technical report, National Renewable Energy Lab.(NREL), Golden CO, 2020



# The Future Grid



## Future

- variable and distributed generation
- limited inertia levels
- weak voltage support
- proprietary control laws (black box)

## Selected challenges

- increased system **uncertainty**
- **sensitivity** to disturbances
- new forms of **instabilities**, induced by inverter-based resources
- need to compensate for **reduced inertia grid strength**

## Research questions:

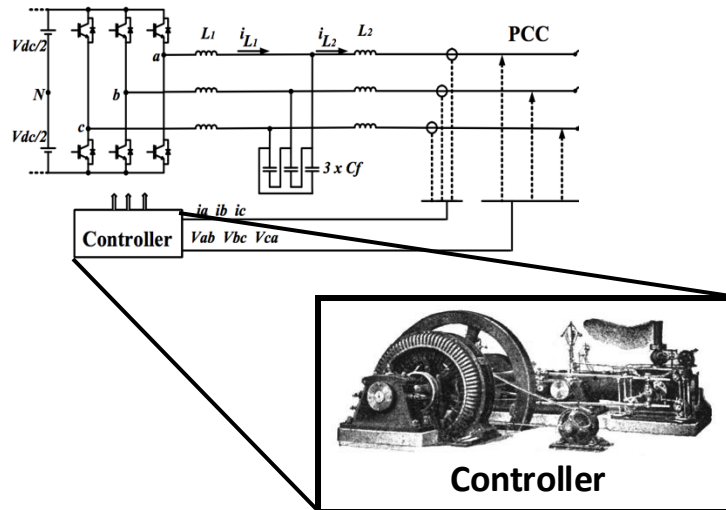
- How should we control a grid with limited inertial/voltage support?
- Should we try to mimic SGs response? Or find new and more efficient control paradigms, suitable for IBRs?

[1] Lin et al. Research roadmap on grid-forming inverters. Technical report, National Renewable Energy Lab.(NREL), Golden CO, 2020

# Inverter-based Control

**Current approach:** Use inverter-based control to mimic generators response

## Virtual Synchronous Generator



## Telecom Analogy

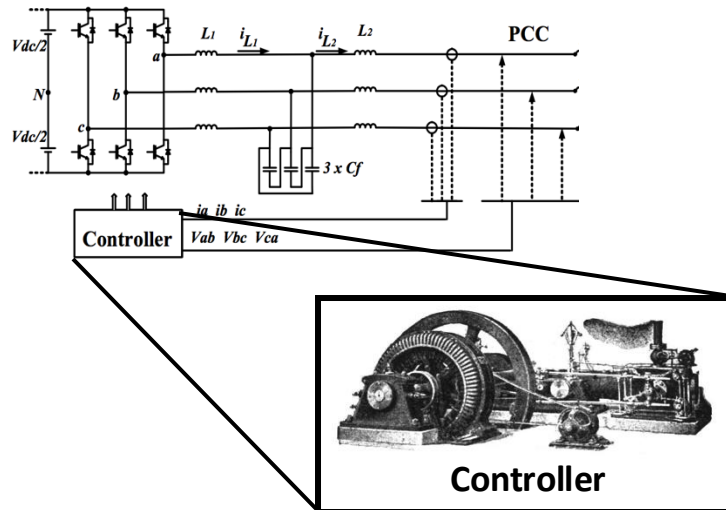




# Inverter-based Control

**Current approach:** Use inverter-based control to mimic generators response

## Virtual Synchronous Generator



## Telecom Analogy



It works, but perhaps  
there is  
something better...

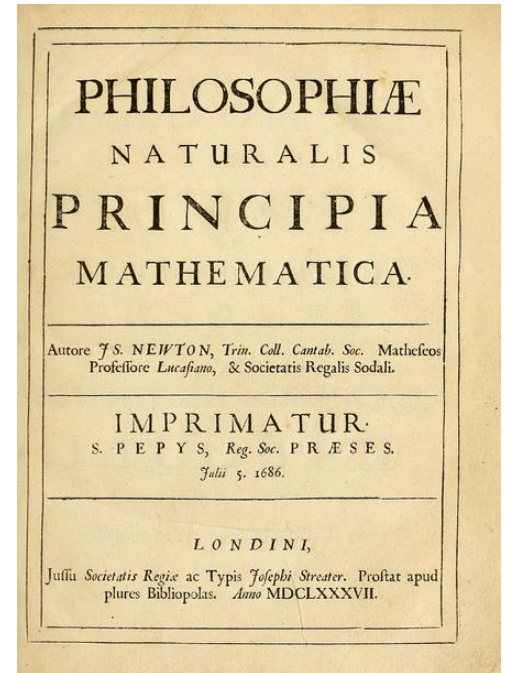
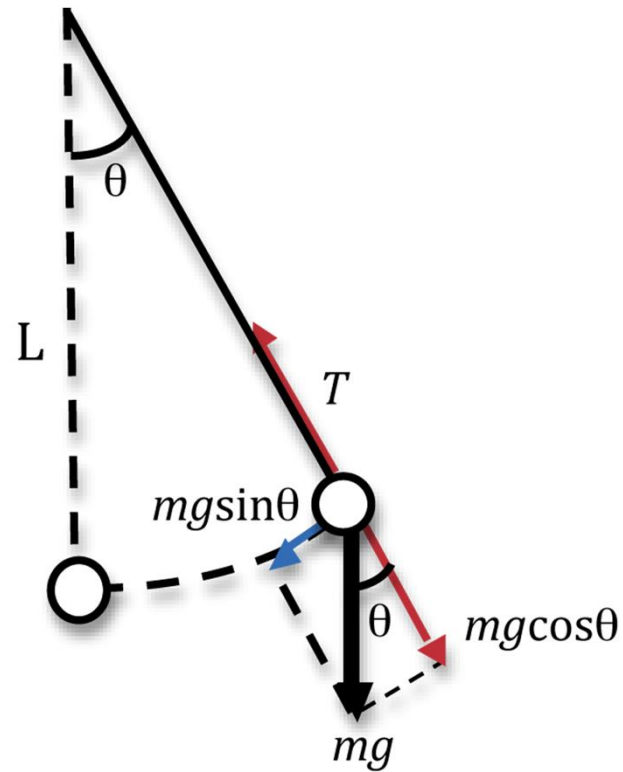
# Outline

- **Merits and trade-offs of low inertia**
  - Control Perspective: Lighter systems are easier to control!
- **Scale-free Stability Analysis of Grids**
  - Generalizes passivity notions using network information
- **Analysis of Weakly-Connected Coherent Networks**
  - Generalized Center of Inertia captures IBR dynamics
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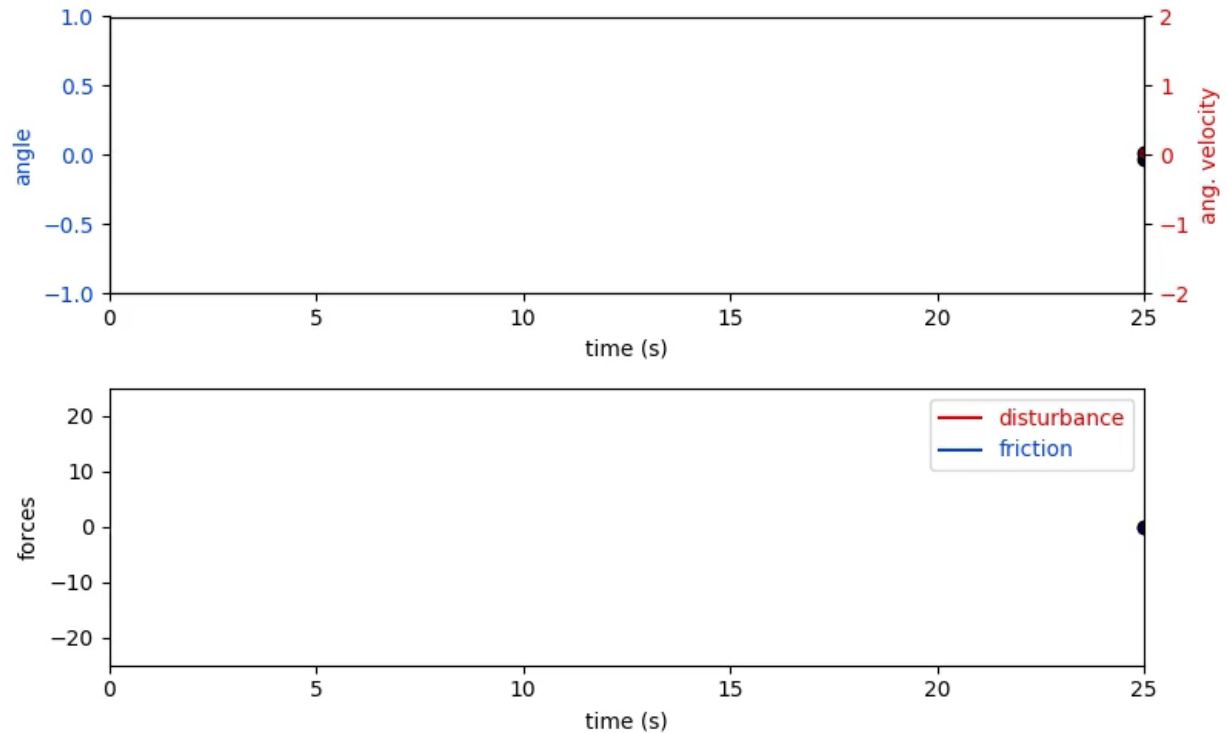
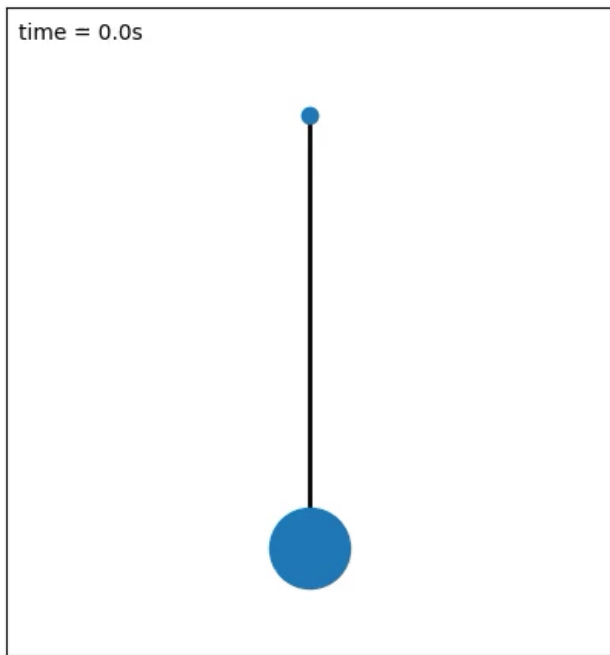
# Merits and Trade-offs of Inertia



$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g \sin \theta + \frac{f}{m}$$

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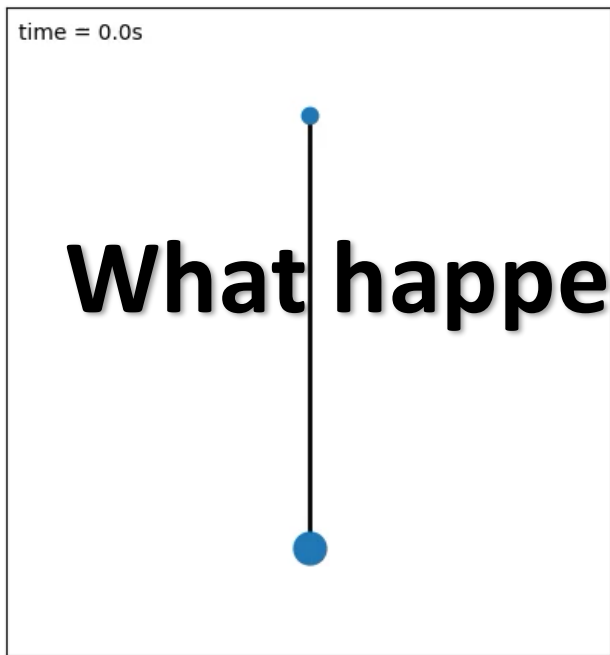


**Pros:** Provides natural disturbance rejection

**Cons:** Hard to regain steady-state

# Merits and Trade-offs of **Low** Inertia

$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g \sin \theta + \frac{f}{m}$$



What happens when one adds **control**?



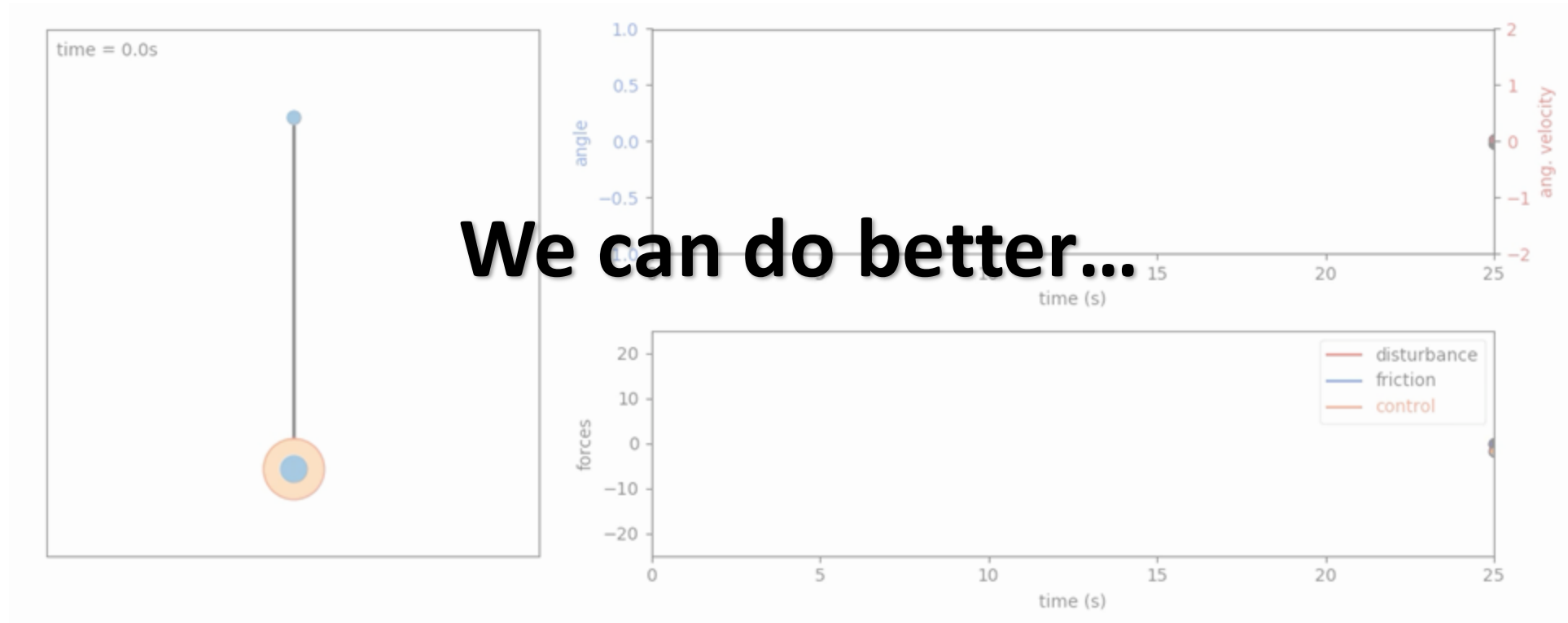
**Cons:** Susceptible to disturbances

**Pros:** Regains steady-state faster



# Control of **Low** Inertia Pendulum

Virtual **Mass** Control:  $m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f - \nu\ddot{\theta}$



## Pros:

Provides disturbance rejection

## Cons:

Hard to regain steady-state + **excessive control effort**

# Control of **Low** Inertia Pendulum

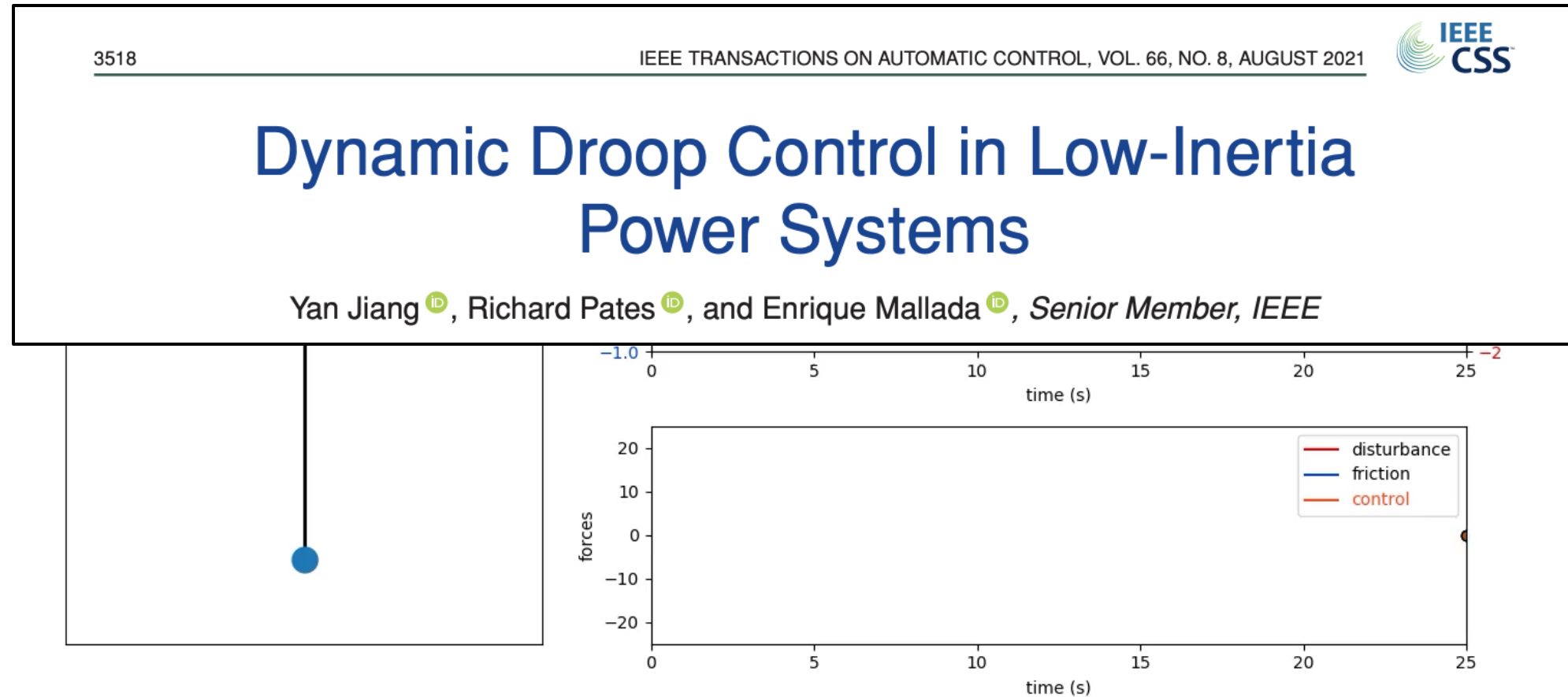


Yan Jiang

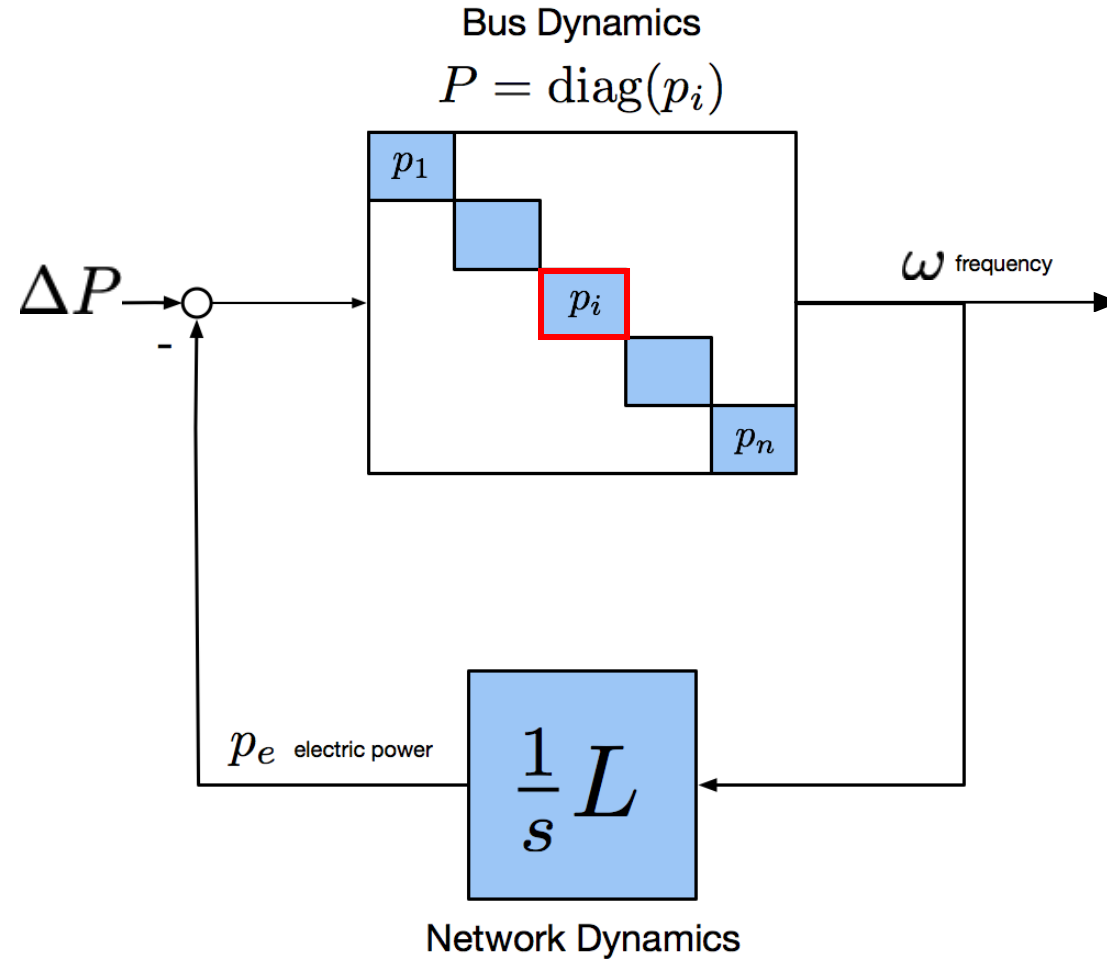


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**Dynamic Droop:**  $m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f + x$



# Power Network Model



Laplacian Matrix

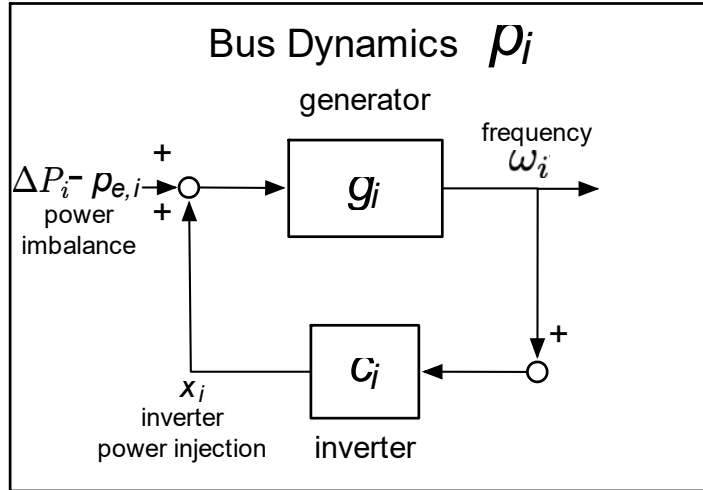
$$L_{ij} = \begin{cases} -B_{ij} & \text{if } ij \in E \\ \sum_k B_{ik} & \text{if } i = j \\ 0 & \text{o.w.} \end{cases}$$

Linearized Power Flows

$$B_{ij} = v_i v_j b_{ij} \cos(\theta_i^* - \theta_j^*)$$

[Bergen Hill '81]

# Bus Dynamics

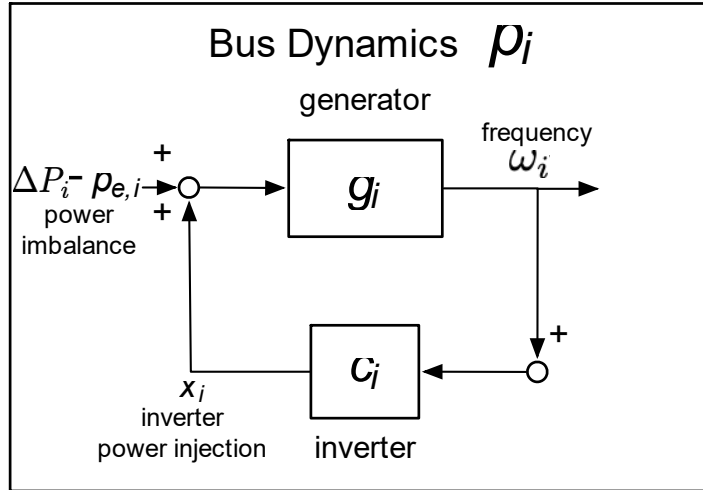


**Generator:**  $g_i : (\Delta P_i - p_{e,i} + x_i) \mapsto \omega_i$

**Model:** Swing Equations + Turbine

$$g_i : \begin{cases} \dot{\theta}_i = \omega_i \\ M_i \dot{\omega}_i = -D_i \omega_i + q_i + (\Delta P_i - p_{e,i} + x_i) \\ \tau_i \dot{q}_i = -R_{g,i}^{-1} \omega_i - q_i \end{cases}$$

# Bus Dynamics



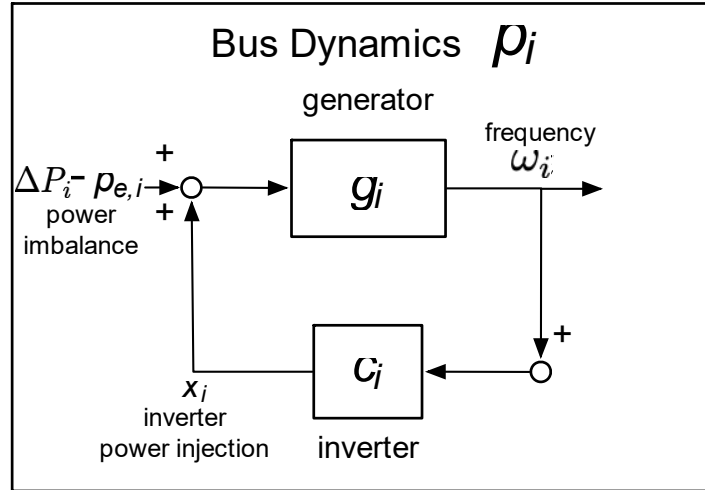
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$$g_i(s) = \frac{1}{M_i s + D_i + \frac{R_{g,i}^{-1}}{\tau_i s + 1}}$$

# Bus Dynamics



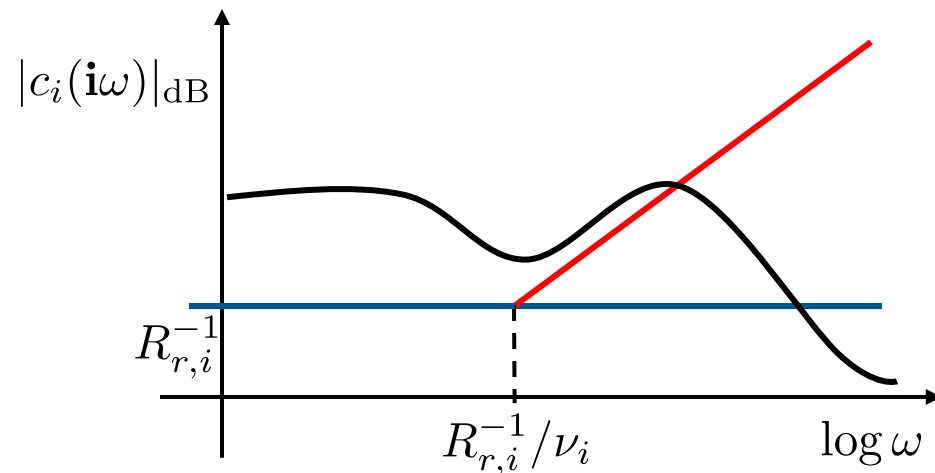
**Grid Following Inverter:**  $C_i : \omega_i \mapsto x_i$

**Droop Control and Virtual Inertia:**

$$C_i : \begin{cases} x_i = -(\nu_i \dot{\omega}_i + R_{r,i}^{-1} \omega_i), \\ C_i(s) = -(\nu_i s + R_{r,i}^{-1} \omega_i) \end{cases}$$

**Closed-loop Bus Dynamics:**

$$p_i : \begin{cases} \dot{\theta}_i = \omega_i \\ (M_i + \nu_i) \dot{\omega}_i = -(D_i + R_{r,i}^{-1}) \omega_i + q_i + (\Delta P_i - p_{e,i}) \\ \tau_i \dot{q}_i = -q_i - R_{g,i}^{-1} \omega_i \end{cases}$$



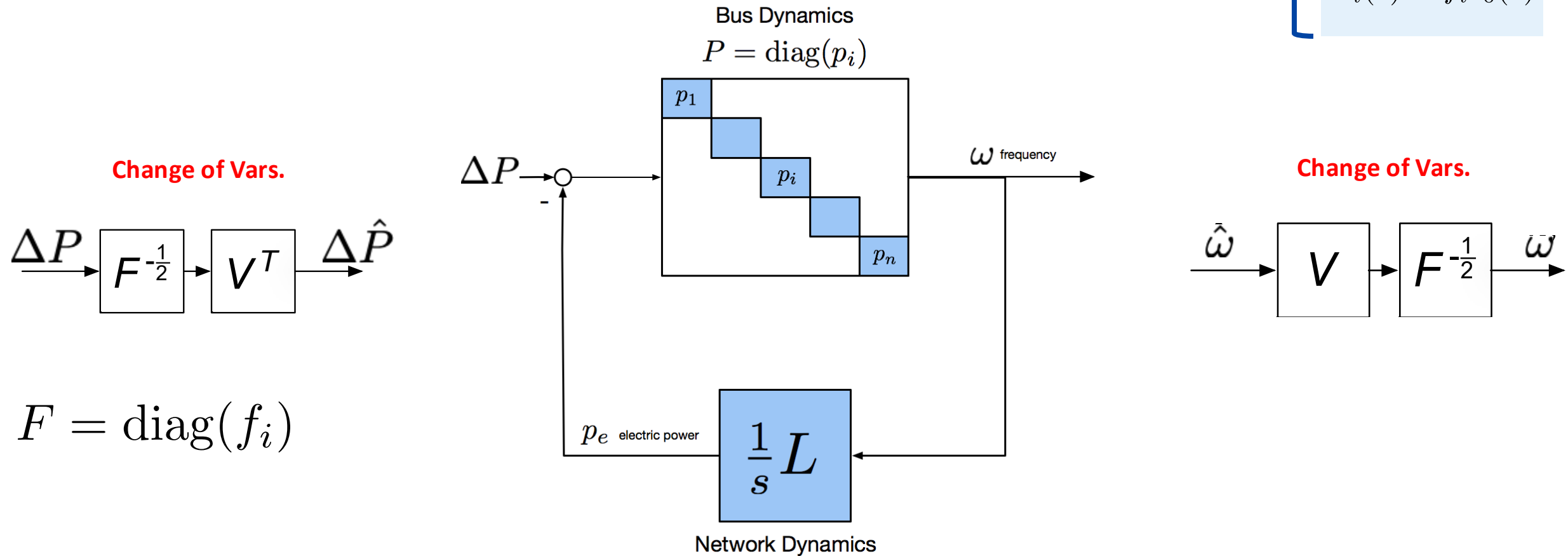


# Modal Decomposition for Multi-Rated Machines

**Assumption:** Let  $f_i$  be the machine relative inertia ( $f_i = \frac{M_i}{\max_j M_j}$ ), and

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$



[Paganini M '17 , Guo Low 18']

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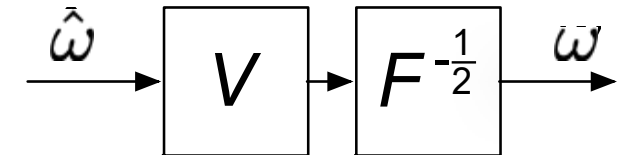
$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

**Center of Inertia**

$$\omega_{\text{CoI}}(t) = \frac{\sum_{i=1}^n M_i \omega_i(t)}{\sum_{i=1}^n M_i}$$

**Change of Vars.**

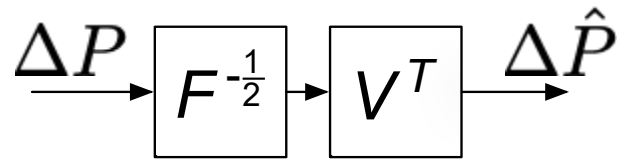


**Sync Error**

$$\tilde{\omega}_i(t) = \omega_i(t) - \omega_{\text{CoI}}(t)$$

[Paganini M '17 , Guo Low 18']

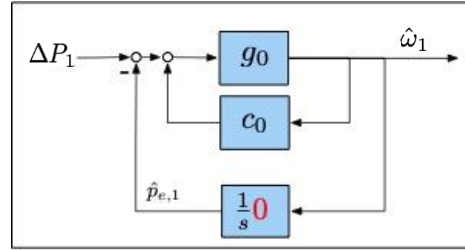
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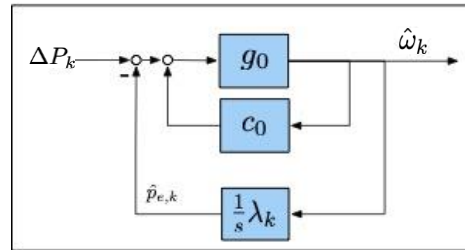
$$F = \text{diag}(f_i)$$

**Eigenvalues of:**  $L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}$

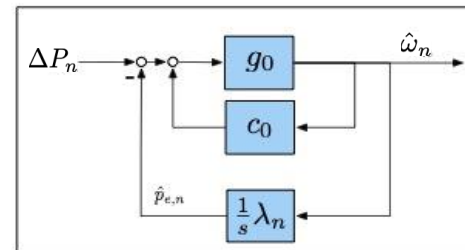
$$0 = \lambda_0 \leq \lambda_1 \leq \dots \leq \lambda_{n-1}$$



⋮



⋮



# Control of **Low** Inertia Pendulum

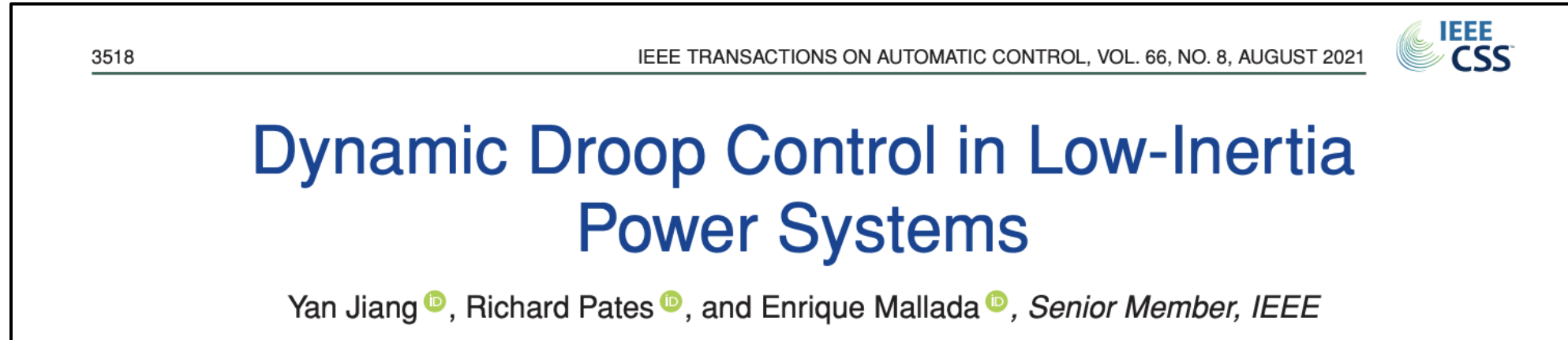


Yan Jiang



Richard Pates

**Dynamic Droop:**  $m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f + x$



### Dynamic Droop Benefits

- Overshoot Elimination in Nadir
- Disturbance Rejection
- Noise Attenuation
- Reduce Inter-area Oscillations

### Caveat

- Control design limited to co-located resources (SGs and **GFL**-IBRs)
- Restrictive assumptions: Proportional dynamics ( $p_i(s) = f_i p_0(s)$ )

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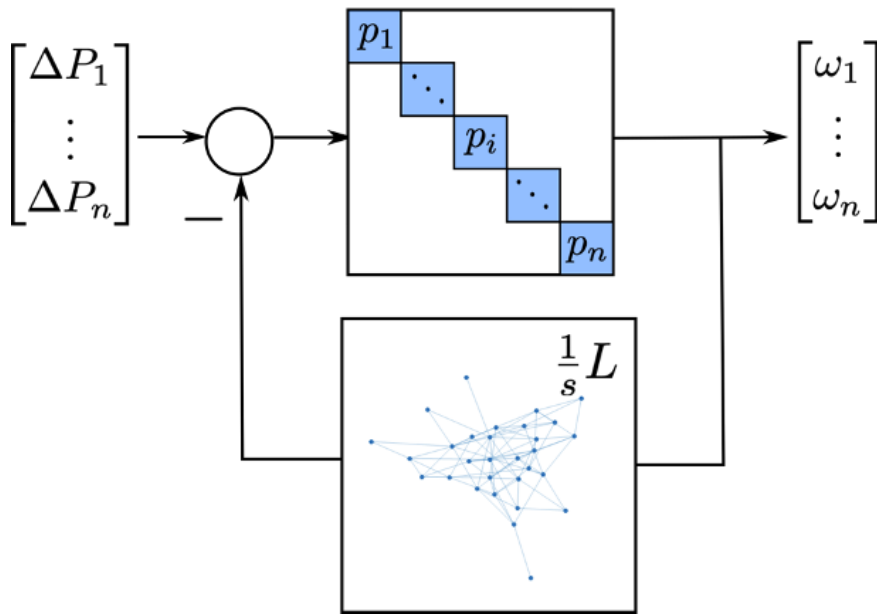
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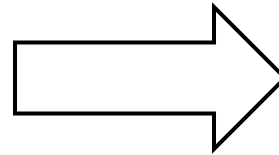
# Decentralized Stability Analysis in Power Grids [TCNS 19]



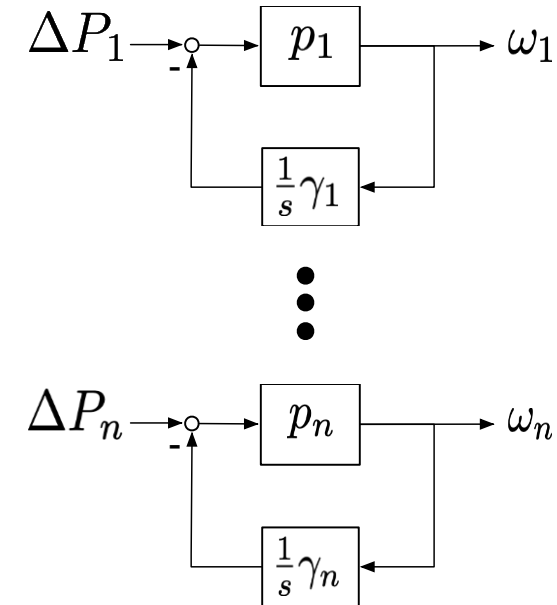
Richard Pates



1. When does this interconnection is stable?



2. Can we analysis and control design based on **local** rules?



## Problem Setup:

- Linearized power flows, lossless  

$$L_{ij} = -b_{ij}v_i v_j \cos(\theta_i^* - \theta_j^*)$$
- Bus  $i$ : arbitrary siso transfer function:  

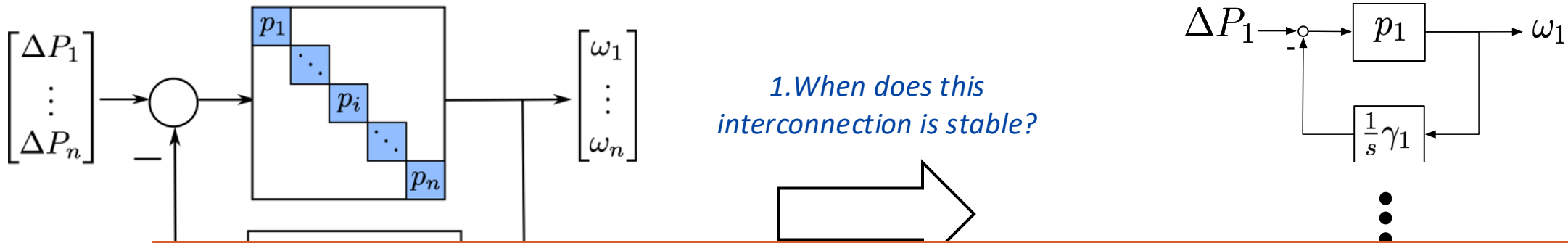
$$\omega_i = p_i(s) \Delta P_i \quad (\text{SGs or GFM-IBRs})$$



# Decentralized Stability Analysis in Power Grids [TCNS 19]



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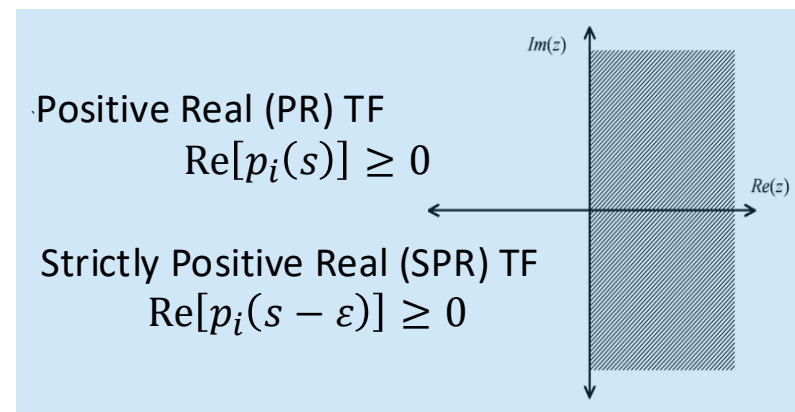


Can we use **network information** to relax passivity conditions?

## Standard Approach: Passivity

- If  $p_i(s)$  is strictly positive real (SPR), then the interconnection is stable for **all networks  $L$** !

**Converse:** for **unknown network ( $L$ )**, passivity is also **necessary**. [TCNS 19]



# Classical Result: Absolute Stability

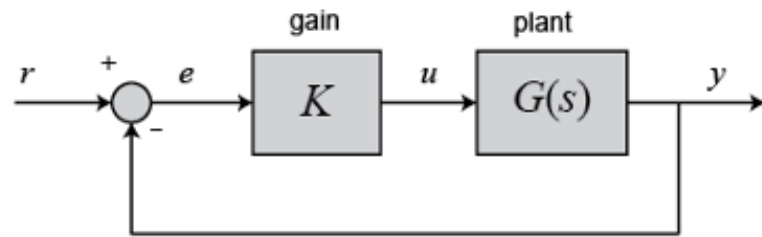
IEEE TRANSACTIONS ON AUTOMATIC CONTROL

## Frequency Domain Stability Criteria—Part I

R. W. BROCKETT, MEMBER, IEEE AND J. L. WILLEMS

*Abstract*—The objective of this paper is to illustrate the limita-

II. THE GENERALIZED POPOV THEOREM

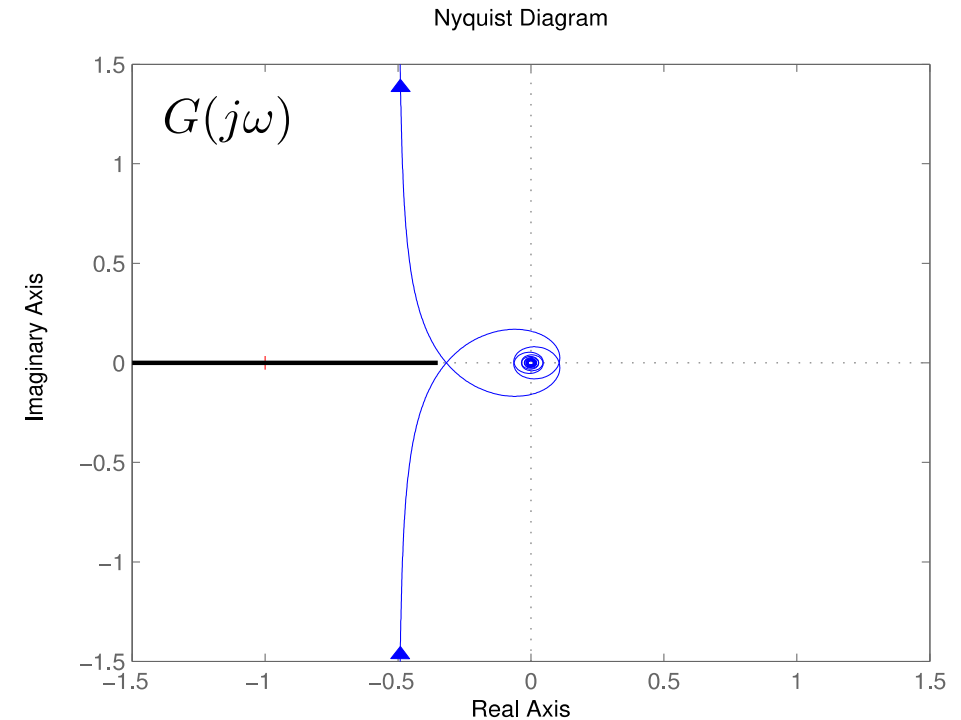


Stable for  $0 \leq K \leq k^*$ ?

**Assume:**  $G(s)$  is stable

**Define:**  $h(s) \in PR$  (passive)

**Test:** If  $h(s)(1 + k^*G(s)) \in SPR$  (strictly)  
then, **yes!**



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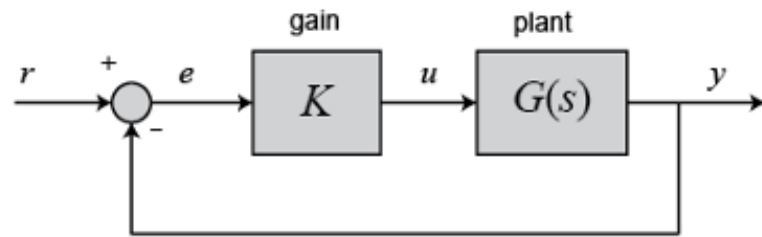
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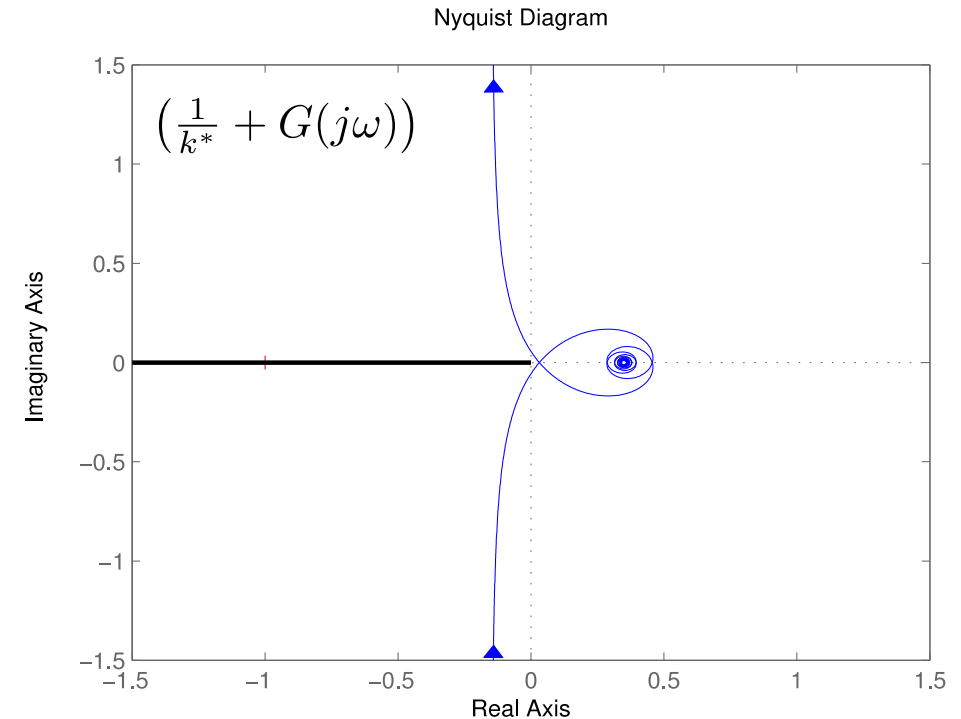


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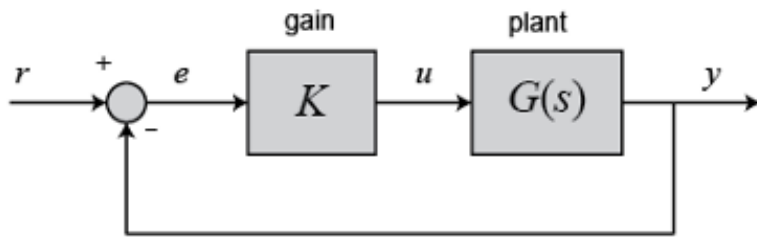
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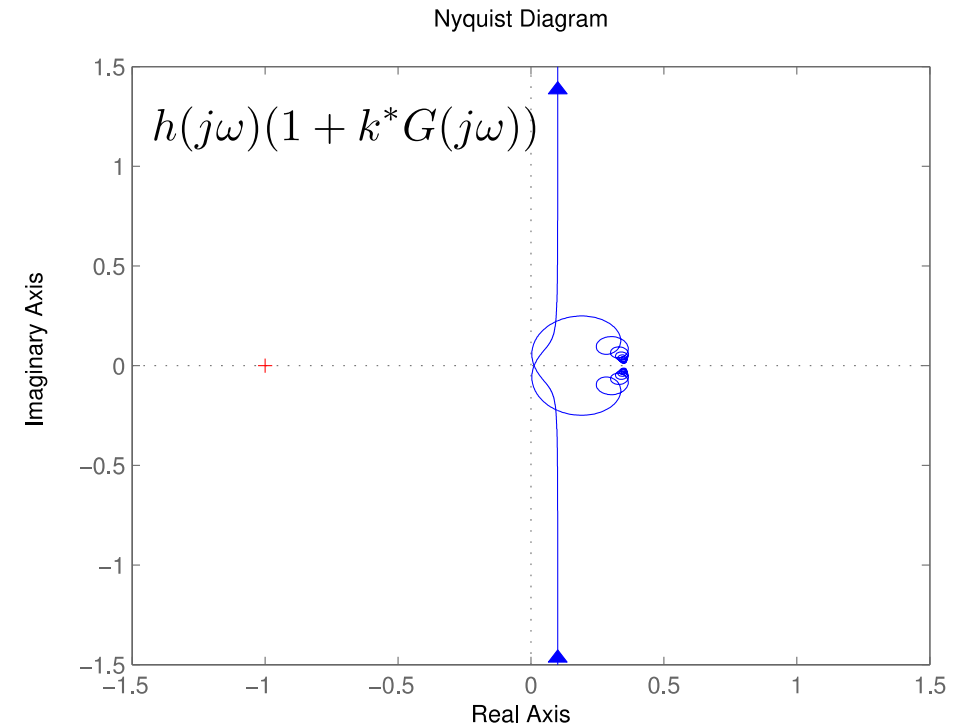


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# Scale-free Stability Analysis

**Key Idea:** Exploit limited network information to relax passivity condition

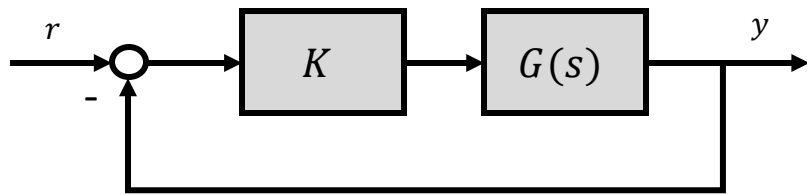
- Let  $\gamma_i$  be a local connectivity bound:  $\sum_{j \in N_i} |L_{ij}| \leq \frac{\gamma_i}{2}$   $L_{ij} = -b_{ij}v_i v_j \cos(\theta_i^* - \theta_j^*)$

**Brockett & Willems '65**

**Assume:**  $G(s)$  is stable

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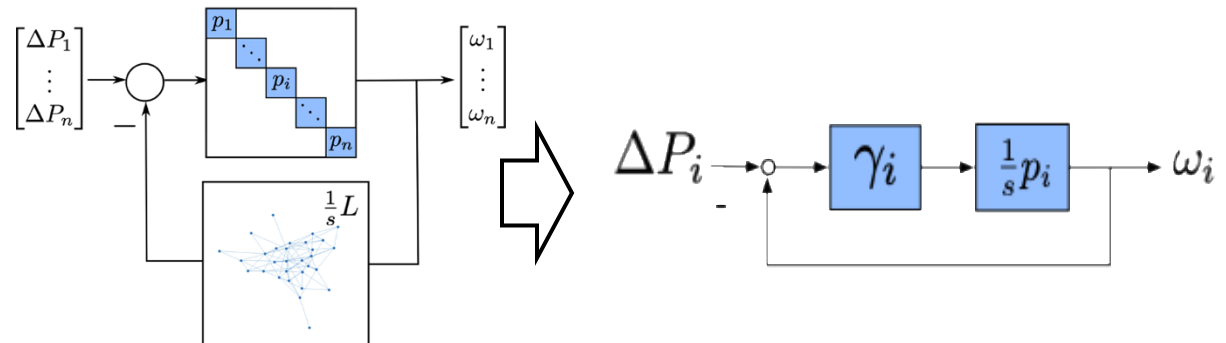


**Pates & M 2019**

**Assume:**  $p_i(s)$  is stable

**Define:**  $h(s) \in PR$  (passive)

**Test:** If  $h(s)\left(1 + \gamma_i \frac{1}{s} p_i(s)\right) \in SPR, \forall i$ , then system stable for networks  $\sum_{j \in N_i} |L_{ij}| \leq \frac{\gamma_i}{2}, \forall i$



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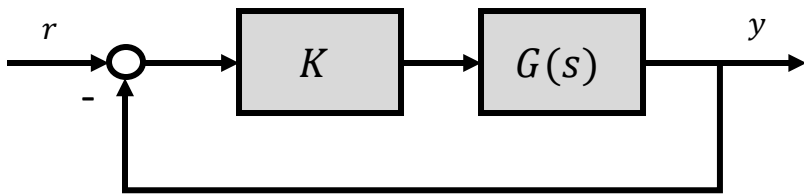
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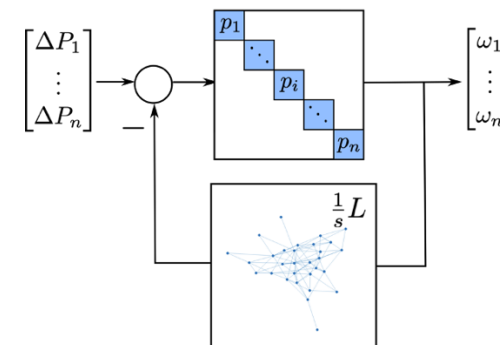


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# Scale-free Stability Analysis

**Key Idea:** Exploit limited network information to relax passivity condition

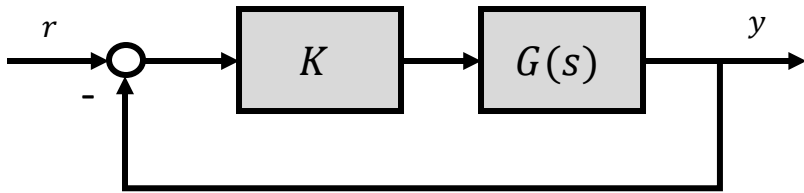
- Let  $\gamma_i$  be a local connectivity bound:  $\sum_{j \in N_i} |L_{ij}| \leq \frac{\gamma_i}{2}$   $L_{ij} = -b_{ij}v_i v_j \cos(\theta_i^* - \theta_j^*)$

**Brockett & Willems '65**

**Assume:**  $G(s)$  is stable

**Define:**  $h(s) \in PR$  (passive)

**Test:** If  $h(s)(1 + k^*G(s)) \in SPR$  (strictly)  
then system is stable for all  $0 \leq K \leq k^*$

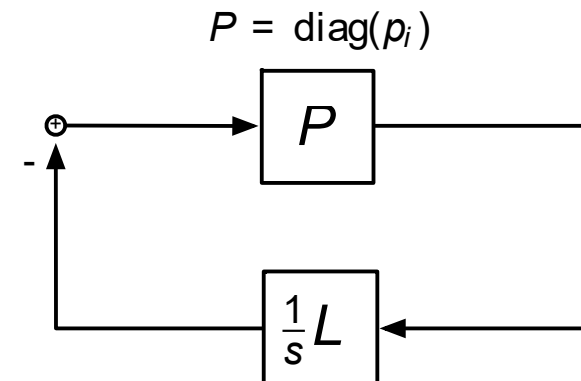


**Pates & M 2019**

**Assume:**  $p_i(s)$  is stable

**Define:**  $h(s) \in PR$  (passive)

**Test:** If  $h(s) \left(1 + \gamma_i \frac{1}{s} p_i(s)\right) \in SPR, \forall i$ , then  
system stable for networks  $\sum_{j \in N_i} |L_{ij}| \leq \frac{\gamma_i}{2}, \forall i$



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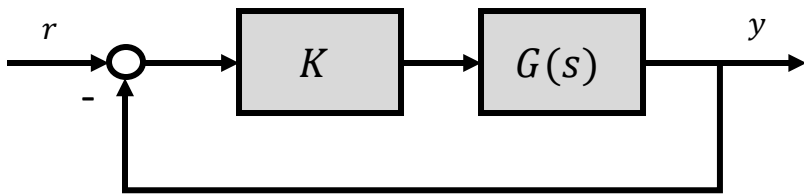
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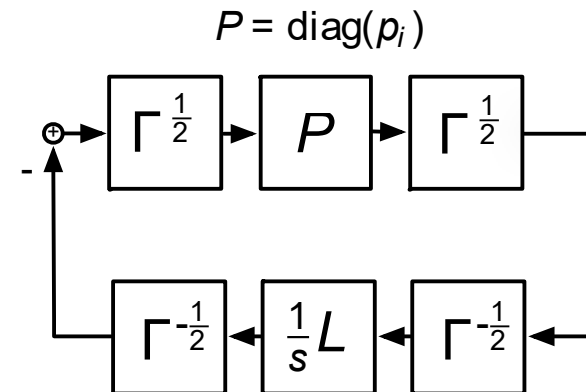


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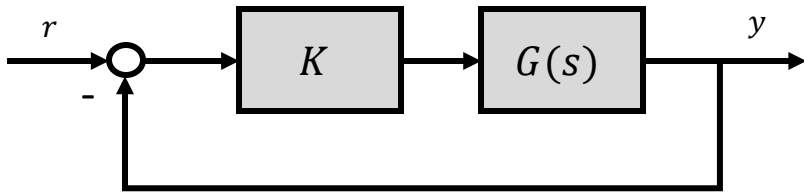
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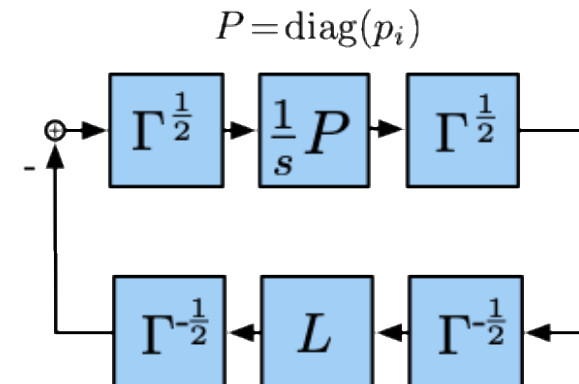


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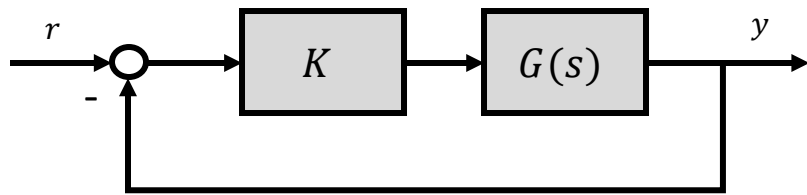
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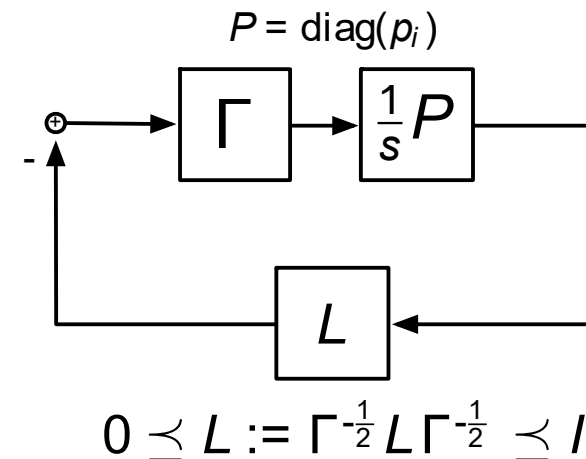


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# Examples

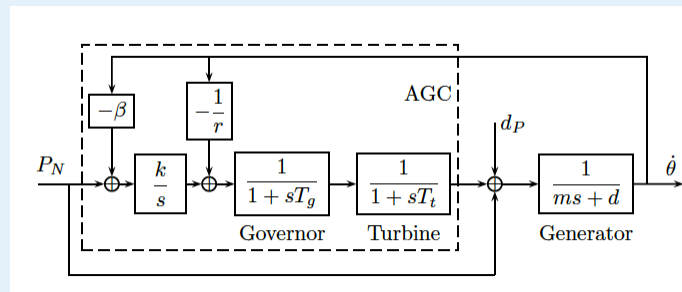
## Delay Robustness of Swing Equations

Let  $p_i(s) = \frac{1}{M_i s + D_i e^{-\tau_i s}}$ .

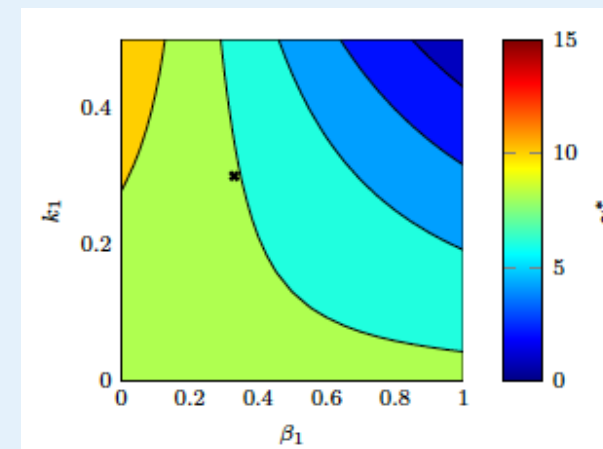
Given  $\tau^* < \frac{\pi}{2}$ , then, for any network such that  $2 \sum_{j \in N_i}^n L_{ij} < \gamma^*$  with  $\gamma^* \approx \frac{\pi M_i (\frac{\pi}{2} - \tau^*)}{2 \left( \frac{M_i \tau^*}{D_i} \right)^2}$

the delayed swing equations are stable for whenever  $\tau_i \leq \tau^* \frac{M_i}{D_i}$

## Automatic Generation Control



$m$	$d$	$T_g$	$T_t$	$r$	$\beta$	$k$
0.16	0.02	0.08	0.40	3.00	0.33	0.30



# Outline

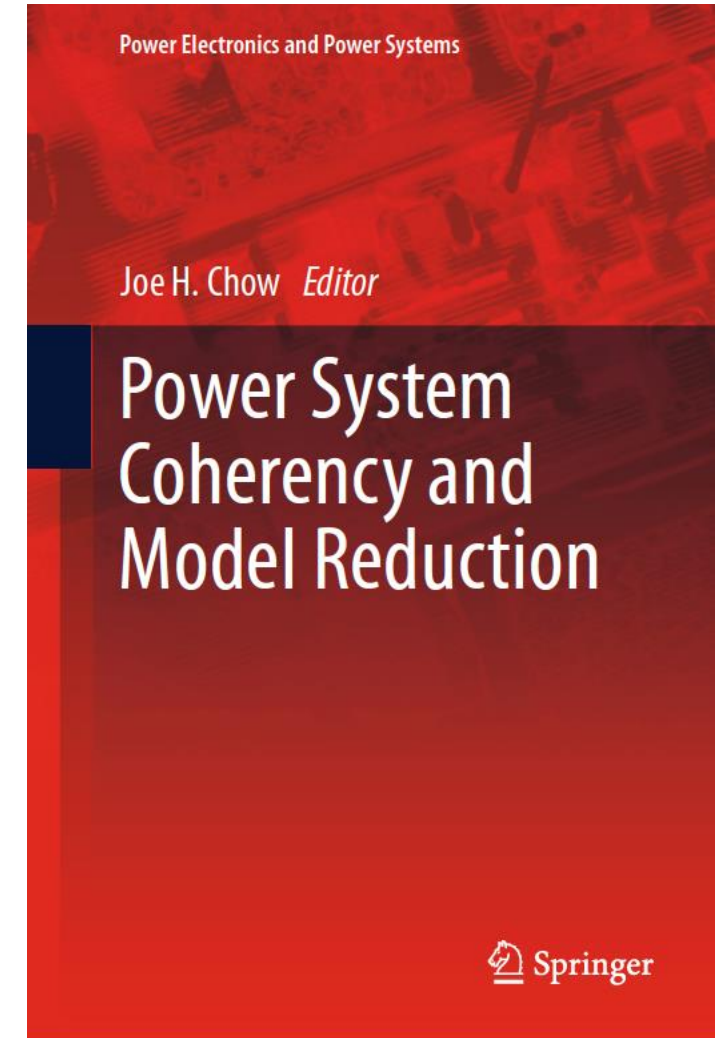
- **Merits and trade-offs of low inertia**
  - Control Perspective: Lighter systems are easier to control!
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  - Generalized Center of Inertia captures IBR dynamics
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# Coherence in Power Networks

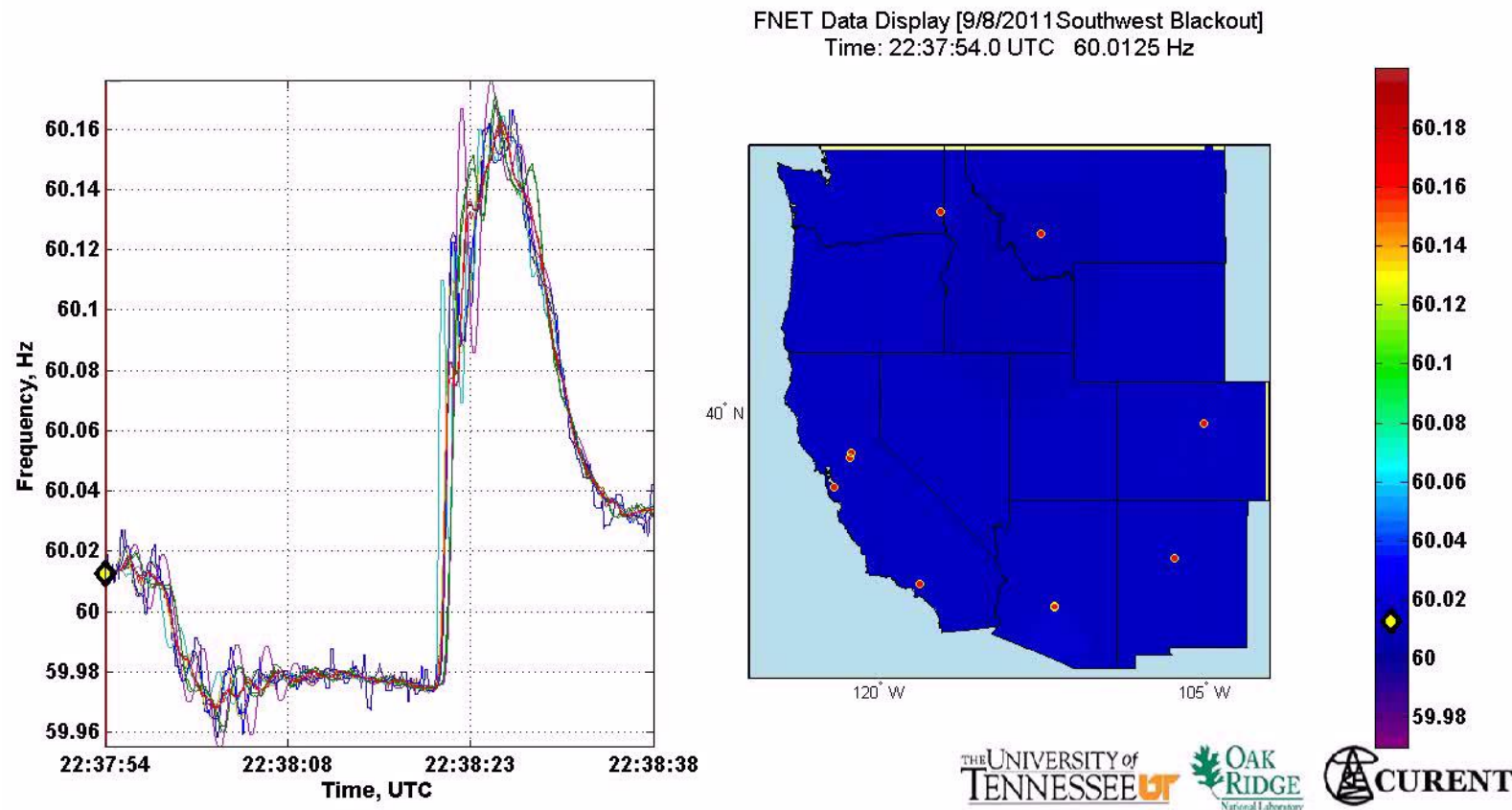
- Studied since the 70s
  - Podmore, Price, Chow, Kokotovic, Verghese, Pai, Schweppe,...
- Enables aggregation/model reduction
  - Speed up transient stability analysis
- Many important questions
  - How to identify coherent modes?
  - How to accurately reduce them?
  - What is the cause?
- Many approaches
  - Timescale separations (Chow, Kokotovic,)
  - Krylov subspaces (Chaniotis, Pai '01)
  - Balanced truncation (Liu et al '09)
  - Selective Modal Analysis (Perez-Arriaga, Verghese, Schweppe '82)



**Goal: Understand how IBR presence affect classical coherence studies**



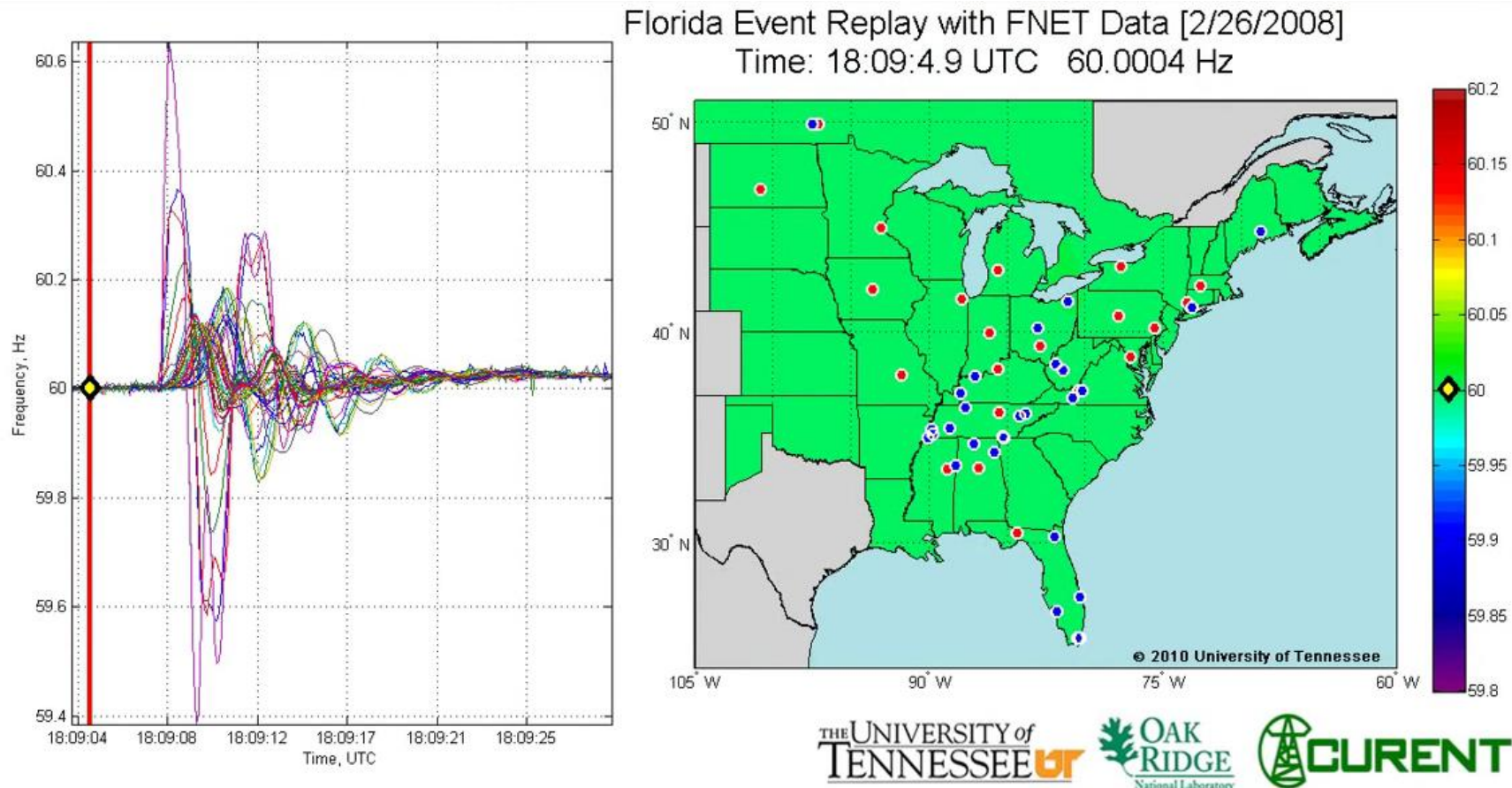
# Case Study 1: Network Coherence



## Key Questions:

- How does coherence emerge, and what does it depend on?
- How to characterize the coherent response in the presence of IBRs?

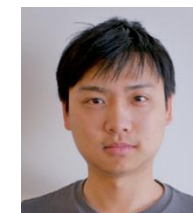
## Case Study 2: Coherent Inter-area Modes



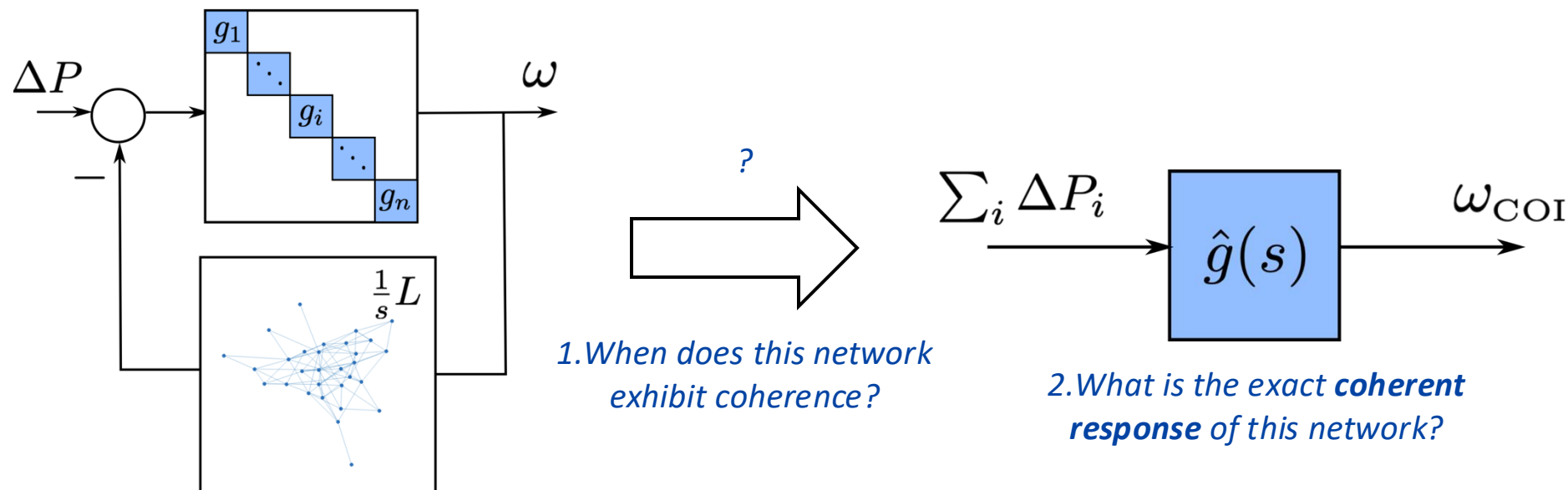
### Key Questions:

- How to identify coherent areas?
- Can we model the inter-area oscillations?

# Analysis of Coherent Dynamics [CDC 19, Auto 25]



Hancheng Min Richard Pates



1. When does this network exhibit coherence?

2. What is the exact **coherent response** of this network?

## • Problem Setup:

- Linearized power flows  $L_{ij}$
- Bus  $i$ : arbitrary siso tf:  
 $\omega_i = g_i(s) \Delta P_i$  (SGs or IBRs)

Example I: SG + Turbine

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{R_i^{-1}}{\tau s + 1}}$$

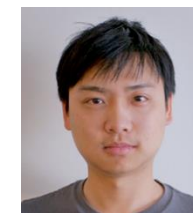
Example II: IBRs

$$g_i(s) = \frac{1}{v_i s + R_i^{-1}}$$

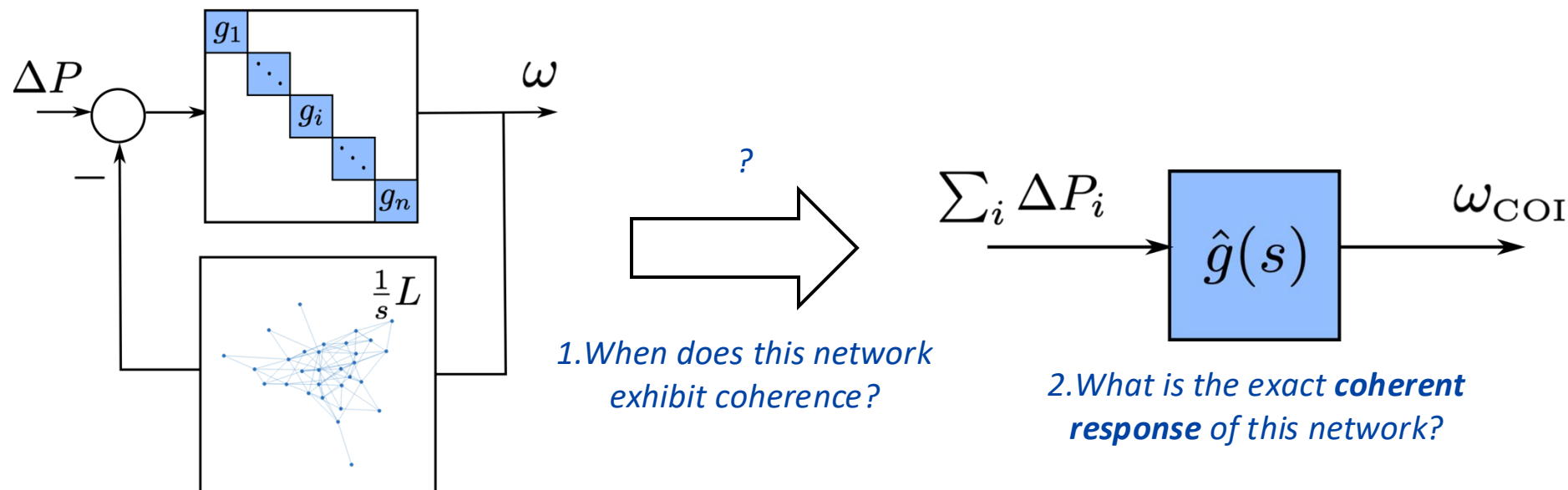
[CDC 19] Min, M. Dynamics concentration of large-scale tightly-connected networks. **Conference on Decision and Control 2019**

[Automatica 25] Min, Pates, M. A frequency domain analysis of slow coherency in networked systems. **Automatica 2025**

# Analysis of Coherent Dynamics [CDC 19, Auto 25]



Hancheng Min Richard Pates



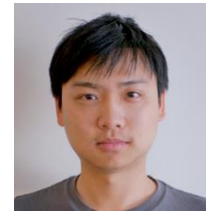
1. Coherence can be understood as a **low rank** property the **closed-loop transfer matrix**
2. It emerges as the **effective algebraic connectivity**  $\left| \frac{1}{s_0} \lambda_2(L) \right|$  increases
3. The coherent dynamics is given by the **harmonic sum** of bus dynamics

$$\hat{g}(s) = \left( \sum_{i=1}^n g_i^{-1}(s) \right)^{-1}$$

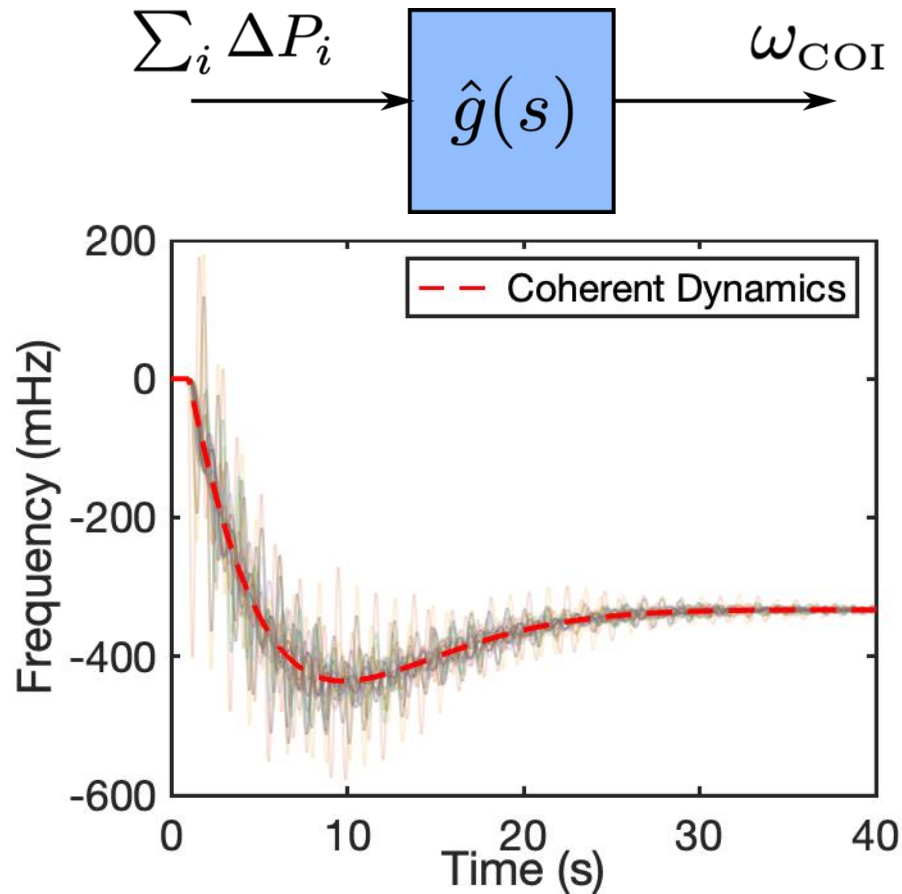
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# Generalized Center of Inertia [CDC 19, Auto 25]



Hancheng Min Richard Pates



$$\hat{g}(s) = \left( \sum_{i=1}^n g_i^{-1}(s) \right)^{-1}$$

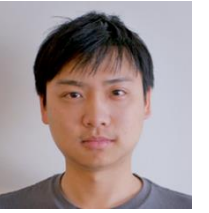
- **Coherent Dynamics:  $\hat{g}(s)$**
- Representation of aggregate response
- Accuracy of approximation:
  - is frequency dependent
  - increases with network connectivity
- Provides excellent template for reduced order models (via balance-truncations)
- More details [LCSS 20]

[CDC 19] Min, M. Dynamics concentration of large-scale tightly-connected networks. **Conference on Decision and Control 2019**

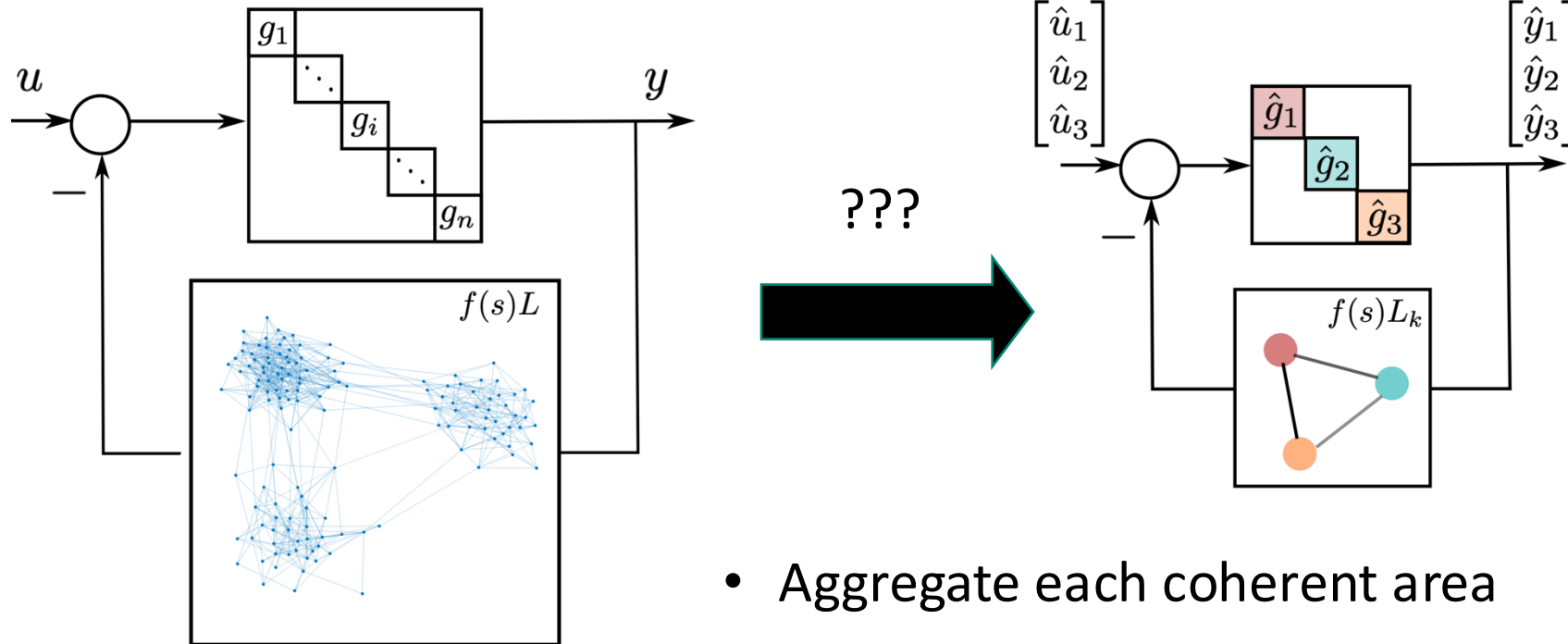
[LCSS 20] Min, Paganini, M. Accurate reduced-order models for heterogeneous coherent generators. **IEEE LCSS 2020**

[Auto 25] Min, Pates, M. A frequency domain analysis of slow coherency in networked systems. **Automatica 2025**

# Weakly-Connected Coherent Networks



Hancheng Min

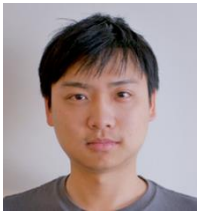


- Aggregate each coherent area
- Inter-area oscillation can be modeled as the interaction among aggregate nodes

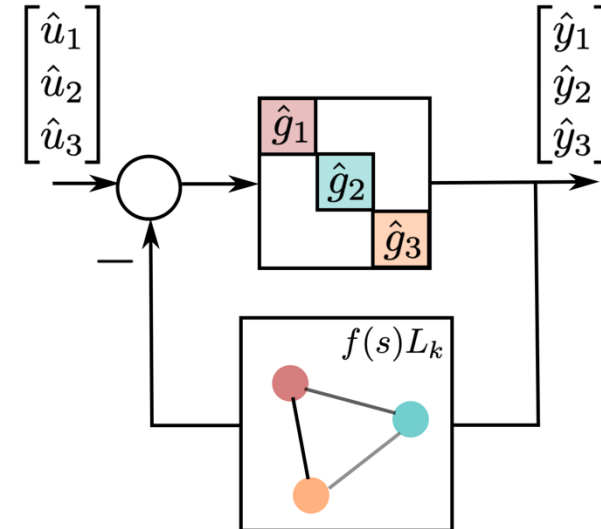
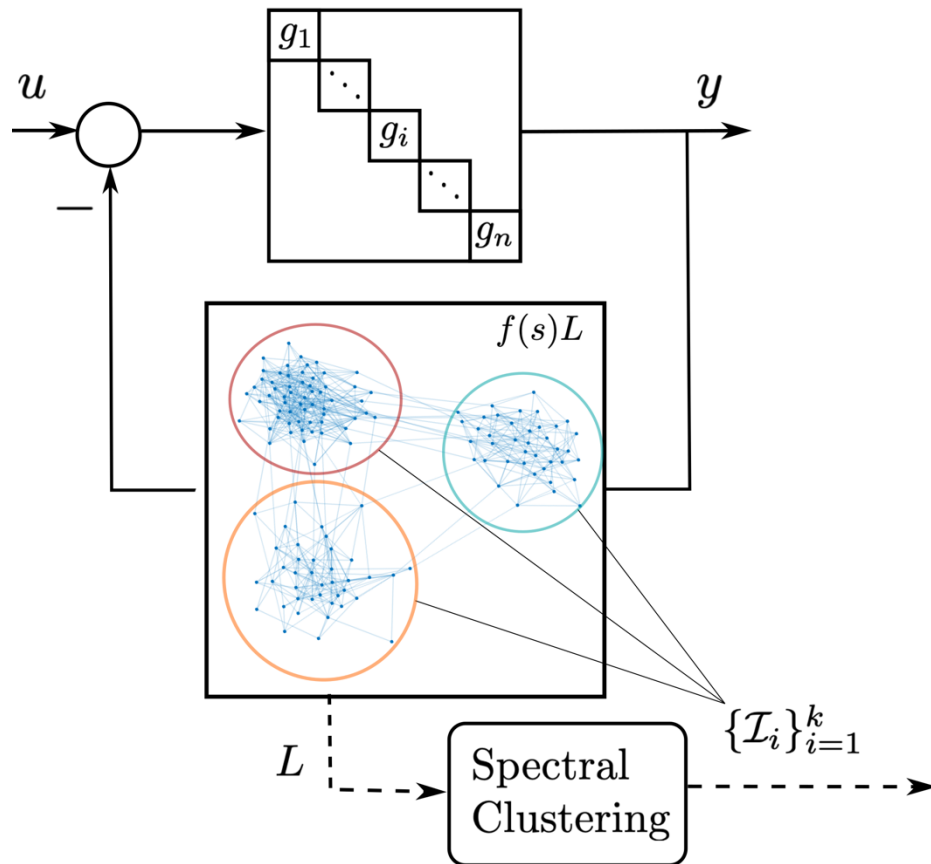


# Structure-preserving Network Reduction

## Step 1: Identifying coherent areas



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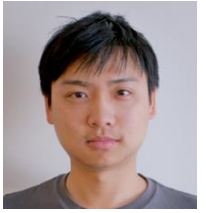


*Tightly-connected  
Networks are coherent*



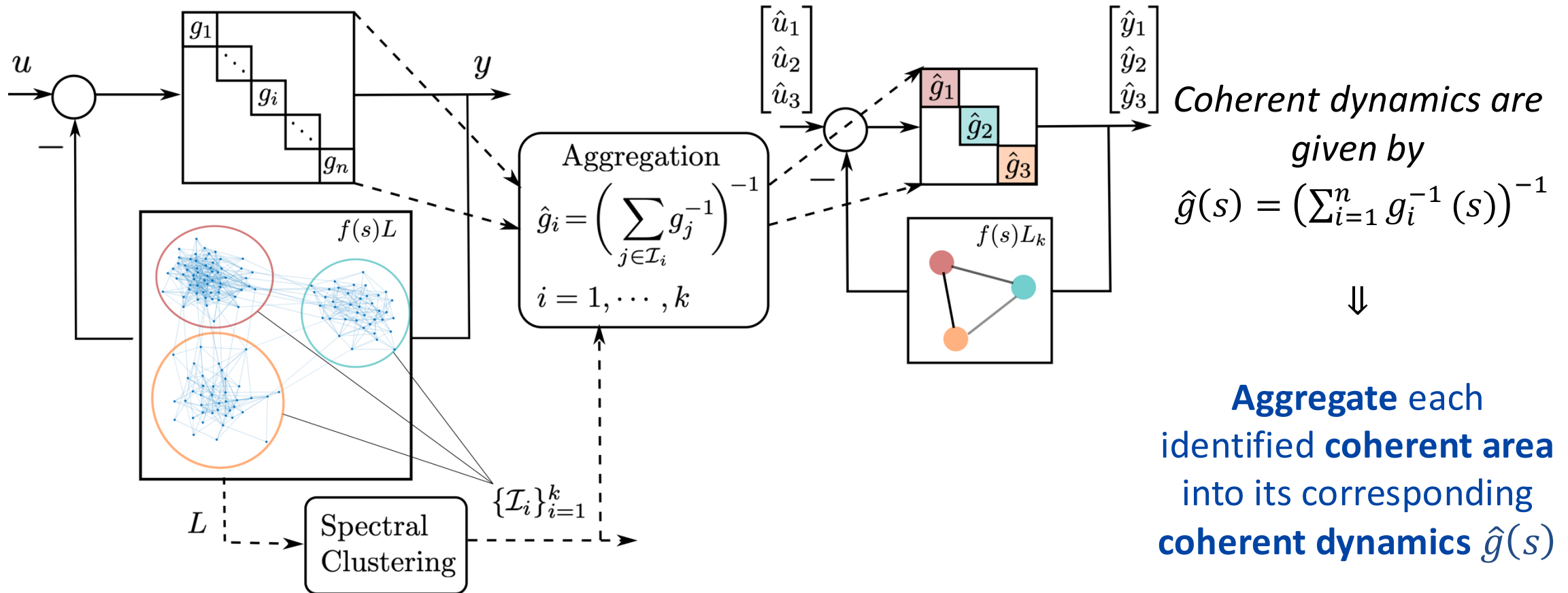
**Use spectral clustering  
algorithm to find  
tightly-connected  
subnetworks/areas**

# Structure-preserving Network Reduction



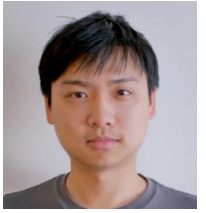
Hancheng Min

## Step 2: Aggregate coherent areas



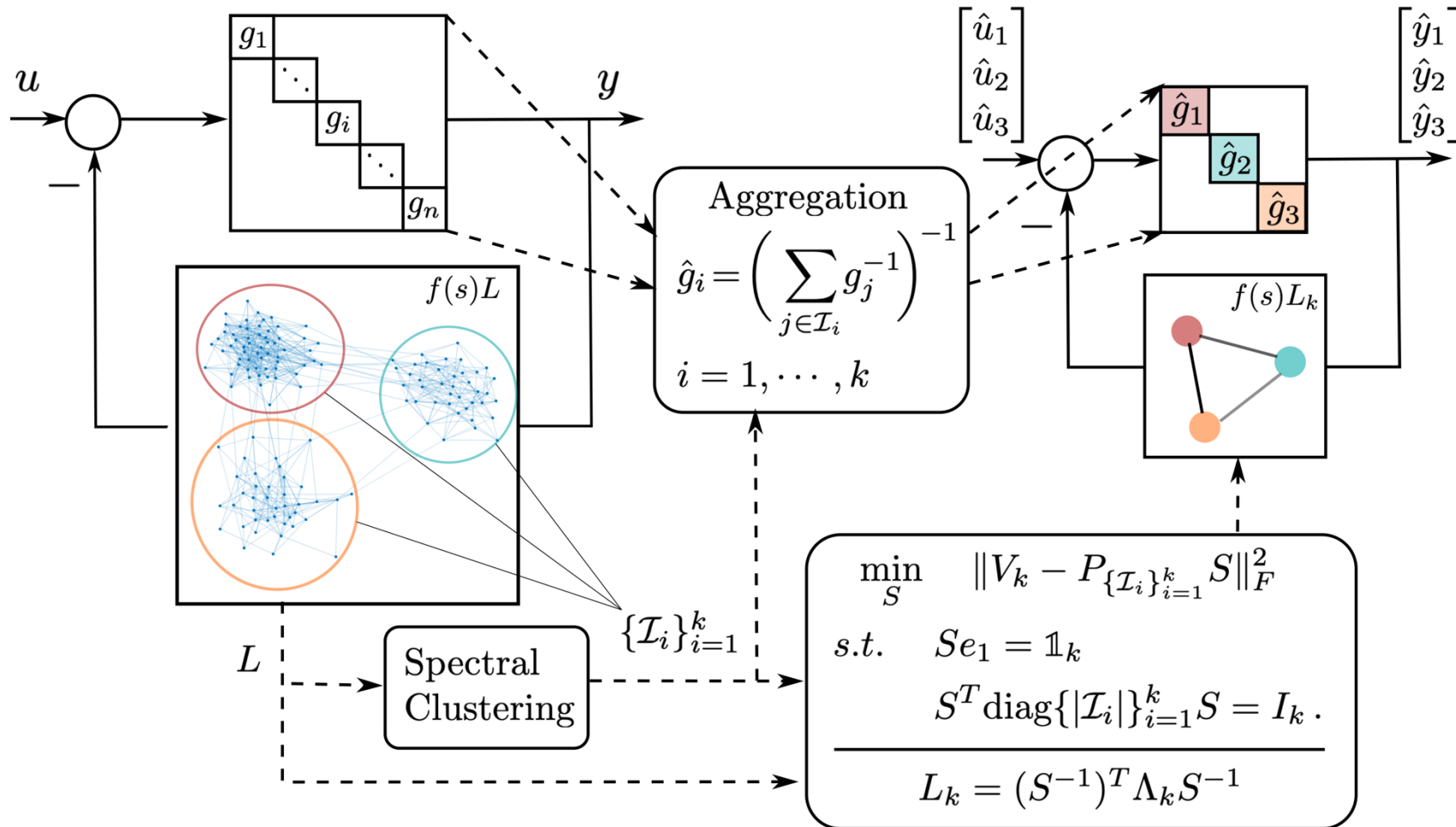


# Structure-preserving Network Reduction



Hancheng Min

Step 3: Model the **interaction** among aggregate nodes



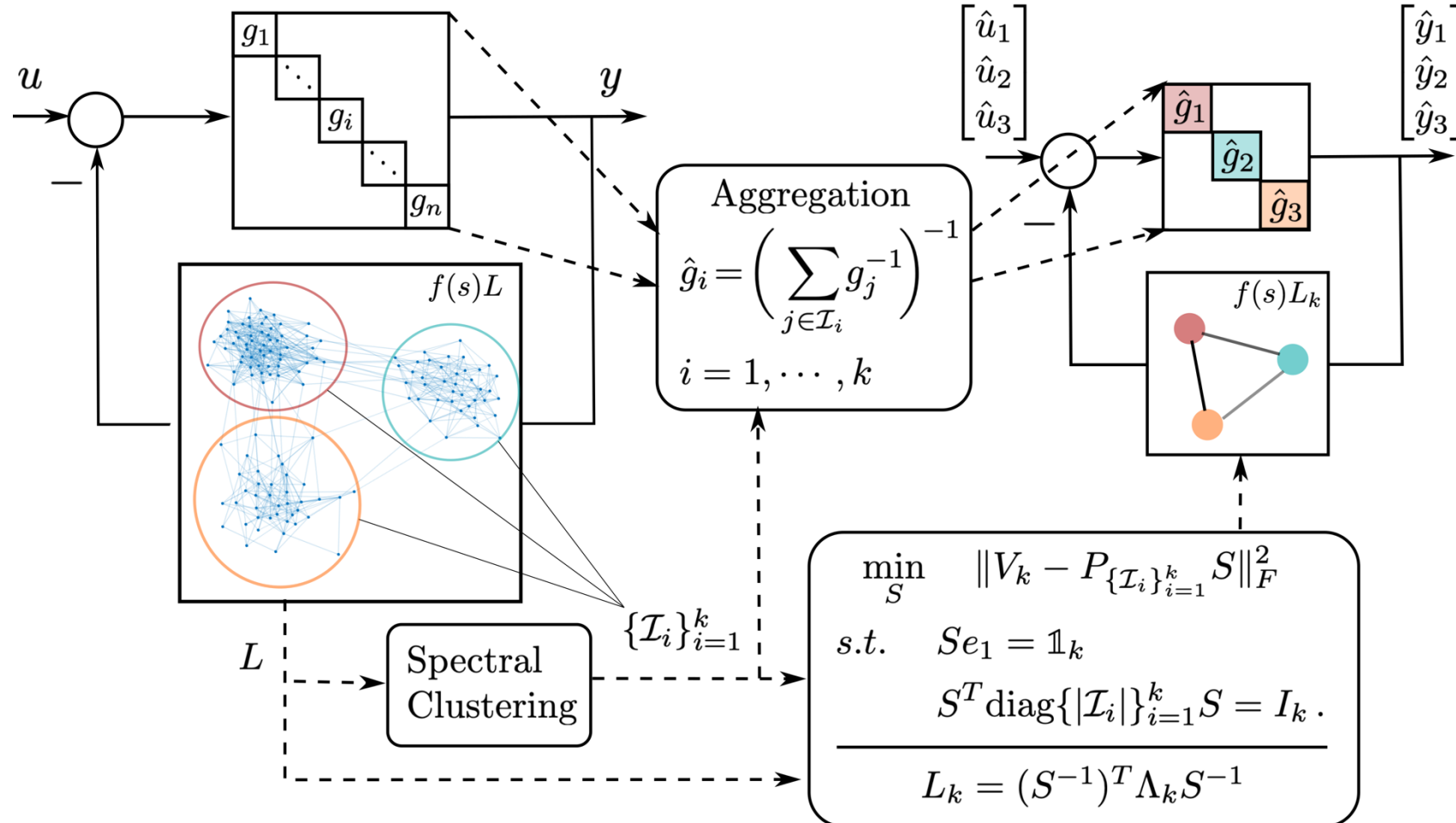
**Construct the reduced network  $L_k$  by solving an optimization problem (it has closed-form solution)**

# Approximation Errors



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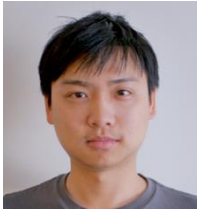
$$\|T(s_0) - \hat{T}_k(s_0)\|_2 = \mathcal{O}\left(\frac{1}{\lambda_{k+1}(L)}\right) + \mathcal{O}\left(\|V_k(L) - P_{\{I_i\}_{i=1}^k} S\|_2\right)$$



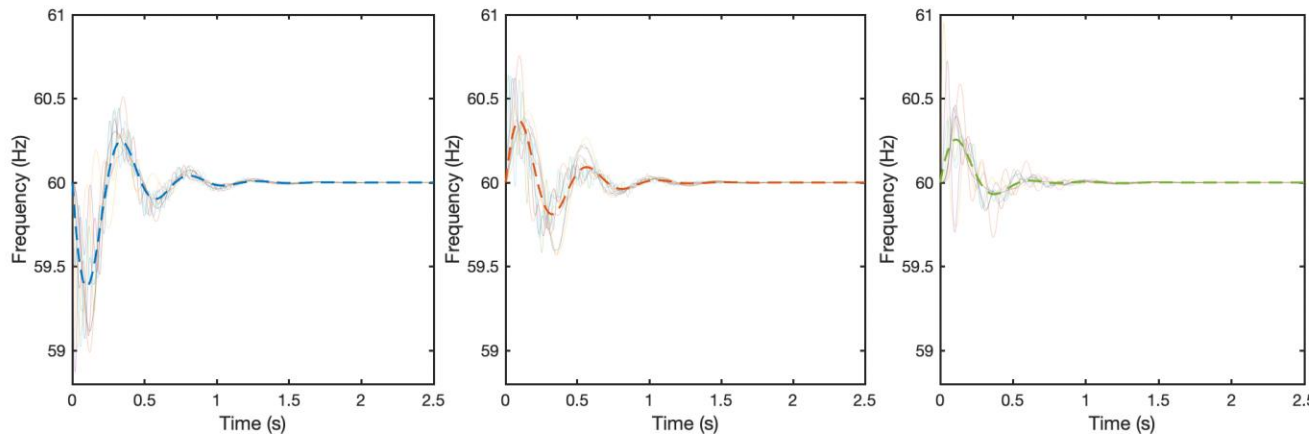
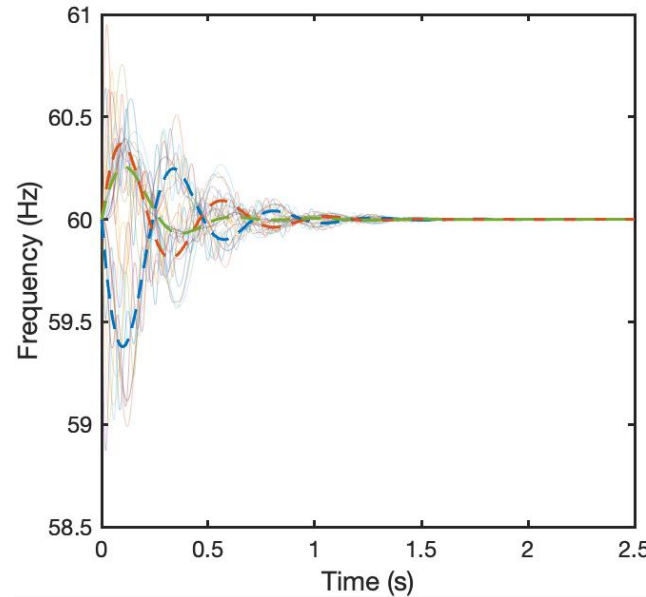
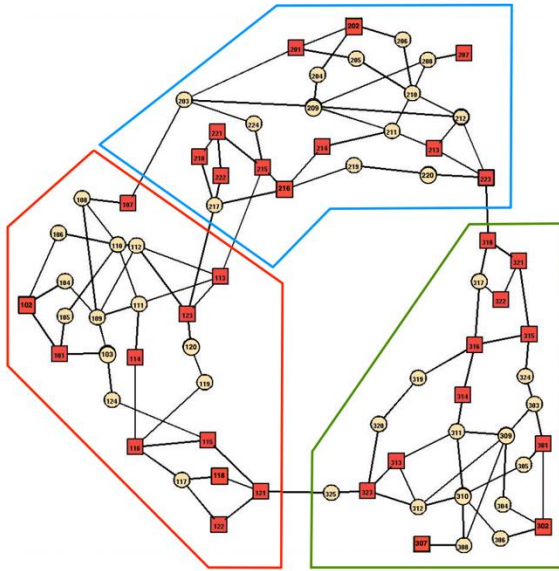
Approximation error depends on:

- Whether the network has a multi-cluster structure
- Whether the SC algorithm finds the right clusters
- How well one model the interaction

# Numerical validation – RTS 96 test case



Hancheng Min



- The IEEE reliability test system: 1996
- 3 areas, 33 generators in total
- Different rotor angles across each area at initialization
- Solid lines: actual frequency response  
Dashed lines: reduced model

# Outline

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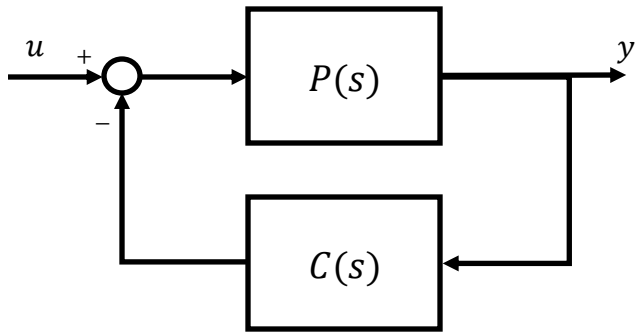
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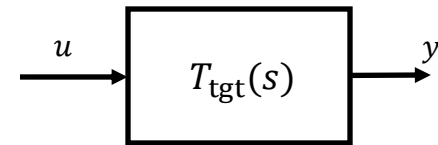
# Model Matching Control

Use control dynamics to shape system response

Real System



Desired Response



$$T_{yu}(s) = \frac{P(s)}{1 + P(s)C(s)} \quad " = " \quad T_{tgt}(s)$$

$$\text{Models match when: } C(s) = \frac{P(s) - T_{tgt}(s)}{T_{tgt}(s)P(s)}$$

# Grid Shaping Control

Use model matching control to shape system response

Grid-following IBRs

Grid-forming IBRs

# Grid-shaping with GFL IBRs [TPS 21]



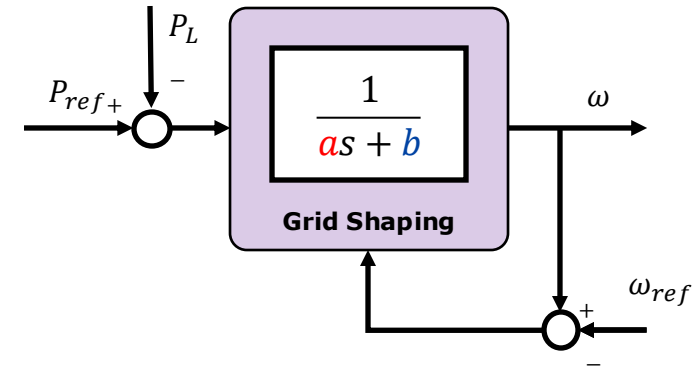
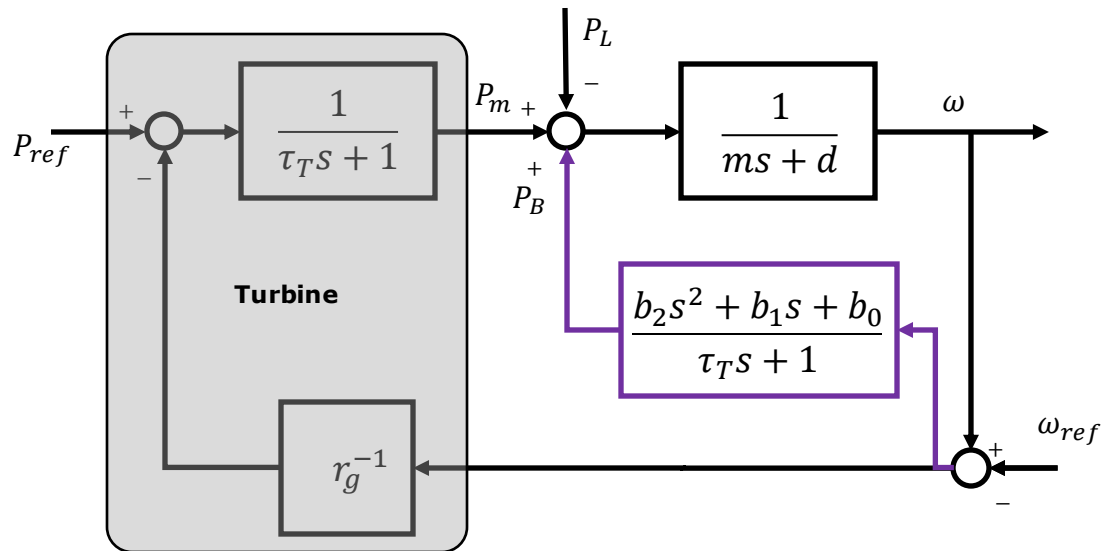
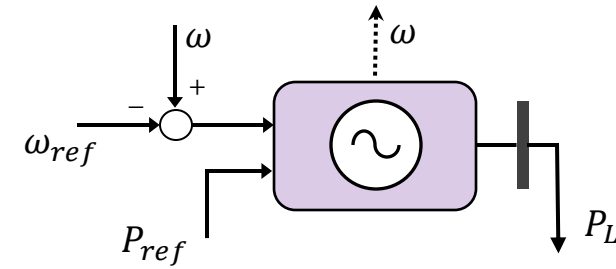
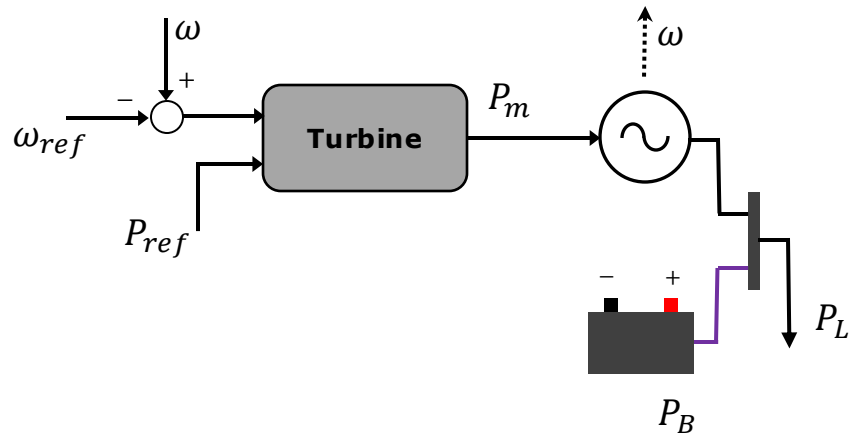
Yan Jiang



Eliza Cohn



Petr Vorobev



**Tunable Performance:**

$$\text{RoCoF} = \frac{1}{a} \Delta P, \quad \Delta \omega = \frac{1}{b} \Delta P$$



# Grid-shaping with GFL IBRs [TPS 21]



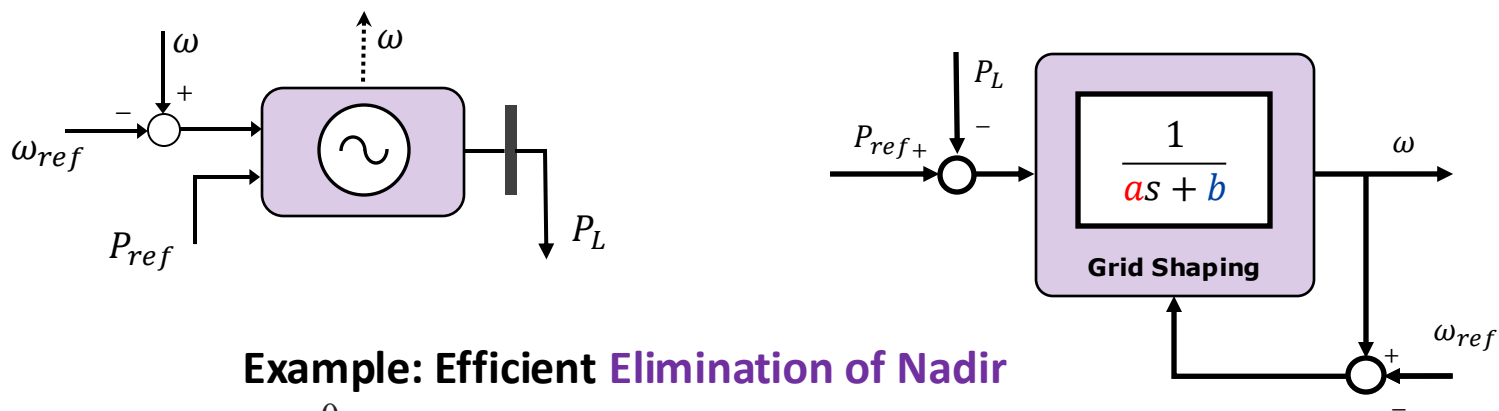
Yan Jiang



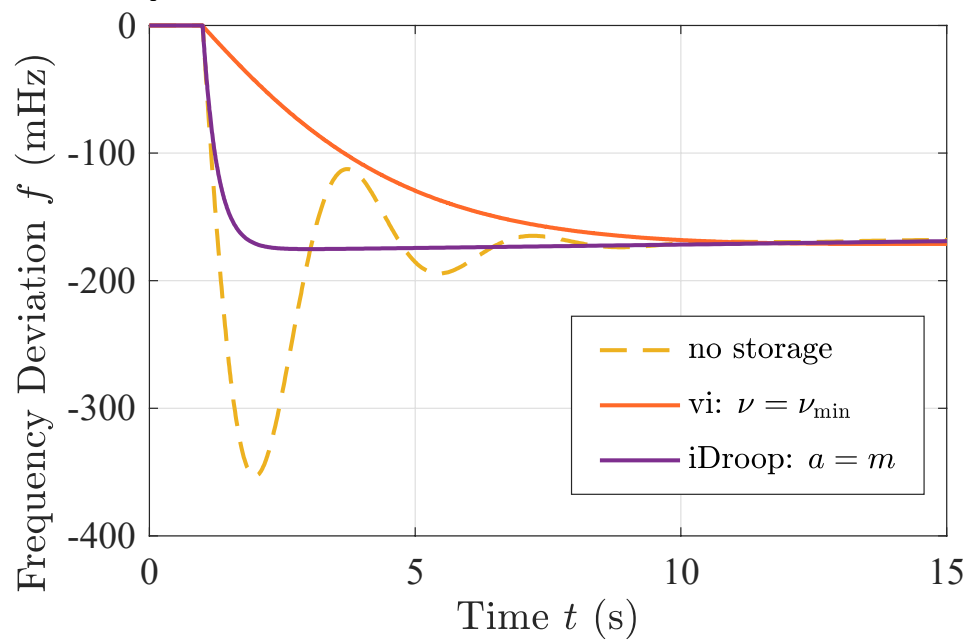
Eliza Cohn



Petr Vorobev

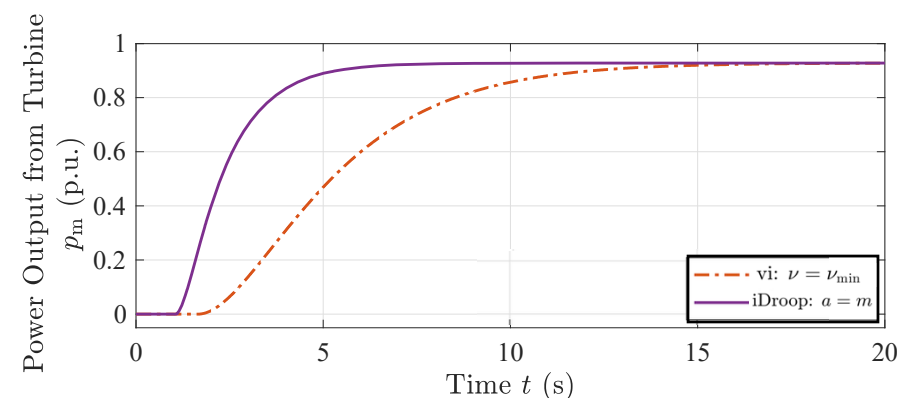
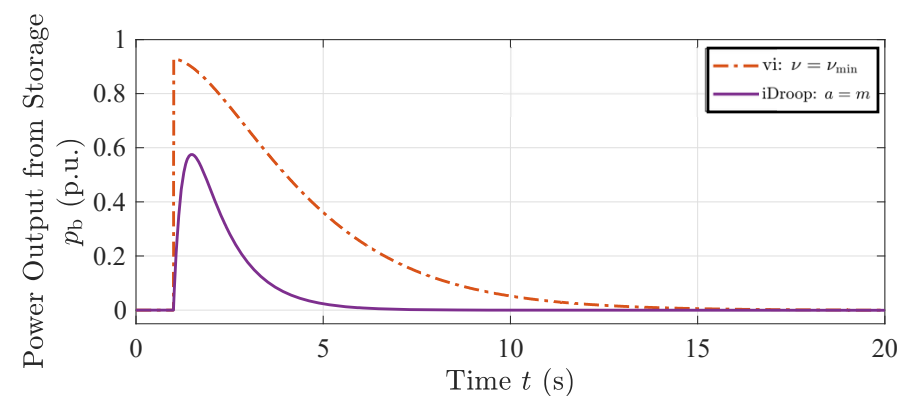


## Example: Efficient Elimination of Nadir



## Tunable Performance:

$$\text{RoCoF} = \frac{1}{a} \Delta P, \quad \Delta \omega = \frac{1}{b} \Delta P$$



# Grid-shaping with GFL IBRs [TPS 21]



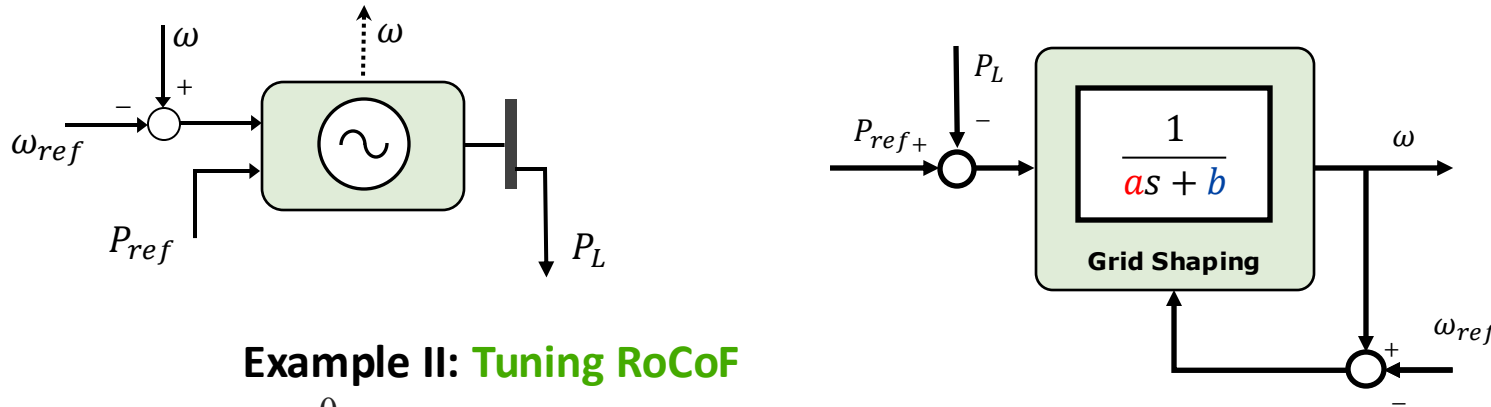
Yan Jiang



Eliza Cohn



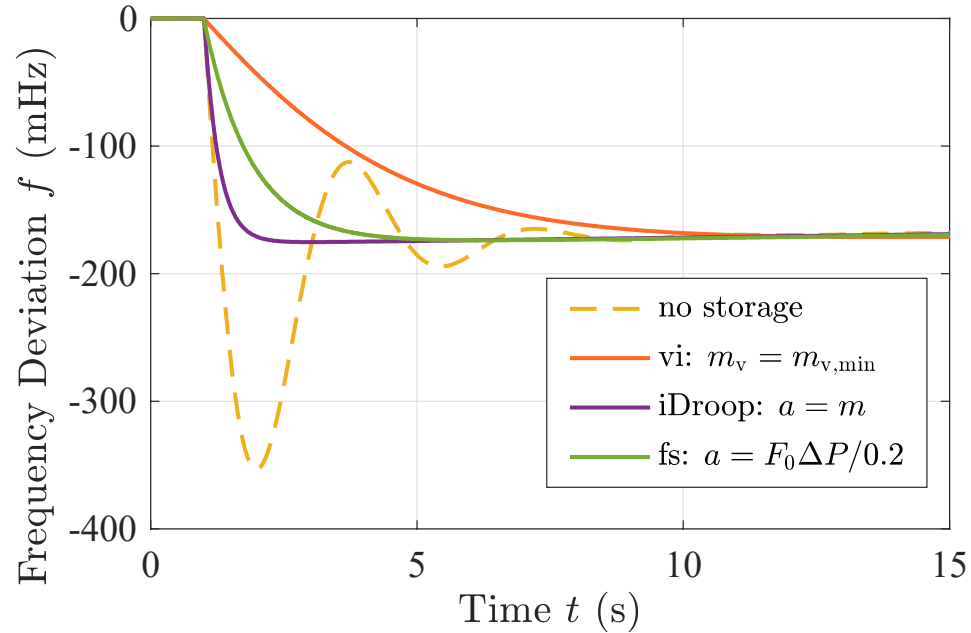
Petr Vorobev



**Tunable Performance:**

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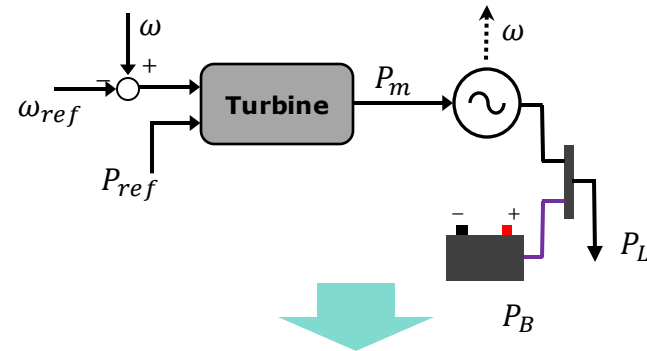
## Example II: Tuning RoCoF



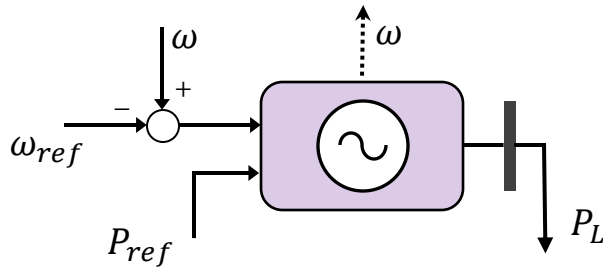
# Grid Shaping Control

Use model matching control to shape system response

## Grid-following IBRs



## Grid-forming IBRs



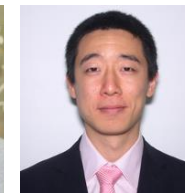
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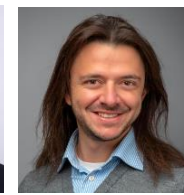
# GFM Grid-shaping Through Lines [LCSS 23]



B. K. Poolla



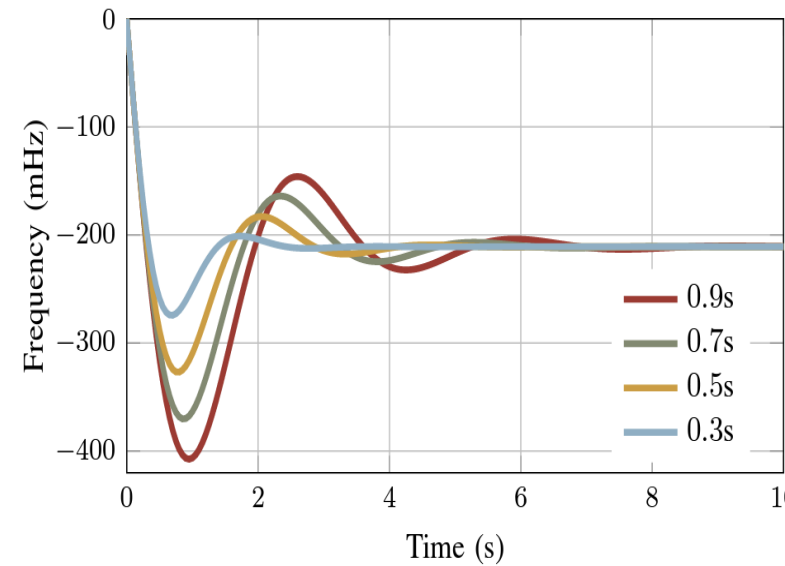
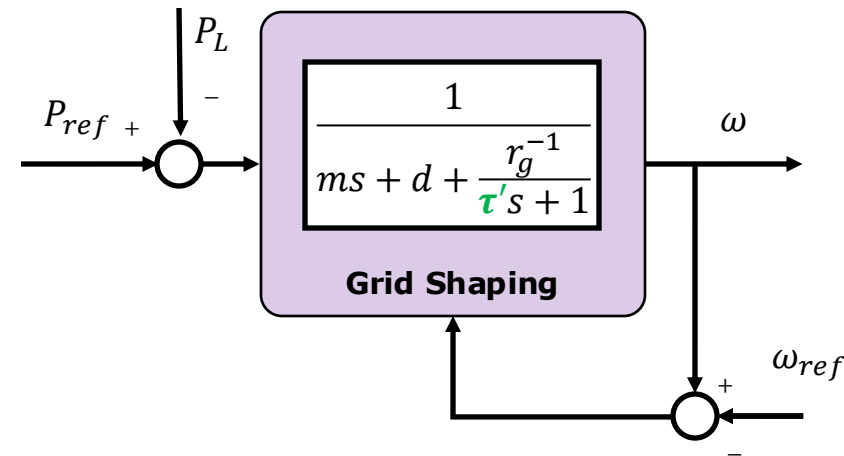
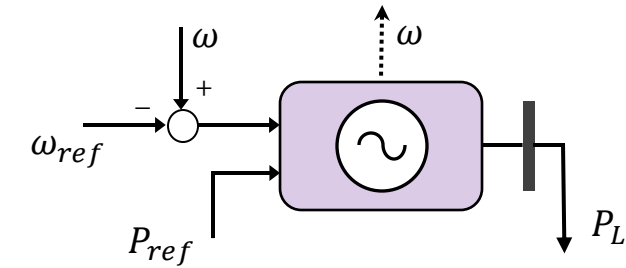
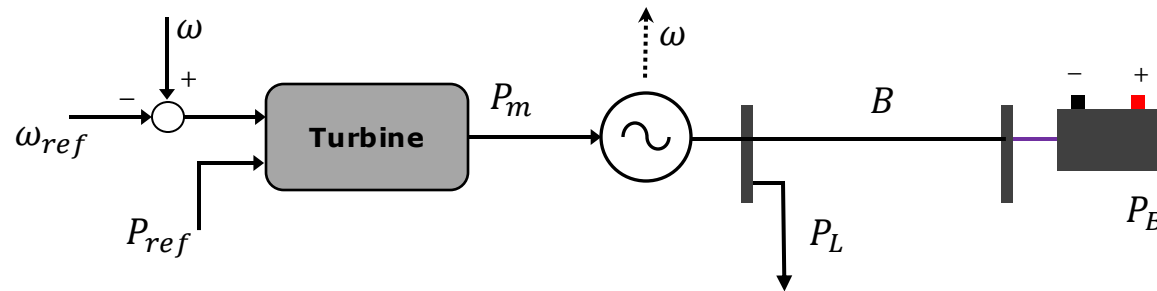
Y. Lin



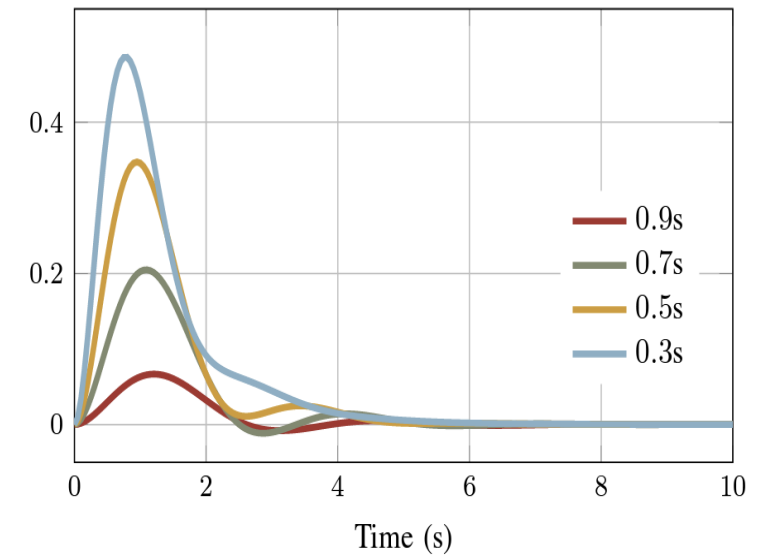
A. Bernstein



D. Groß



Frequency response for a 1 p.u. load step



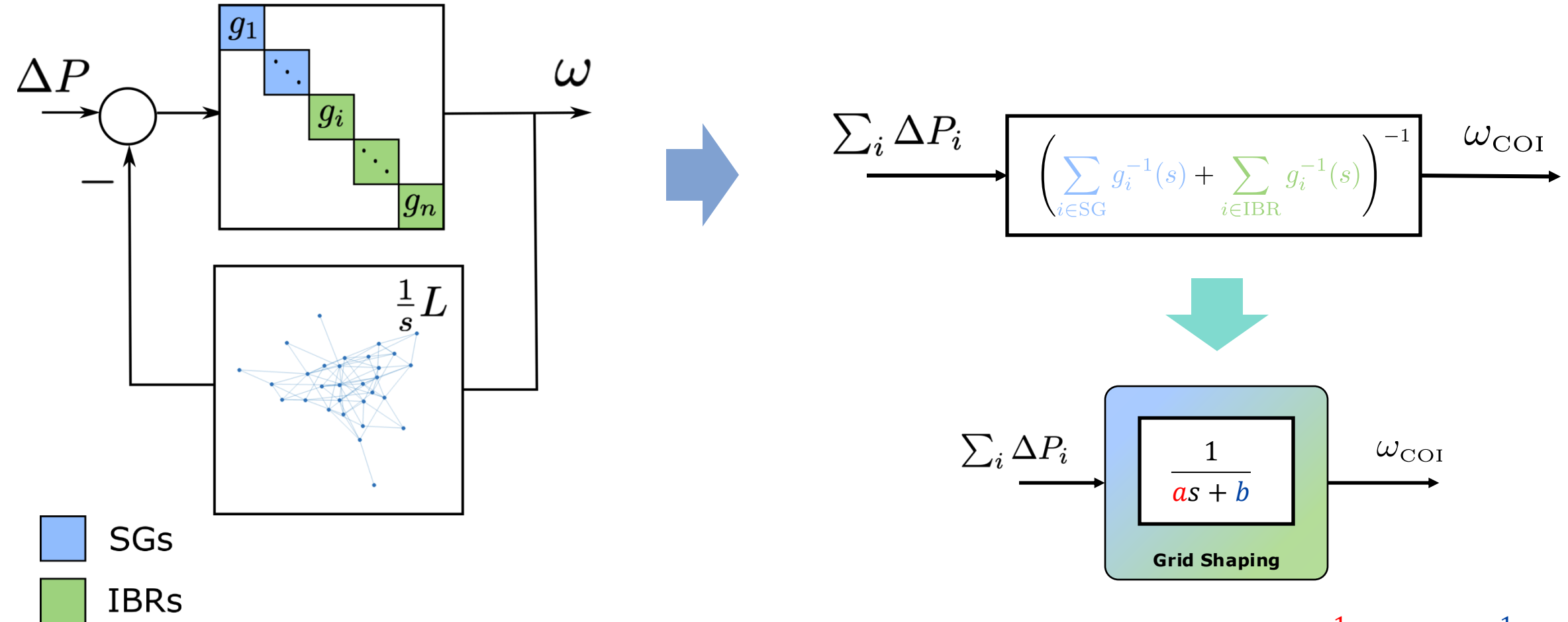
IBR power injection for a 1 p.u. load step

**Tunable Performance:**

E.g.: Turbine Time Constant =  $\tau'$



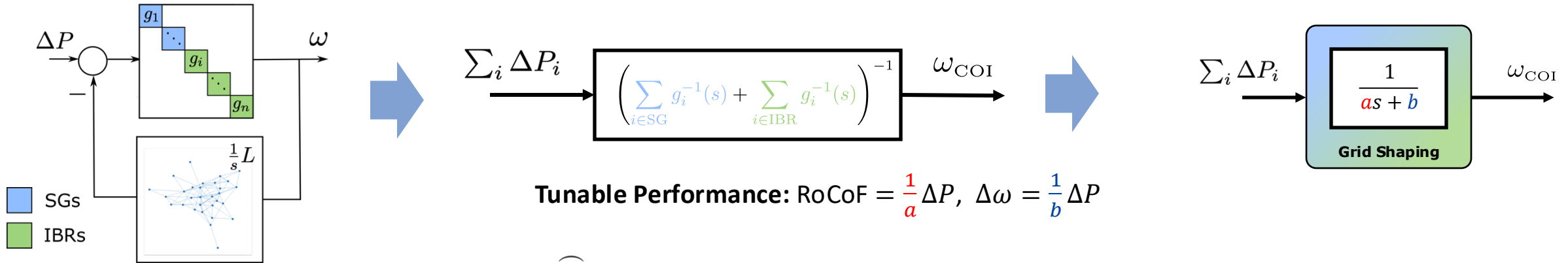
# GFM System-wide Grid-shaping [LCSS 20]



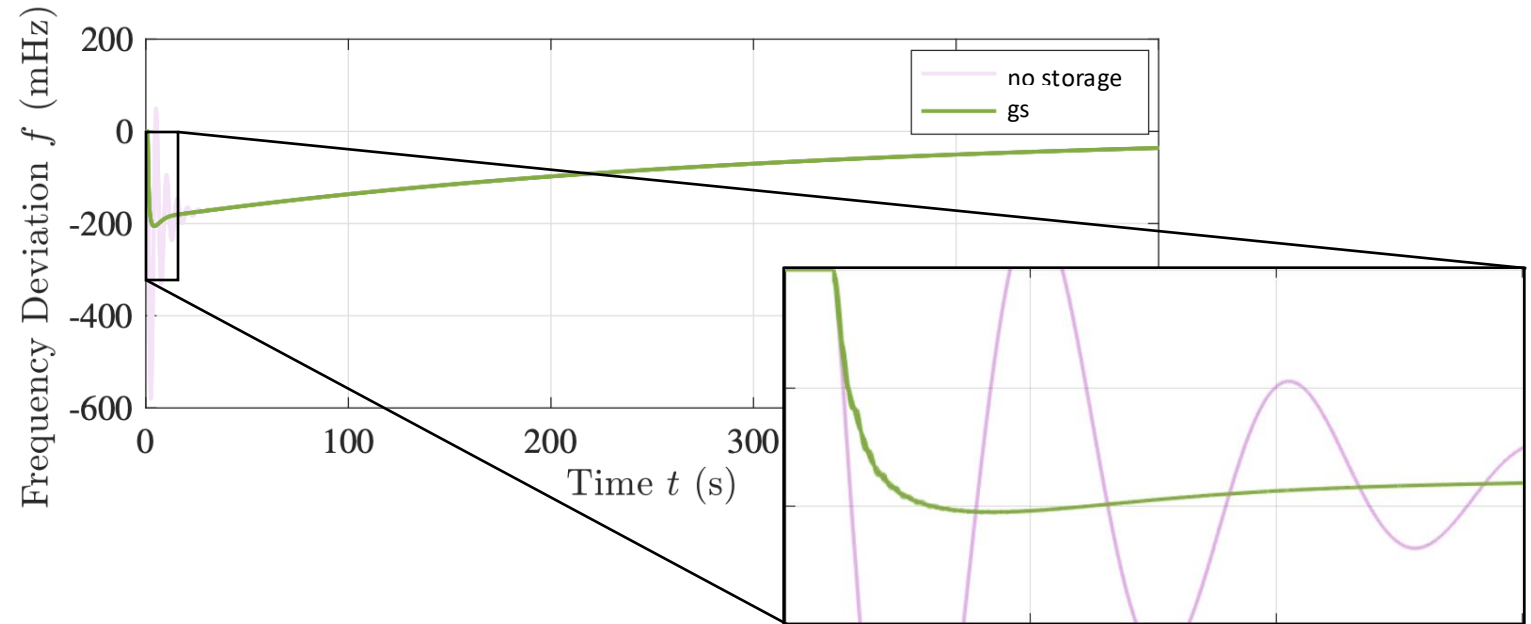
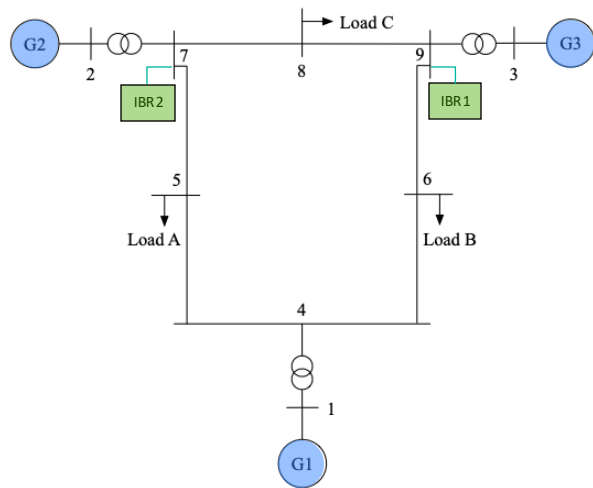
**Tunable Performance:**  $\text{RoCoF} = \frac{1}{a} \Delta P$ ,  $\Delta \omega = \frac{1}{b} \Delta P$



# GFM System-wide Grid-shaping [LCSS 20]



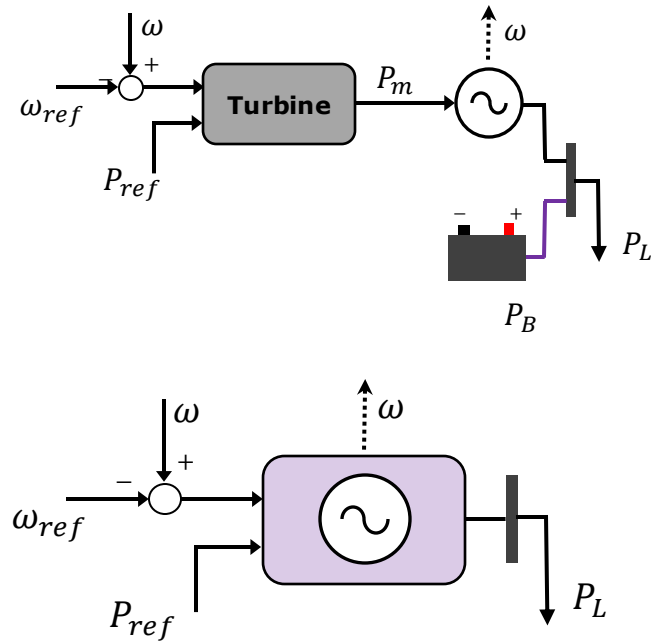
**Tunable Performance:**  $\text{RoCoF} = \frac{1}{a} \Delta P$ ,  $\Delta \omega = \frac{1}{b} \Delta P$



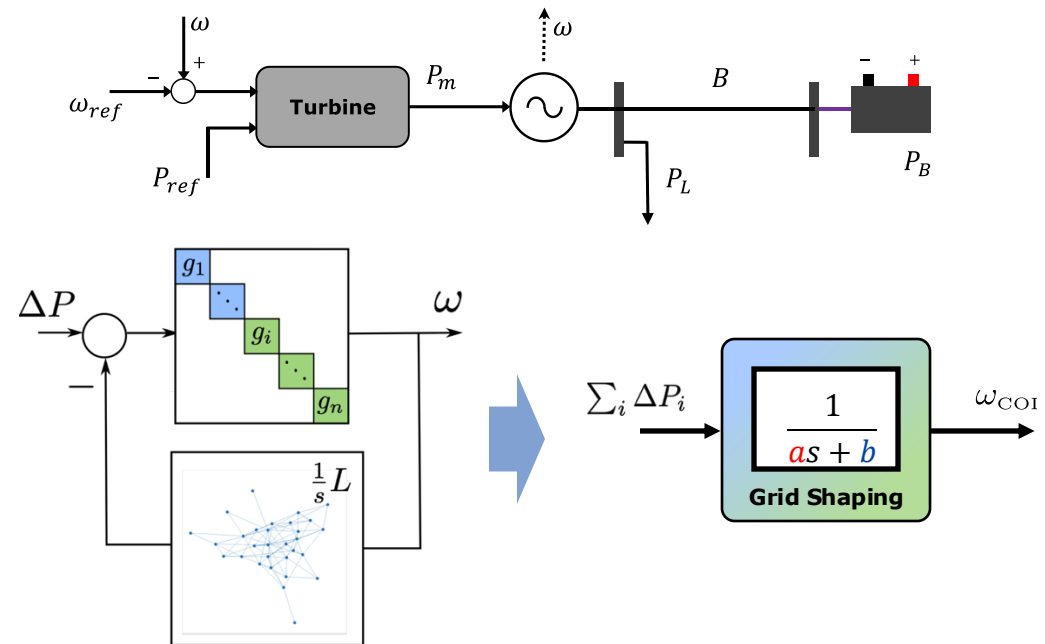
# Grid Shaping Control

Use model matching control to shape system response

## Grid-following IBRs



## Grid-forming IBRs



Tunable Performance:  $\text{RoCoF} = \frac{1}{a} \Delta P$ ,  $\Delta\omega = \frac{1}{b} \Delta P$ ,  $\tau'$ , ...

# Summary

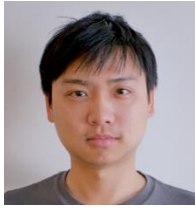
- **Merits and trade-offs of low inertia**
  - Control Perspective: Lighter systems are easier to control!
  - Smarter controller can provide multiple benefits in Nadir, RoCoF, inter-area oscillations, and disturbance rejection, with less effort
- **Scale-free Stability Analysis of Grids**
  - Generalizes passivity notions using network information
  - Decentralized test based on local models
  - Compatible with  $H_\infty$ -synthesis methods
- **Analysis of Weakly-Connected Coherent Networks**
  - Generalized Center of Inertia captures IBR dynamics
  - Provide a new tunable target to meet system specs
  - Coherent modes identified via spectral clustering
- **Grid Shaping Control**
  - Grid-following/forming control framework for future grids
  - Leverages IBRs to *shape* the coherent response



# Thanks!



Yan Jiang



Hancheng Min



Eliza Cohn



Petr Vorobev



Richard Pates



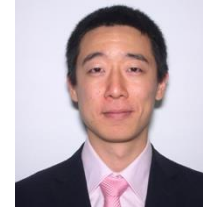
Fernando Paganini



Dominic Groß



Bala K. Poolla



Yashen Lin



Andrey Bernstein

## Merits and trade-offs of low inertia

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## Analysis of Weakly-Connected Coherent Networks

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