Grid Shaping Control for High-IBR Power Systems

Stability Analysis and Control Design

Enrique Mallada

Johns Hopkins University

RSRG Seminar, Caltech

April 3, 2025

Acknowledgements

Students


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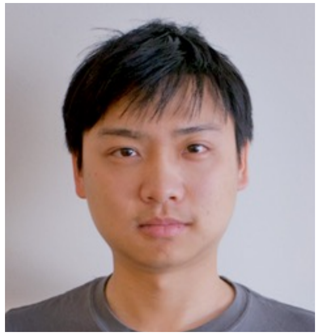
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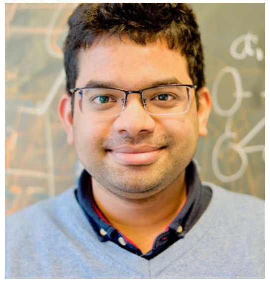
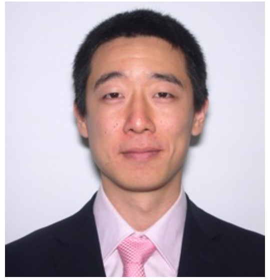

Bala K. Poolla


Yashen Lin


Andrey Bernstein

Decarbonization of electricity is key to mitigate climate change

California lifts renewable energy target to 73% by 2032

The California Public Utilities Commission raised renewable energy procurement targets, plans for a more aggressive decarbonization plan, and includes increased reliability provisions.

FEBRUARY 14, 2022 RYAN KENNEDY

New York mandates 70% renewable energy by 2030

By Kelsey Misbrener | October 15, 2020

Oregon bill targets 100% clean power by 2040, with labor and environmental justice on board

After Democratic cap-and-trade bills faltered in the face of GOP revolts, an electricity-focused, consensus-driven bill gains ground in Oregon.

23 June 2021

Vermont House passes 75% by 2032 renewable energy mandate

Published March 11, 2015

ENVIRONMENT

Maryland bill mandating 50% renewable energy by 2030 to become law, but without Gov. Larry Hogan's signature

By Scott Dance
Baltimore Sun • May 22, 2019 at 6:40 pm

Virginia becomes the first state in the South to target 100% clean power

The state's Democratic majority is doing what Democratic majorities do.

By David Roberts | @drvolts | Updated Apr 13, 2020, 2:56pm EDT

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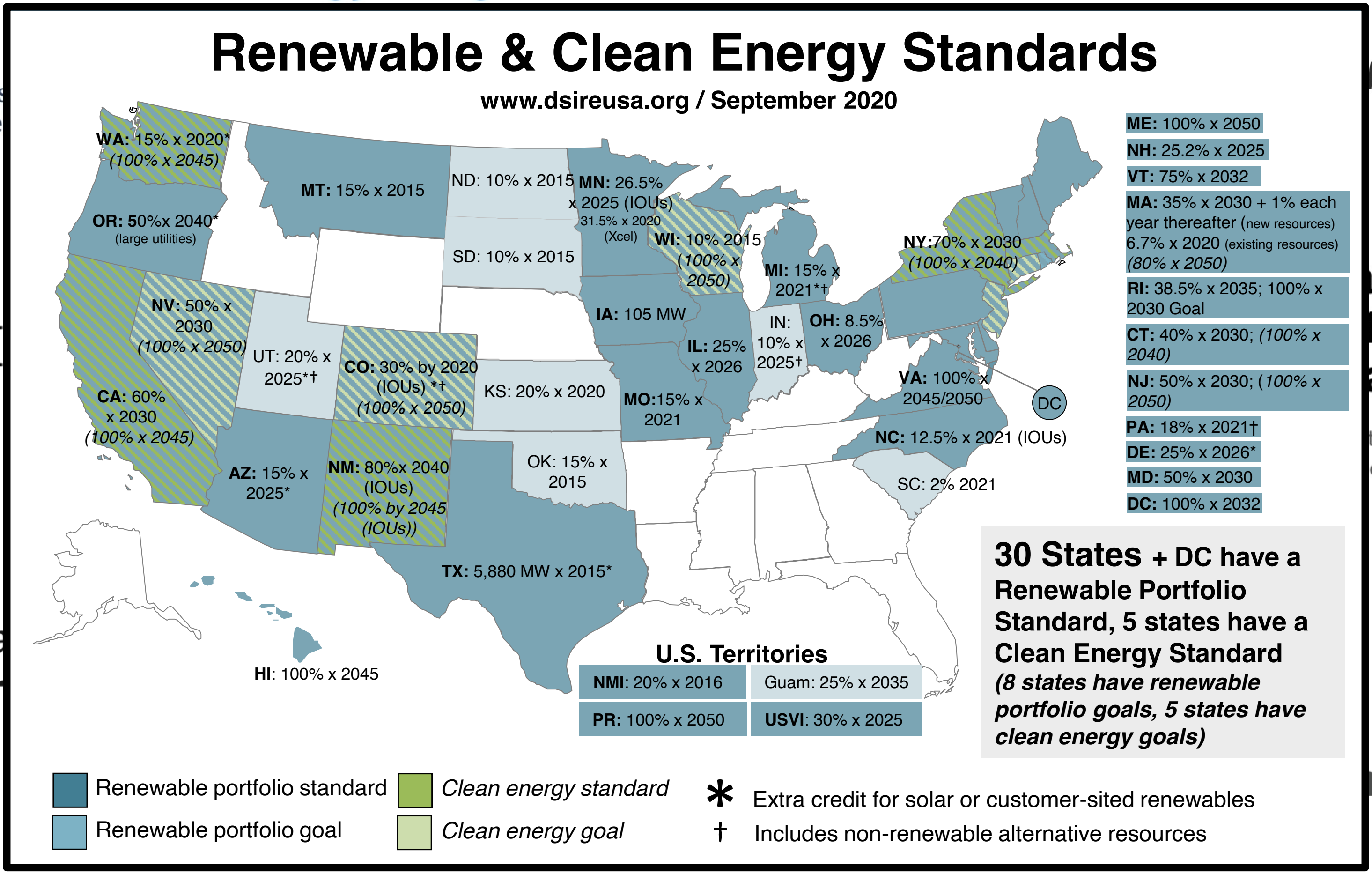

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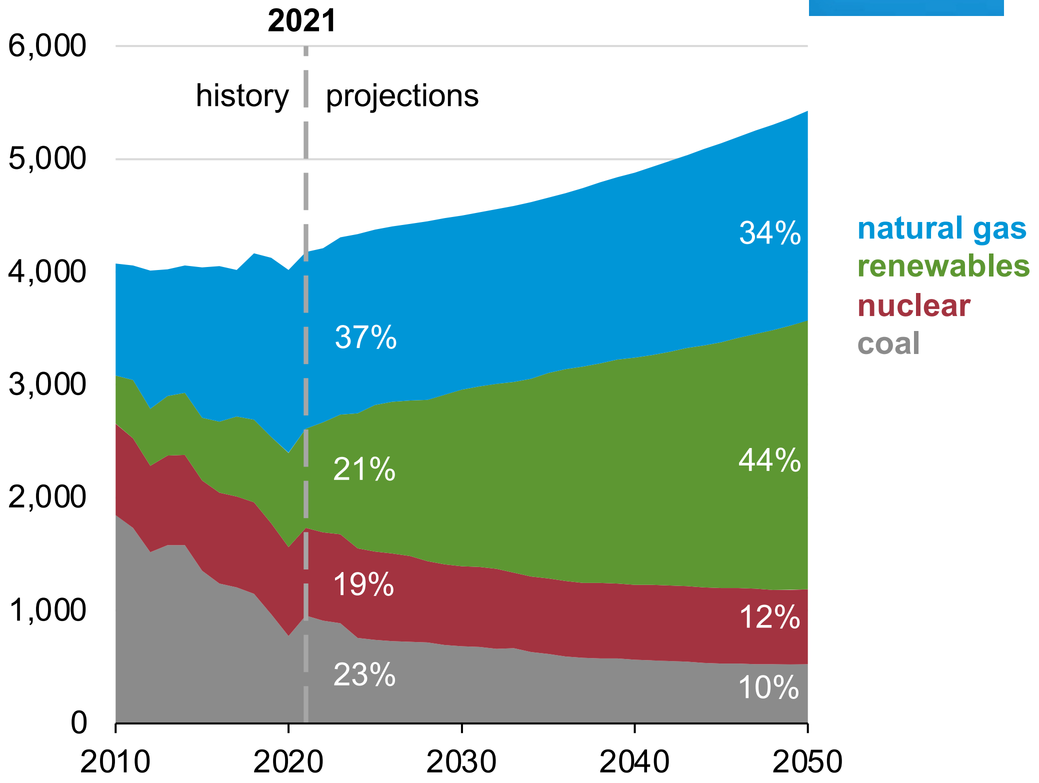
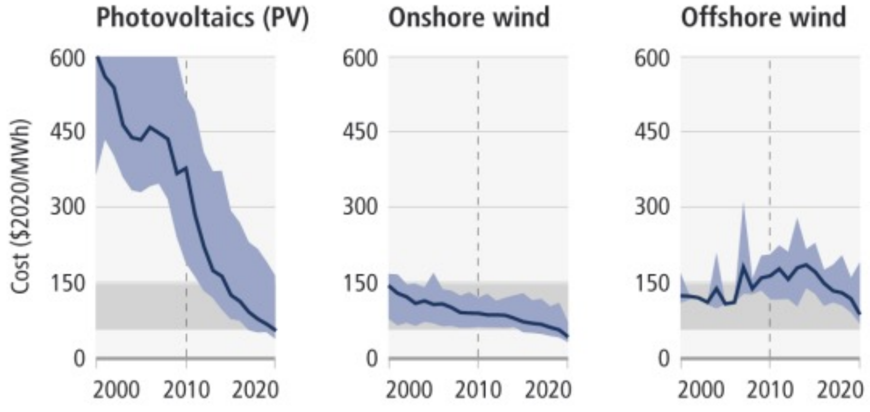
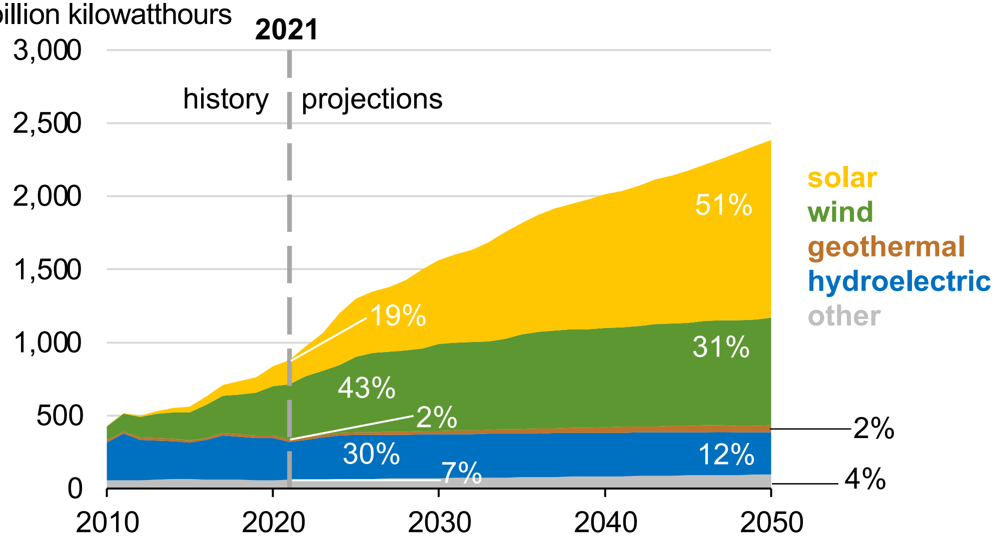
Decarbonization of electricity is key to mitigate climate change

U.S. electricity generation from selected fuels
AEO2022 Reference case
billion kilowatthours

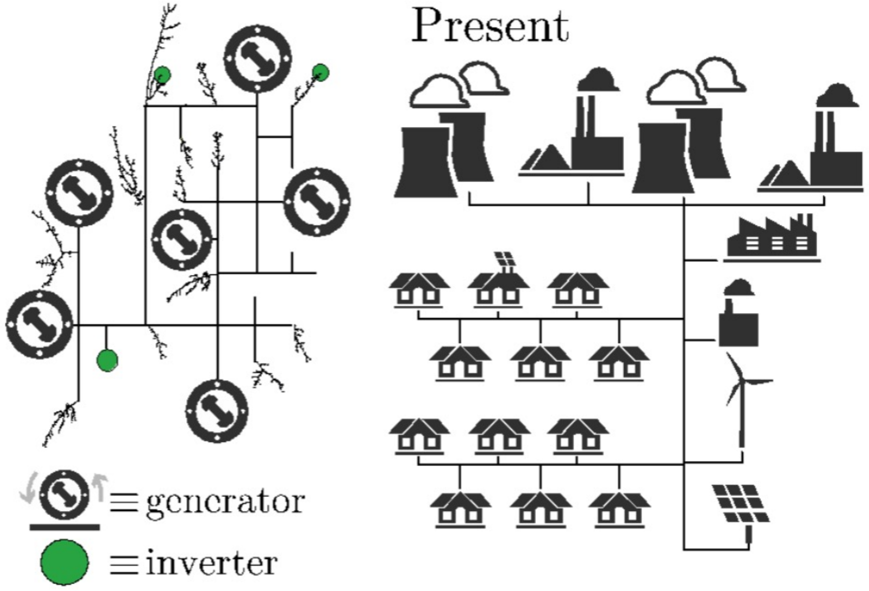




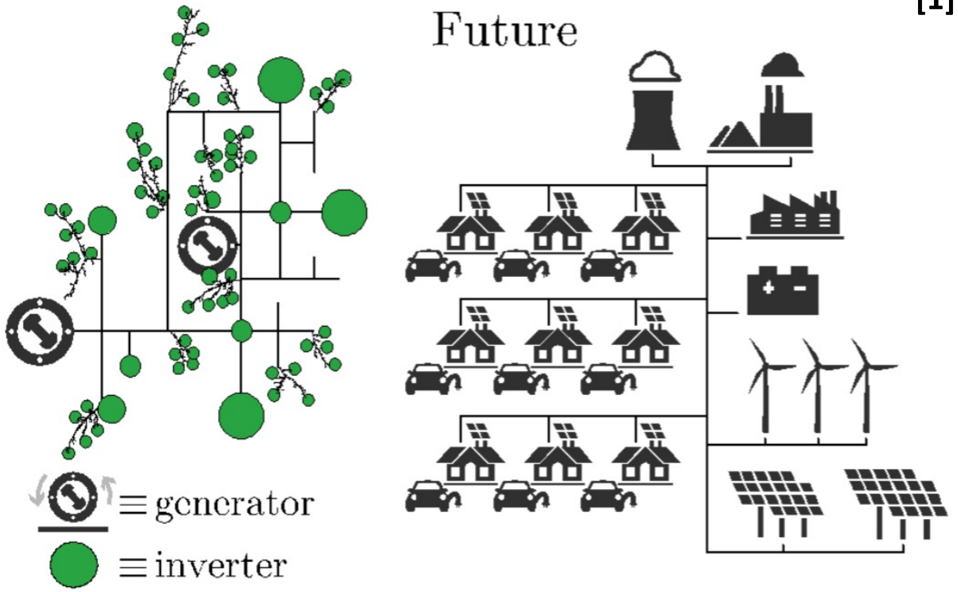
U.S. renewable electricity generation, including end use
AEO2022 Reference case
billion kilowatthours



The Future Grid



*[1]



Present grid

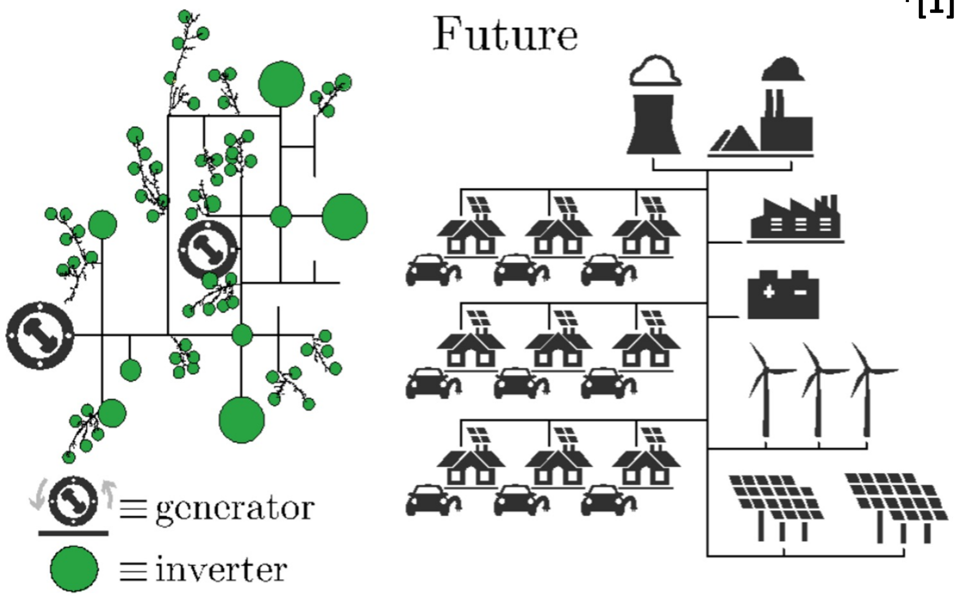
- dispatchable generation
- high inertial response
- strong voltage support
- well known physics

Future

- variable and distributed generation
- limited inertia levels
- weak voltage support
- proprietary control laws (black box)

[1] Lin et al. Research roadmap on grid-forming inverters. Technical report, National Renewable Energy Lab.(NREL), Golden CO, 2020

The Future Grid



Future

- variable and distributed generation
- limited inertia levels
- weak voltage support
- proprietary control laws (black box)

Selected challenges

- increased system **uncertainty**
- **sensitivity** to disturbances
- new forms of **instabilities**, induced by inverter-based resources
- need to compensate for **reduced inertia grid strength**

Research questions:

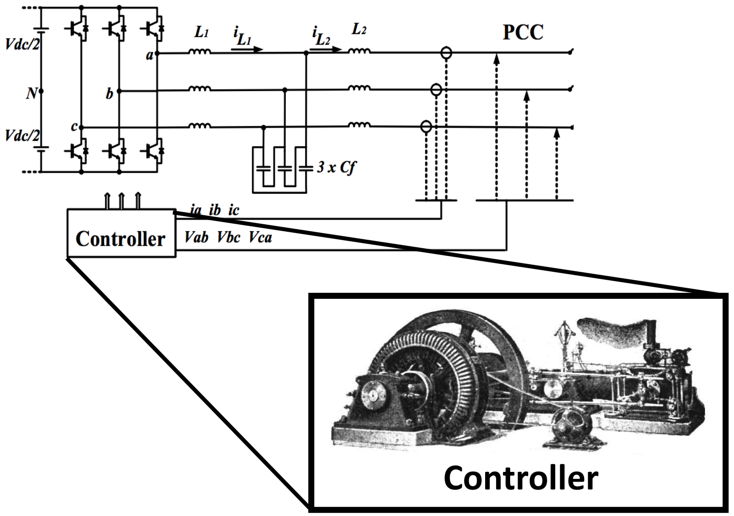
- How should we control a grid with limited inertial/voltage support?
- Should we try to mimic SGs response? Or find new and more efficient control paradigms, suitable for IBRs?

[1] Lin et al. Research roadmap on grid-forming inverters. Technical report, National Renewable Energy Lab.(NREL), Golden CO, 2020

Inverter-based Control

Current approach: Use inverter-based control to **mimic generators response**

Virtual Synchronous Generator

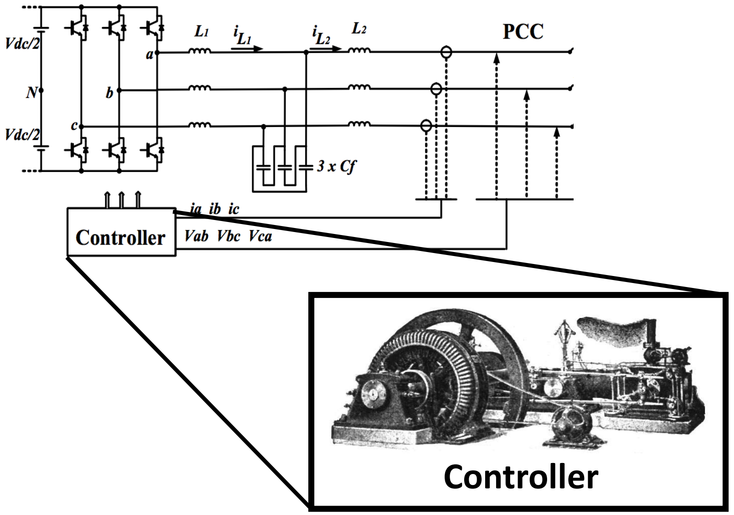


Telecom Analogy

Inverter-based Control

Current approach: Use inverter-based control to **mimic generators response**

Virtual Synchronous Generator



Telecom Analogy

It works, but perhaps there is something better…

Outline

- **Merits and trade-offs of low inertia**
  - Control Perspective: Lighter systems are easier to control!
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  - Generalizes passivity notions using network information
- **Analysis of Weakly-Connected Coherent Networks**
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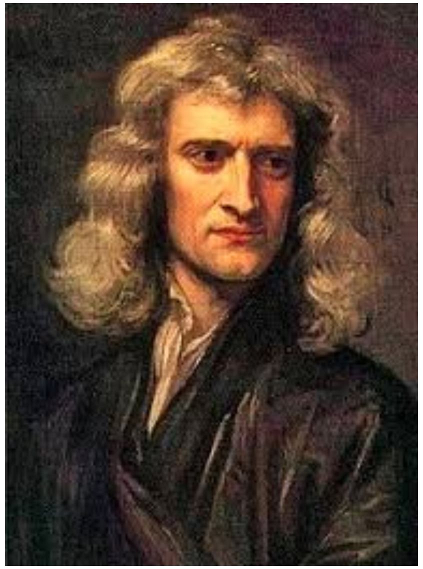
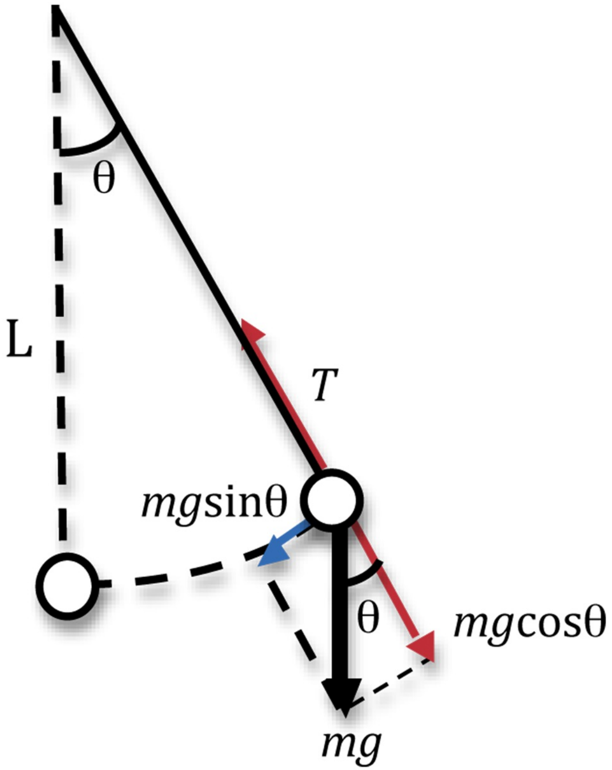
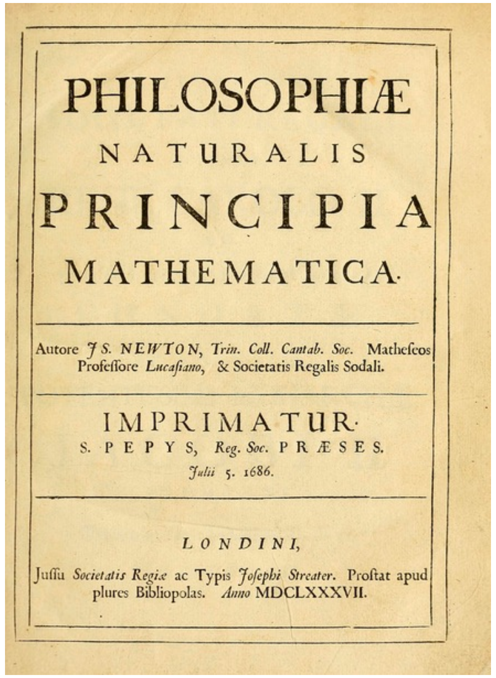
Merits and Trade-offs of Inertia







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S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

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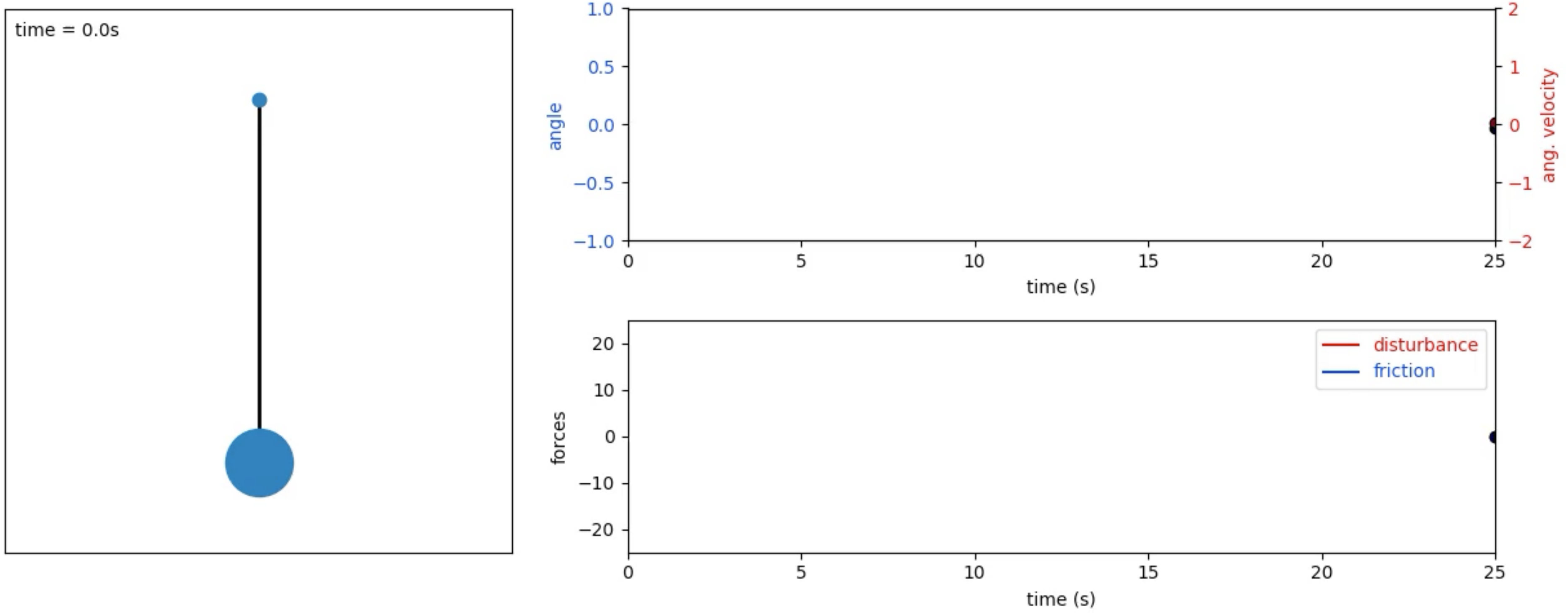
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$

Merits and Trade-offs of Inertia

$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$

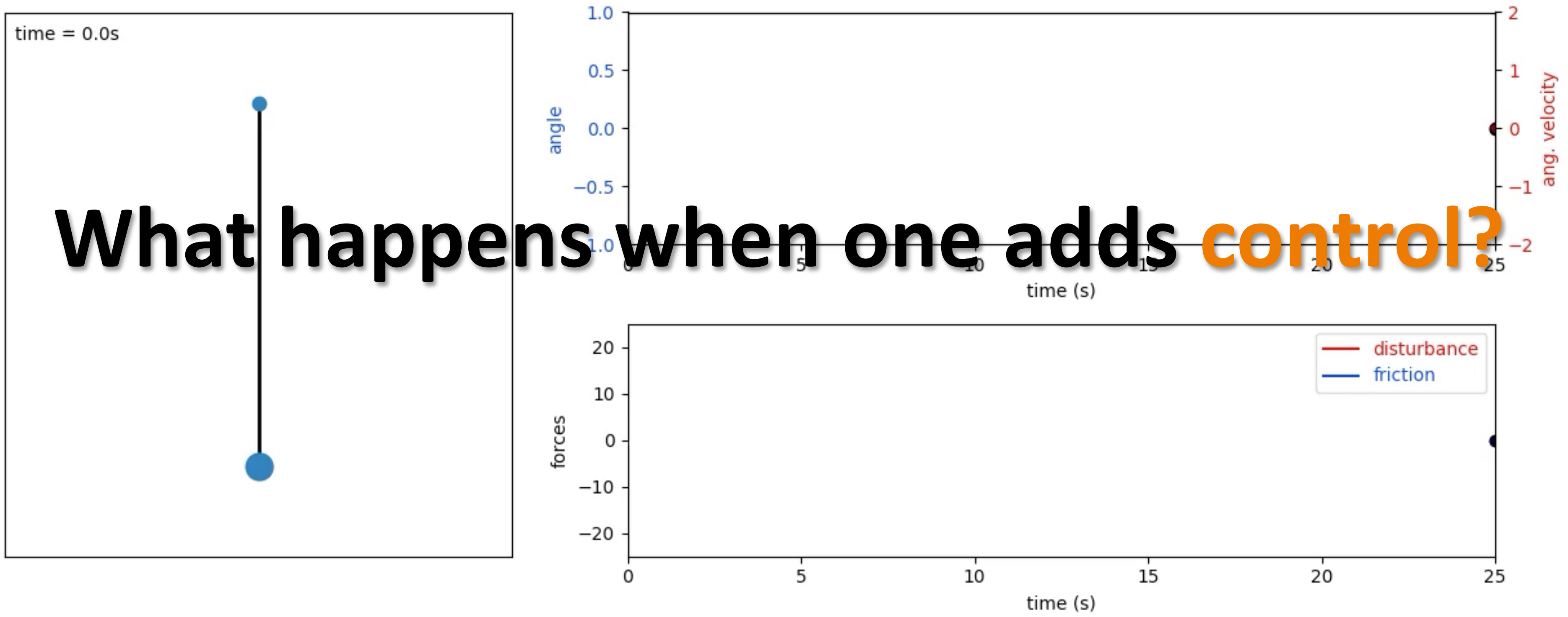


Pros: Provides natural disturbance rejection

Cons: Hard to regain steady-state

Merits and Trade-offs of Low Inertia

$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$



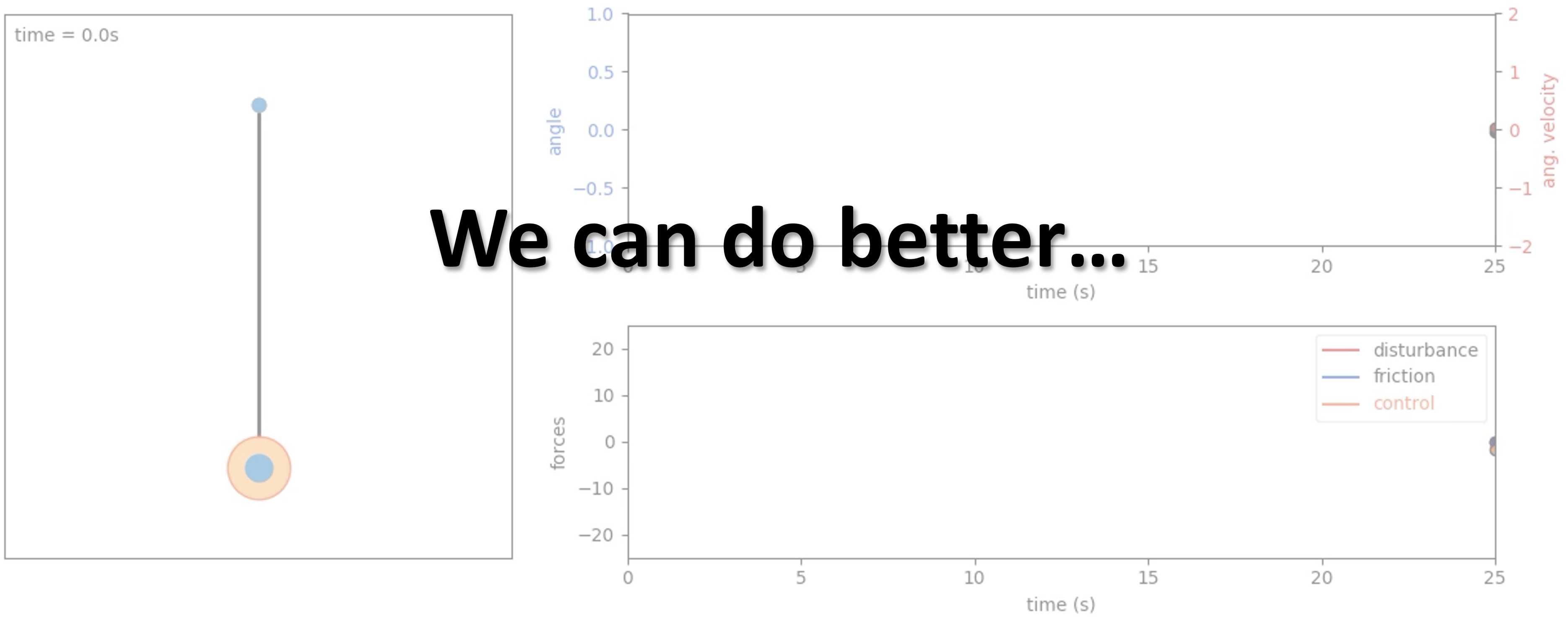
What happens when one adds control?

Cons: Susceptible to disturbances

Pros: Regains steady-sate faster

Control of Low Inertia Pendulum

Virtual Mass Control: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - \nu\ddot{\theta}$



We can do better…

Pros:

Provides disturbance rejection

Cons:

Hard to regain steady-state + **excessive control effort**

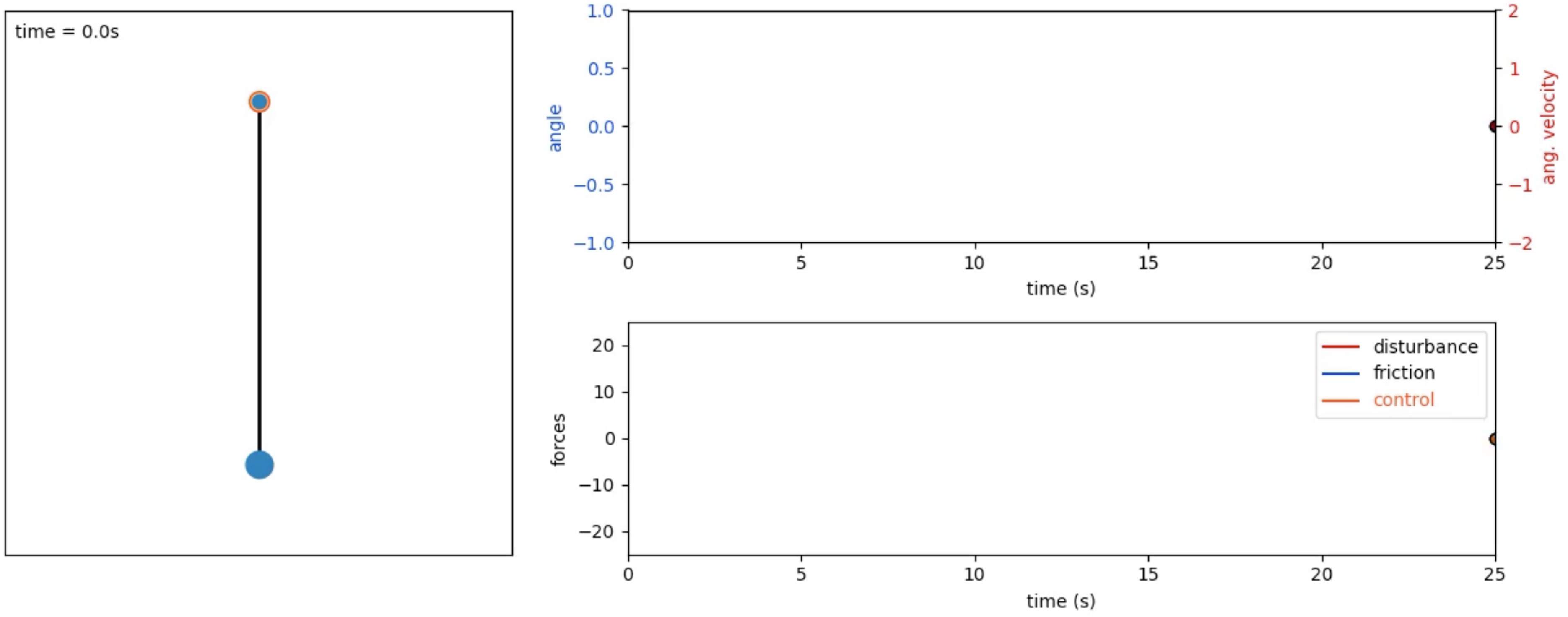
Control of Low Inertia Pendulum

Yan Jiang   Richard Pates

Dynamic Droop:

$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f + x$$

$$\tau'\dot{x} = -x - (r_r^{-1}\dot{\theta} + \tau'\nu'\ddot{\theta})$$



[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Control of Low Inertia Pendulum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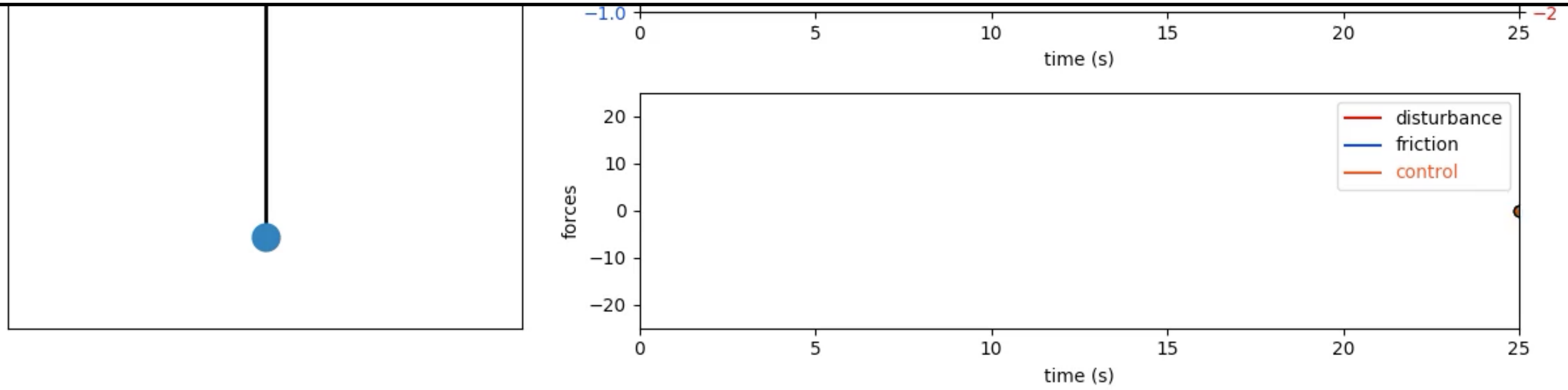
Dynamic Droop: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f + x$

3518 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 66, NO. 8, AUGUST 2021

Dynamic Droop Control in Low-Inertia Power Systems

Yan Jiang, Richard Pates, and Enrique Mallada, *Senior Member, IEEE*



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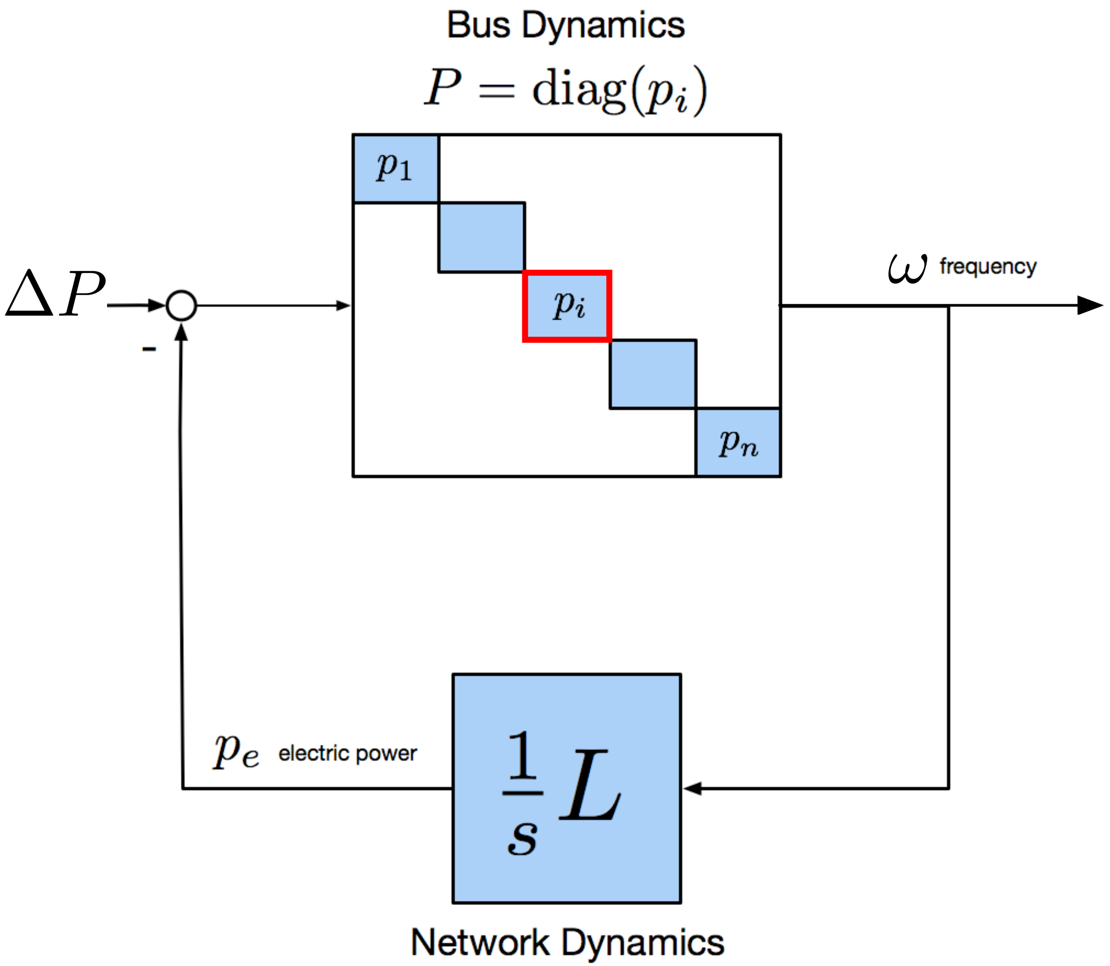
Power Network Model



Laplacian Matrix

$$L_{ij} = \begin{cases} -B_{ij} & \text{if } ij \in E \\ \sum_k B_{ik} & \text{if } i = j \\ 0 & \text{o.w.} \end{cases}$$

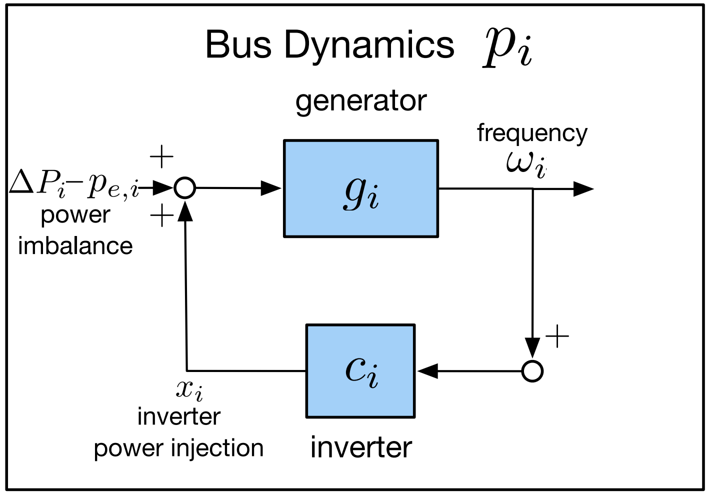
Linearized Power Flows

$$B_{ij} = v_i v_j b_{ij} \cos(\theta_i^* - \theta_j^*)$$

[Bergen Hill ‘81]

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap,* **IEEE Transactions on Automatic Control, 2020**

Bus Dynamics



Generator: $g_i : (\Delta P_i - p_{e,i} + x_i) \mapsto \omega_i$

Model: Swing Equations + Turbine

$$g_i : \begin{cases} \dot{\theta}_i = \omega_i \\ M_i \dot{\omega}_i = -D_i \omega_i + q_i + (\Delta P_i - p_{e,i} + x_i) \\ \tau_i \dot{q}_i = -R_{g,i}^{-1} \omega_i - q_i \end{cases}$$

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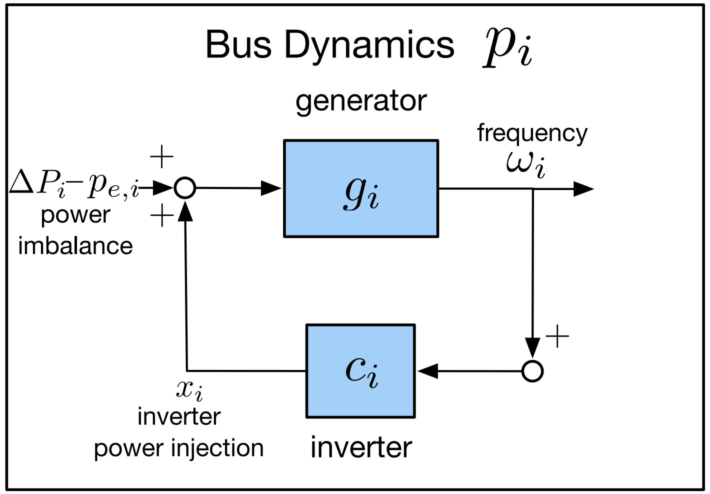

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$$g_i(s) = \frac{1}{M_i s + D_i + \frac{R_{g,i}^{-1}}{\tau_i s + 1}}$$

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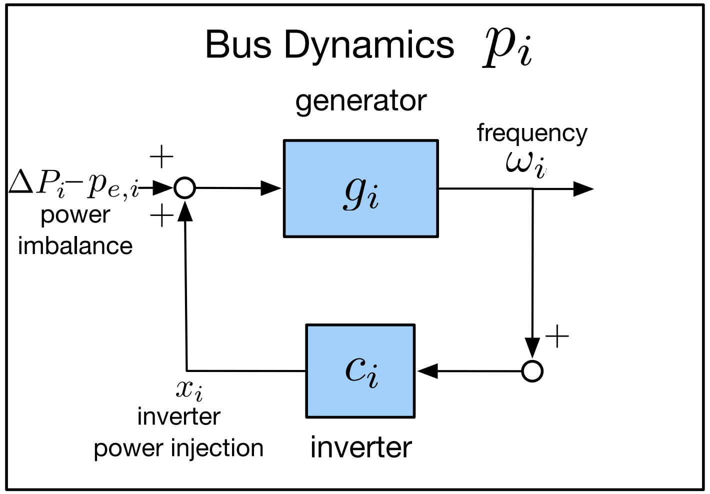
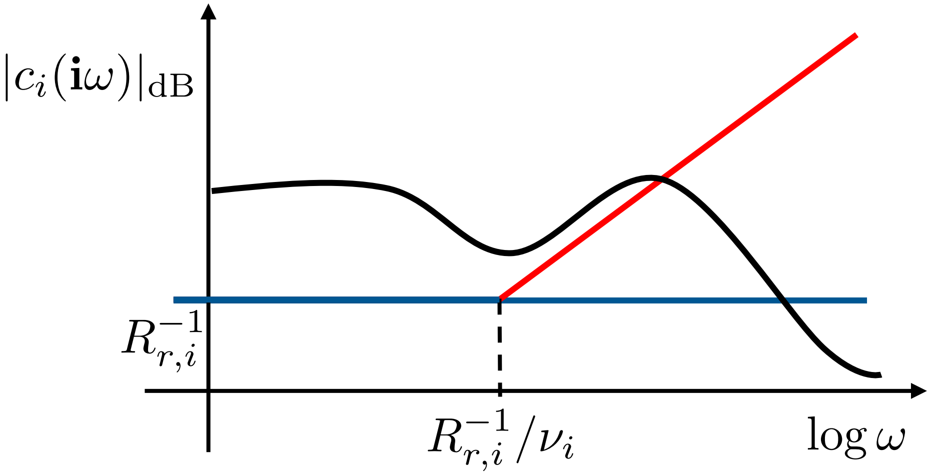




Grid Following Inverter: $c_i : \omega_i \mapsto x_i$

Droop Control and Virtual Inertia:

$$c_i : \left\{ x_i = -(\nu_i \dot{\omega}_i + R_{r,i}^{-1}\omega_i), \qquad c_i(s) = -(\nu_i s + R_{r,i}^{-1}\omega_i) \right.$$

Closed-loop Bus Dynamics:

$$p_i : \begin{cases} \dot{\theta}_i = \omega_i \\ (M_i + \nu_i)\dot{\omega}_i = -(D_i + R_{r,i}^{-1})\omega_i + q_i + (\Delta P_i - p_{e,i}) \\ \tau_i \dot{q}_i = -q_i - R_{g,i}^{-1}\omega_i \end{cases}$$

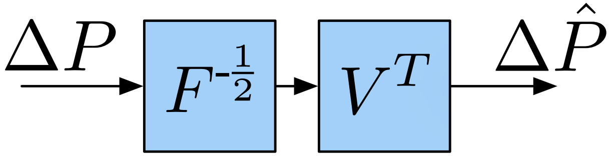
[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap,* **IEEE Transactions on Automatic Control, 2020**

Modal Decomposition for Multi-Rated Machines

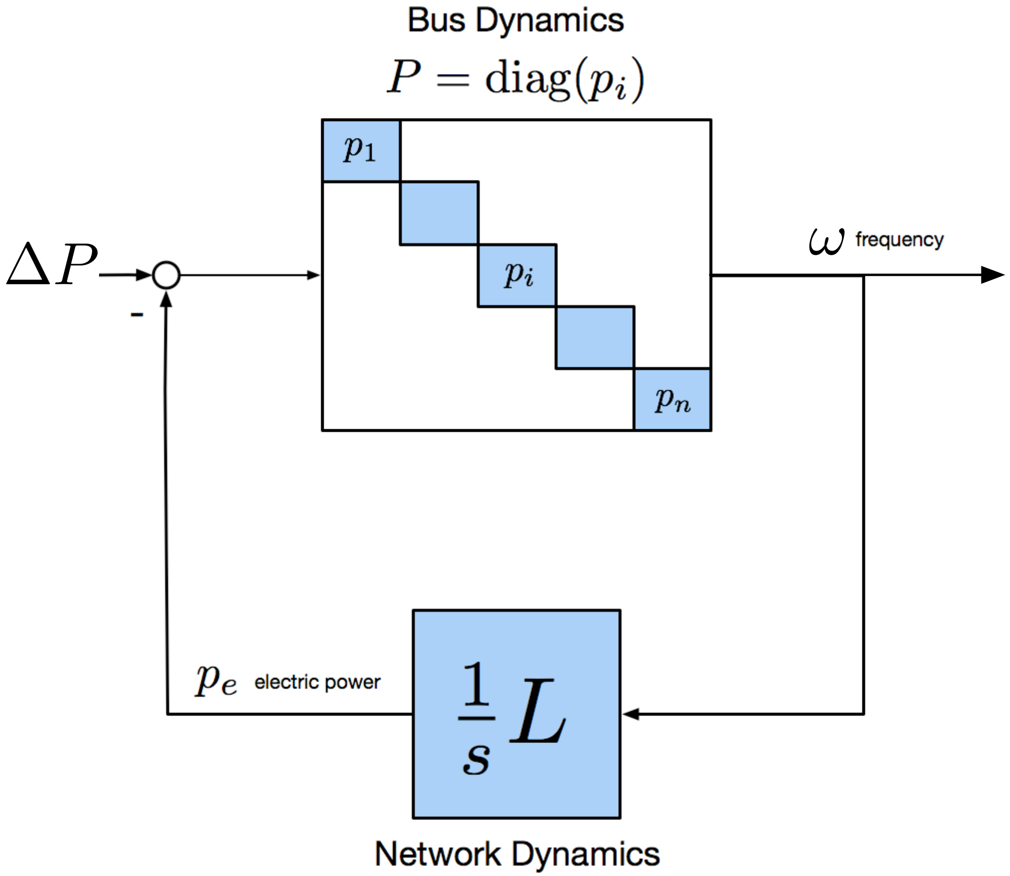
Assumption: Let f_i be the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

$$\begin{cases} g_i(s) = \frac{1}{f_i} g_0(s) \\ c_i(s) = f_i c_0(s) \end{cases}$$

Change of Vars.



$$F = \mathrm{diag}(f_i)$$



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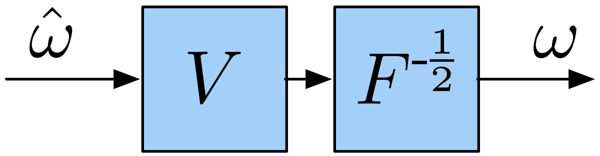

[Paganini M ‘17 , Guo Low 18’]

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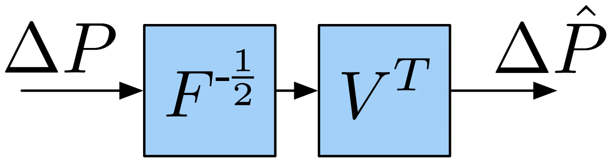
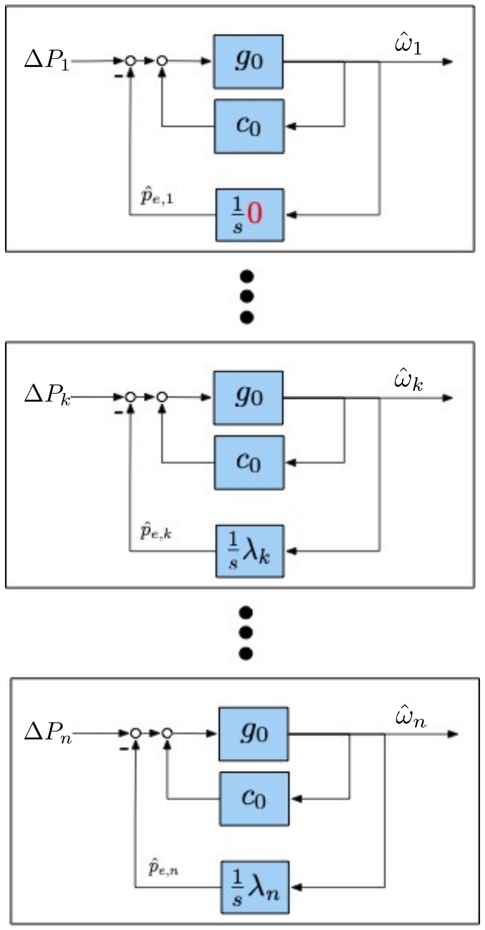


$$F = \mathrm{diag}(f_i)$$

Eigenvalues of: $L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}$

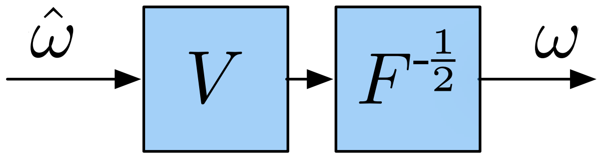
$$0 = \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}$$



Center of Inertia

$$\omega_{\mathrm{CoI}}(t) = \frac{\sum_{i=1}^{n} M_i \omega_i(t)}{\sum_{i=1}^{n} M_i}$$

Change of Vars.



Sync Error

$$\tilde{\omega}_i(t) = \omega_i(t) - \omega_{\mathrm{CoI}}(t)$$

[Paganini M '17 , Guo Low 18']

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, **IEEE Transactions on Automatic Control, 2020**

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IEEE CSS

Dynamic Droop Control in Low-Inertia Power Systems

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Dynamic Droop Benefits

- Overshoot Elimination in Nadir
- Noise Attenuation
- Disturbance Rejection
- Reduce Inter-area Oscillations

Caveat

- Control design limited to co-located resources (SGs and GFL-IBRs)
- Restrictive assumptions: Proportional dynamics $(p_i(s) = f_i p_0(s))$

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Outline

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  - Control Perspective: Lighter systems are easier to control!
- **Scale-free Stability Analysis of Grids**
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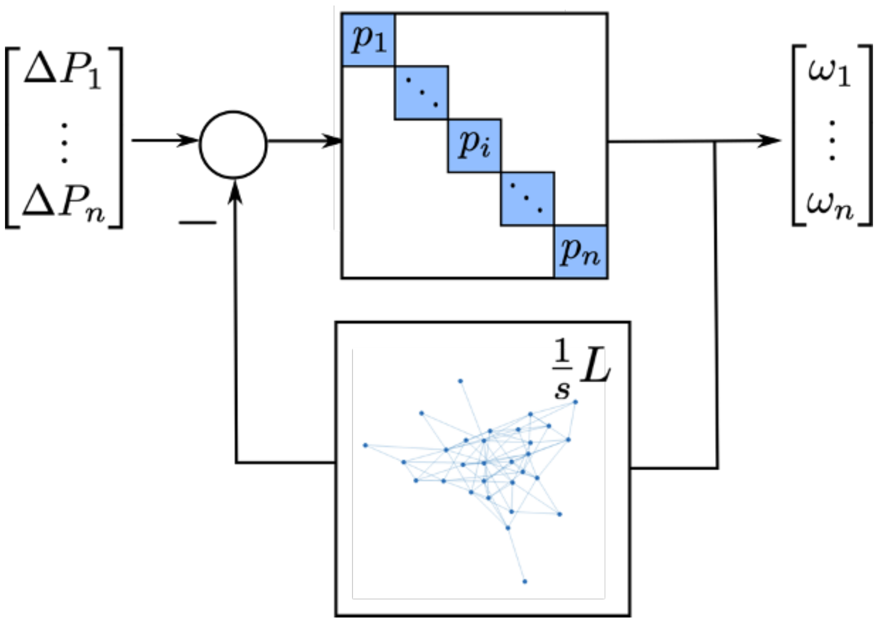
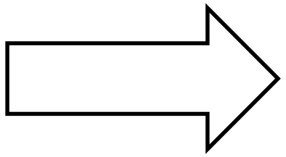
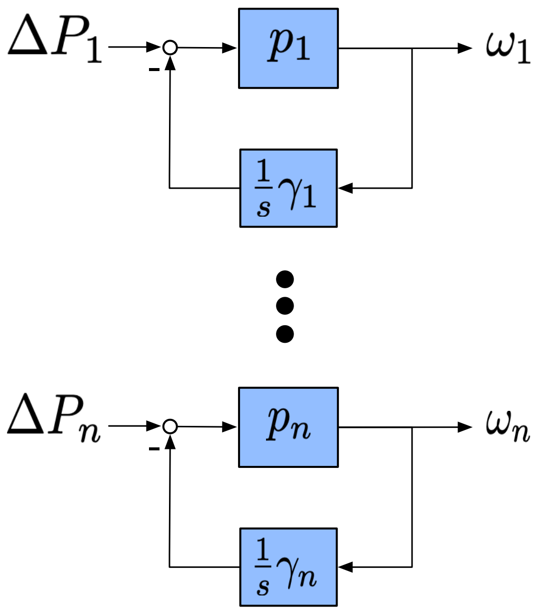
Decentralized Stability Analysis in Power Grids [TCNS 19]



1.When does this interconnection is stable?



2. Can we analysis and control design based on ***local rules****?*



Richard Pates

Problem Setup:

- *Linearized* power flows, *lossless*

$$L_{ij} = -b_{ij} v_i v_j \cos(\theta_i^* - \theta_j^*)$$

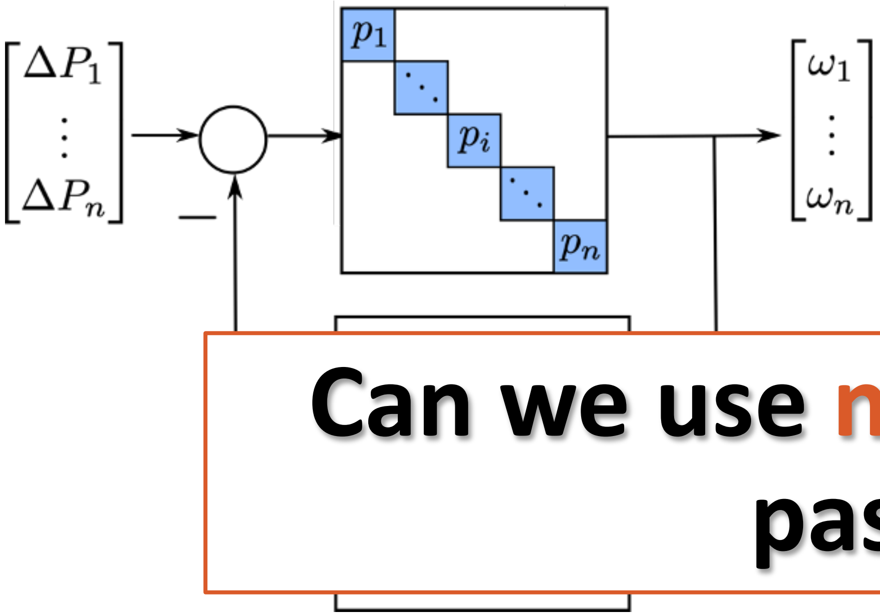
- Bus i: arbitrary *siso* transfer function:

$$\omega_i = p_i(s)\,\Delta P_i \quad \text{(SGs or GFM-IBRs)}$$

[TCNS 19] Pates, M. Robust Scale Free Synthesis for Frequency Regulation in Power Systems. **IEEE Transactions on Control of Network Systems, 2019**

Decentralized Stability Analysis in Power Grids [TCNS 19]

Richard Pates



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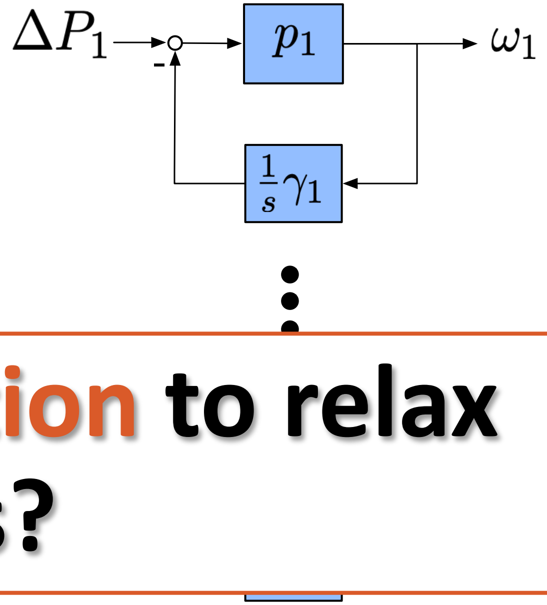


Can we use network information to relax passivity conditions?

Standard Approach: Passivity

- If $p_i(s)$ is strictly positive real (SPR), then the interconnection is stable for **all networks L!**

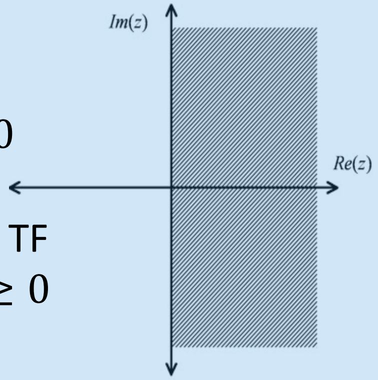
Converse: for **unknown network (L)**, passivity is also **necessary**.[TCNS 19]

Positive Real (PR) TF

$$\mathrm{Re}[p_i(s)] \geq 0$$

Strictly Positive Real TF

$$\mathrm{Re}[p_i(s-\varepsilon)] \geq 0$$



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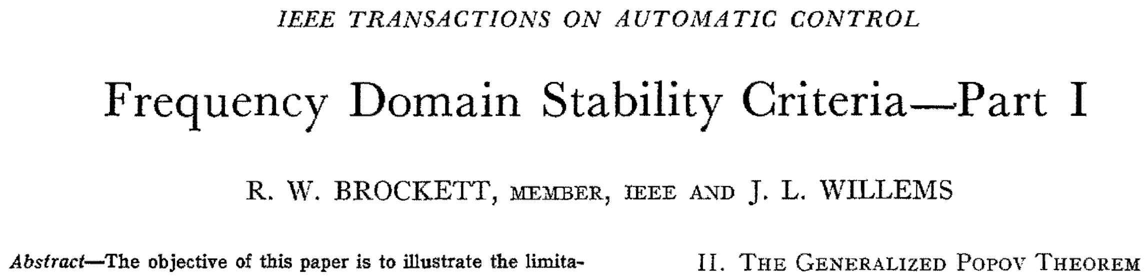
Classical Result: Absolute Stability



IEEE TRANSACTIONS ON AUTOMATIC CONTROL

Frequency Domain Stability Criteria—Part I

R. W. BROCKETT, MEMBER, IEEE AND J. L. WILLEMS

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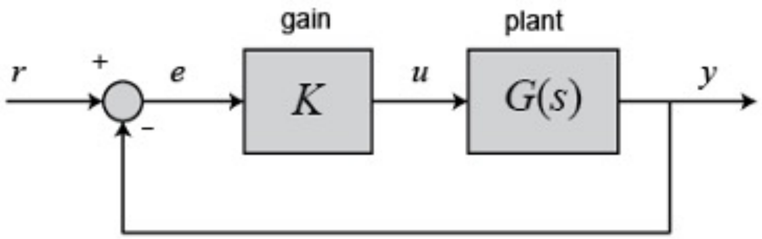

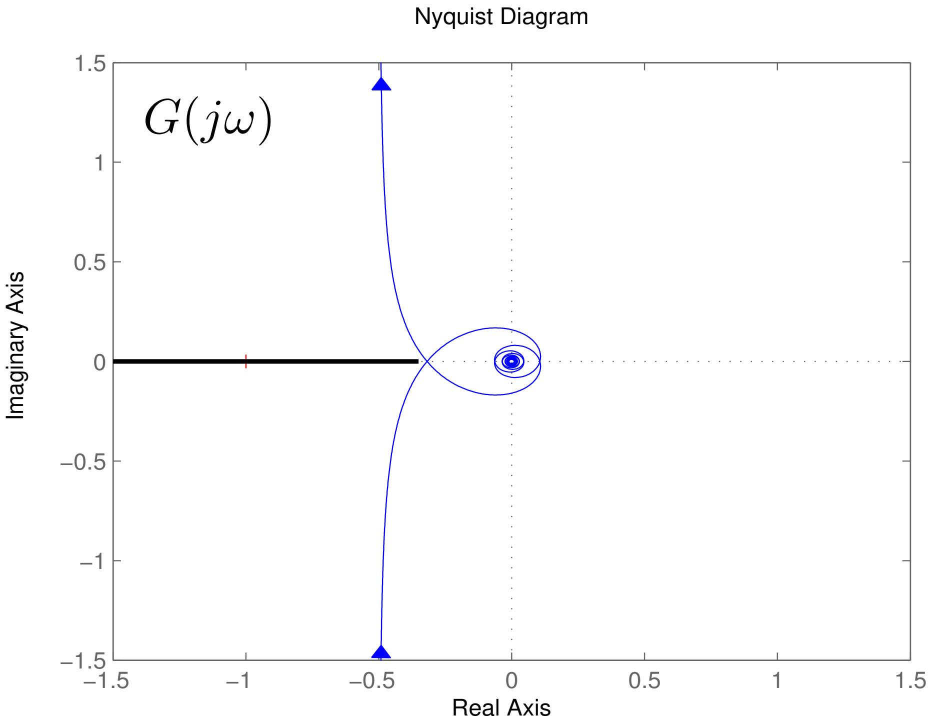

Stable for $0 \leq K \leq k^*$?

Assume: $G(s)$ is stable

Define: $h(s) \in PR$ (passive)

Test: If $h(s)\big(1 + k^* G(s)\big) \in SPR$ (strictly) then, **yes!**

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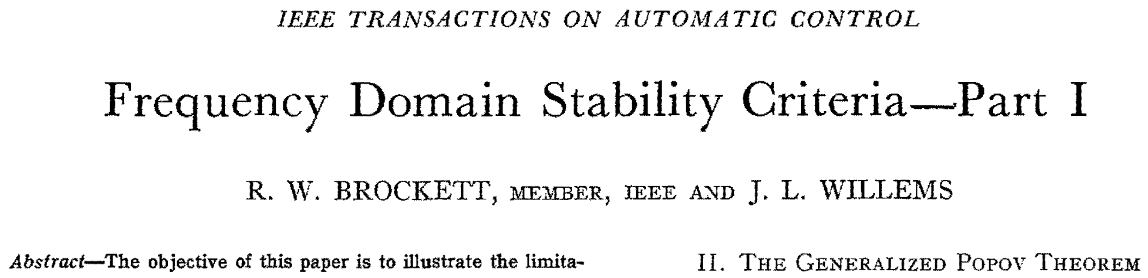
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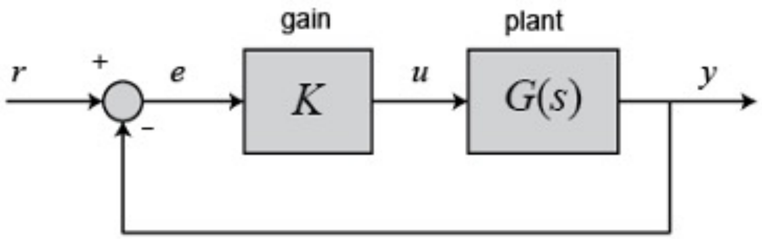

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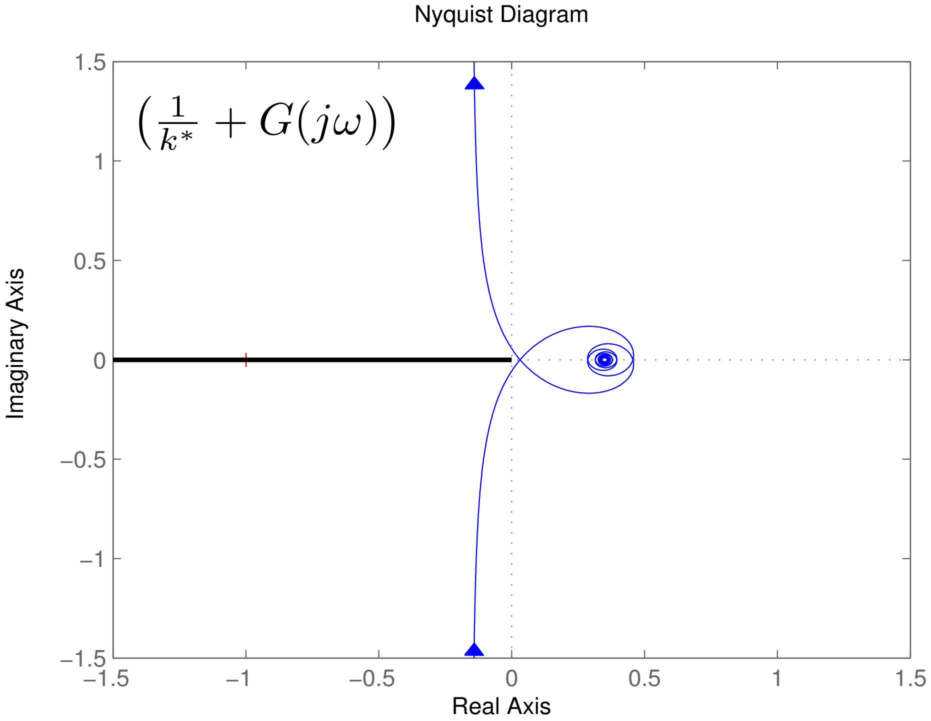

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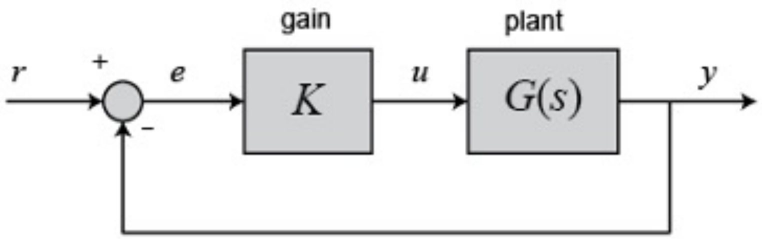

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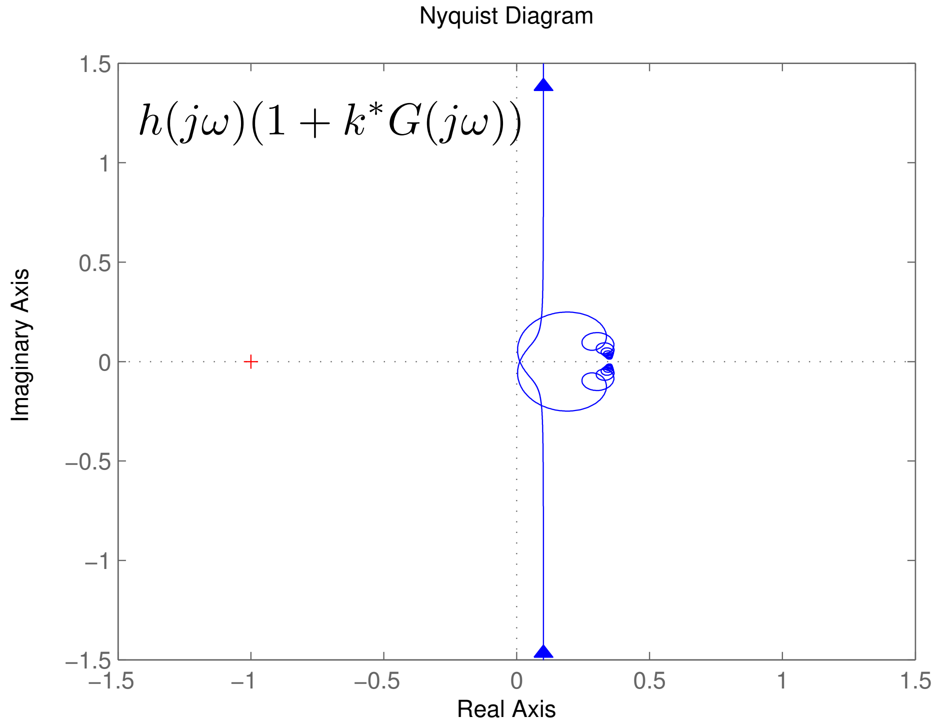

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Scale-free Stability Analysis

Key Idea: Exploit limited network information to relax passivity condition

- Let γ_i be a local connectivity bound: $\sum_{j \in N_i} L_{ij} \leq \frac{\gamma_i}{2}$

$$L_{ij} = -b_{ij} v_i v_j \cos(\theta_i^* - \theta_j^*)$$

Brockett & Willems '65

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Pates & M 2019

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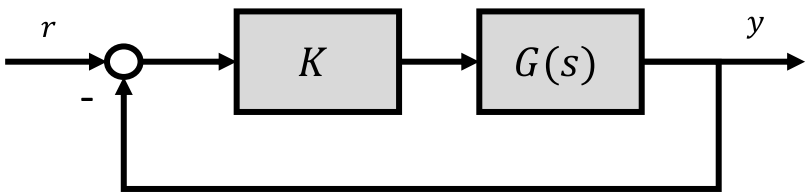

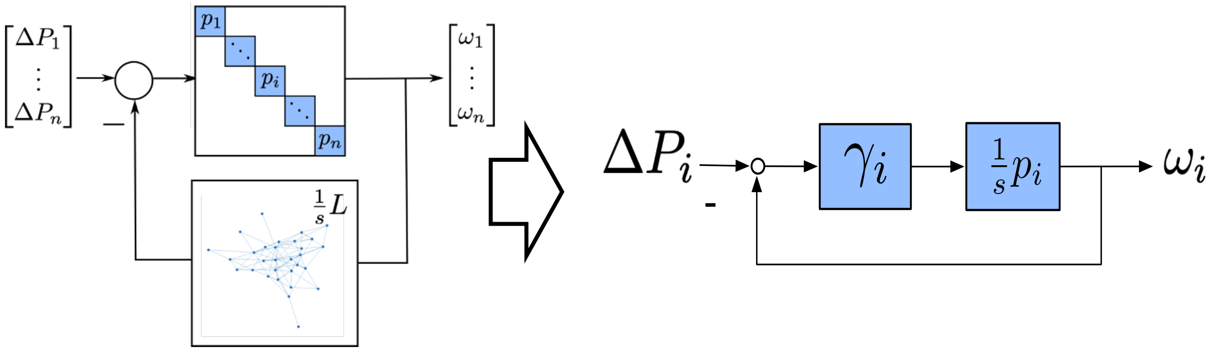

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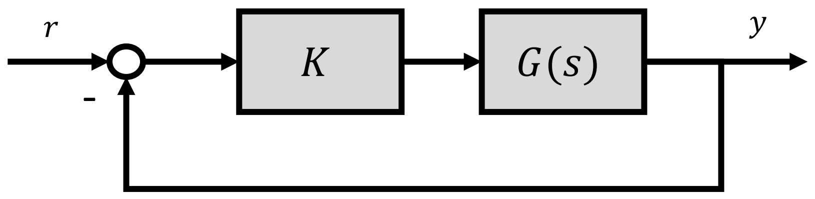

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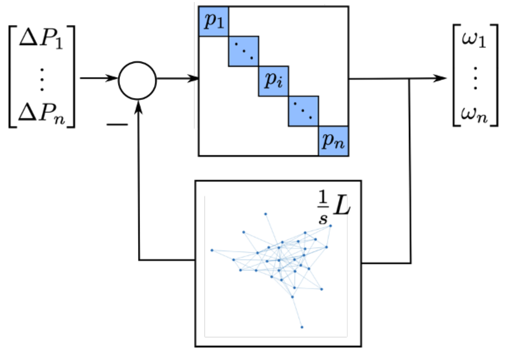

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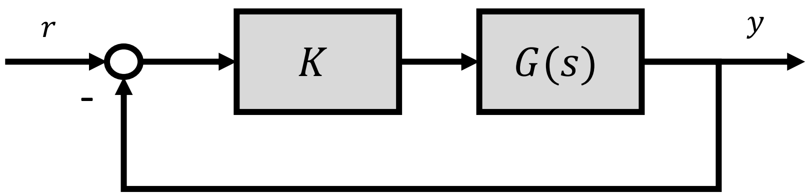

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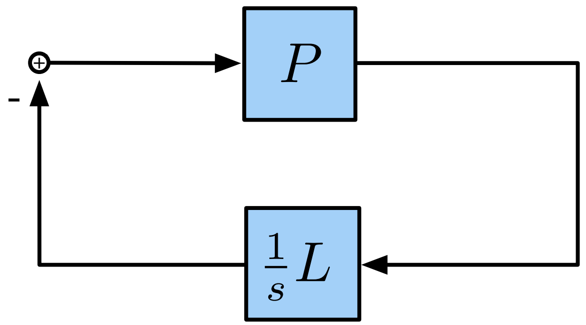

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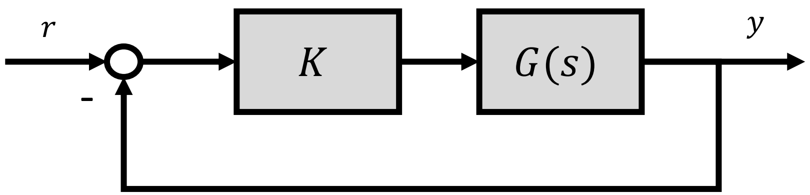

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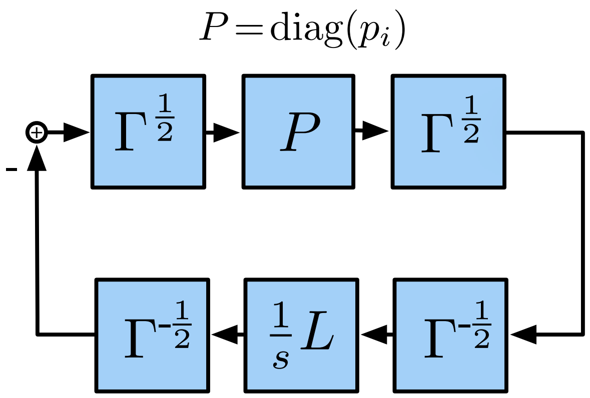

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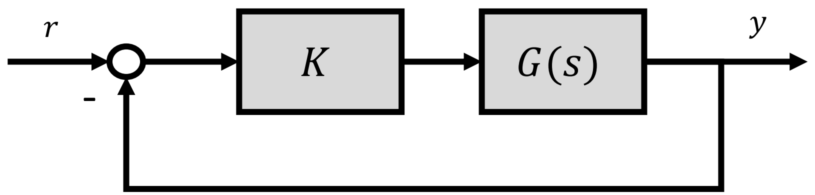

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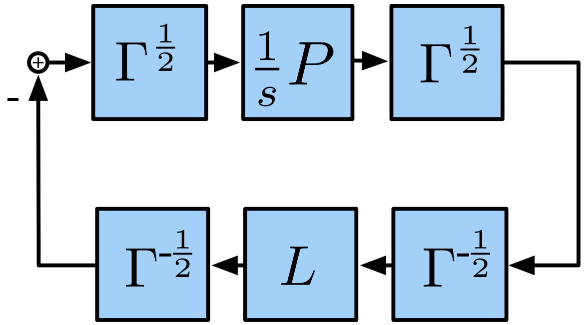

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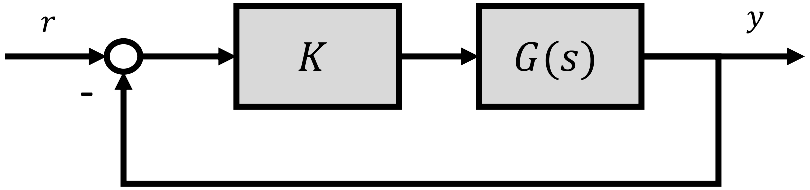

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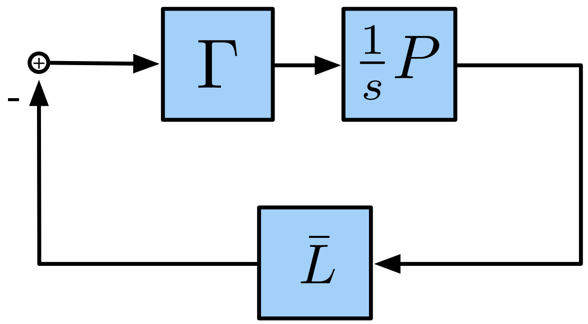


$$0 \preceq \bar{L} := \Gamma^{-\frac{1}{2}} L \Gamma^{-\frac{1}{2}} \preceq I$$

Examples

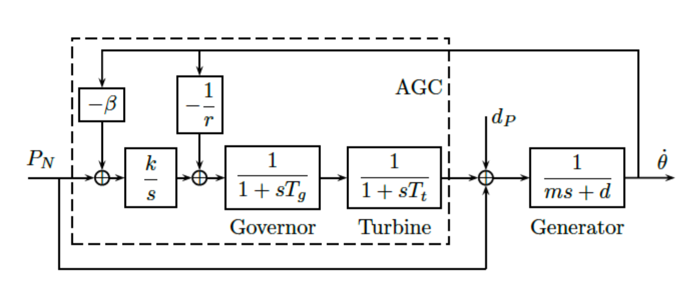
Delay Robustness of Swing Equations

Let $p_i(s) = \dfrac{1}{M_i s + D_i e^{-\tau_i s}}$

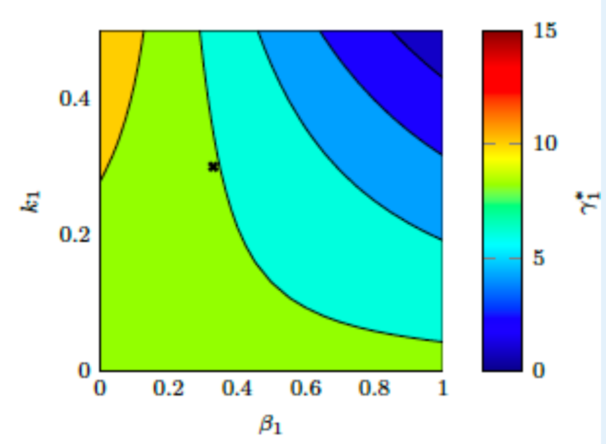
Given $\tau^* < \frac{\pi}{2}$, then, for any network such that $2\sum_{j\in N_i}^{n} L_{ij} < \gamma^*$ with $\gamma^* \approx \dfrac{\pi M_i(\frac{\pi}{2} - \tau^*)}{2\left(\frac{M_i \tau^*}{D_i}\right)^2}$

the delayed swing equations are stable for whenever $\tau_i \leq \tau^* \frac{M_i}{D_i}$

Automatic Generation Control



m	d	T_g	T_t	r	β	k
0.16	0.02	0.08	0.40	3.00	0.33	0.30



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Outline

- **Merits and trade-offs of low inertia**
  - Control Perspective: Lighter systems are easier to control!
- **Scale-free Stability Analysis of Grids**
  - Generalizes passivity notions using network information
- **Analysis of Weakly-Connected Coherent Networks**
  - Generalized Center of Inertia captures IBR dynamics
  - Identification of coherent modes via spectral clustering
- **Grid Shaping Control**
  - Grid-following/forming control framework for controlling future girds

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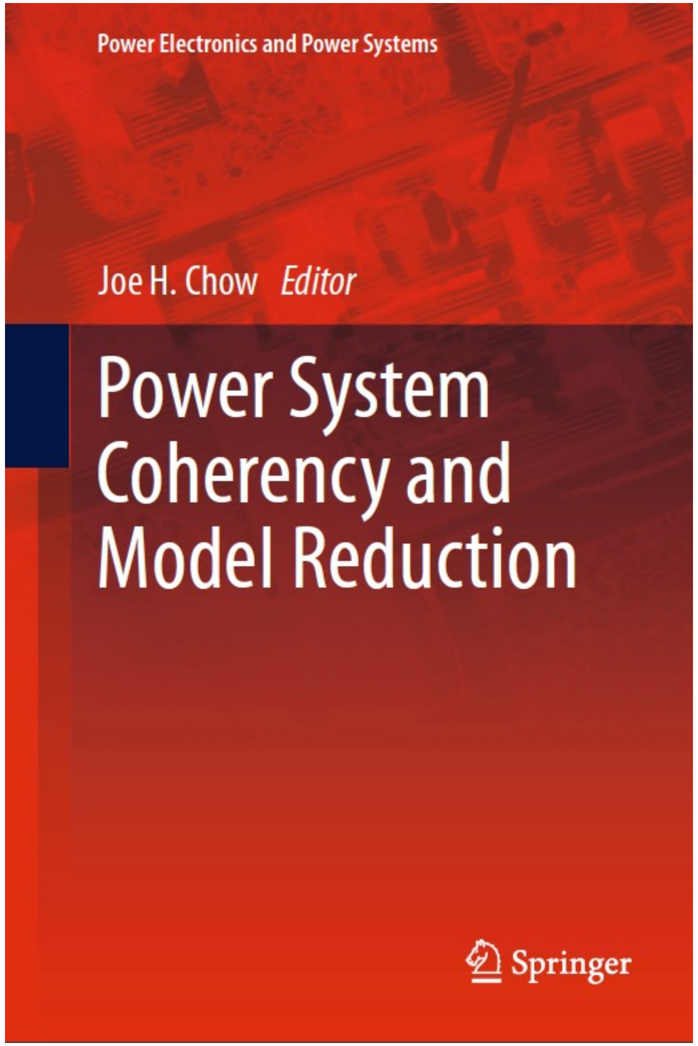
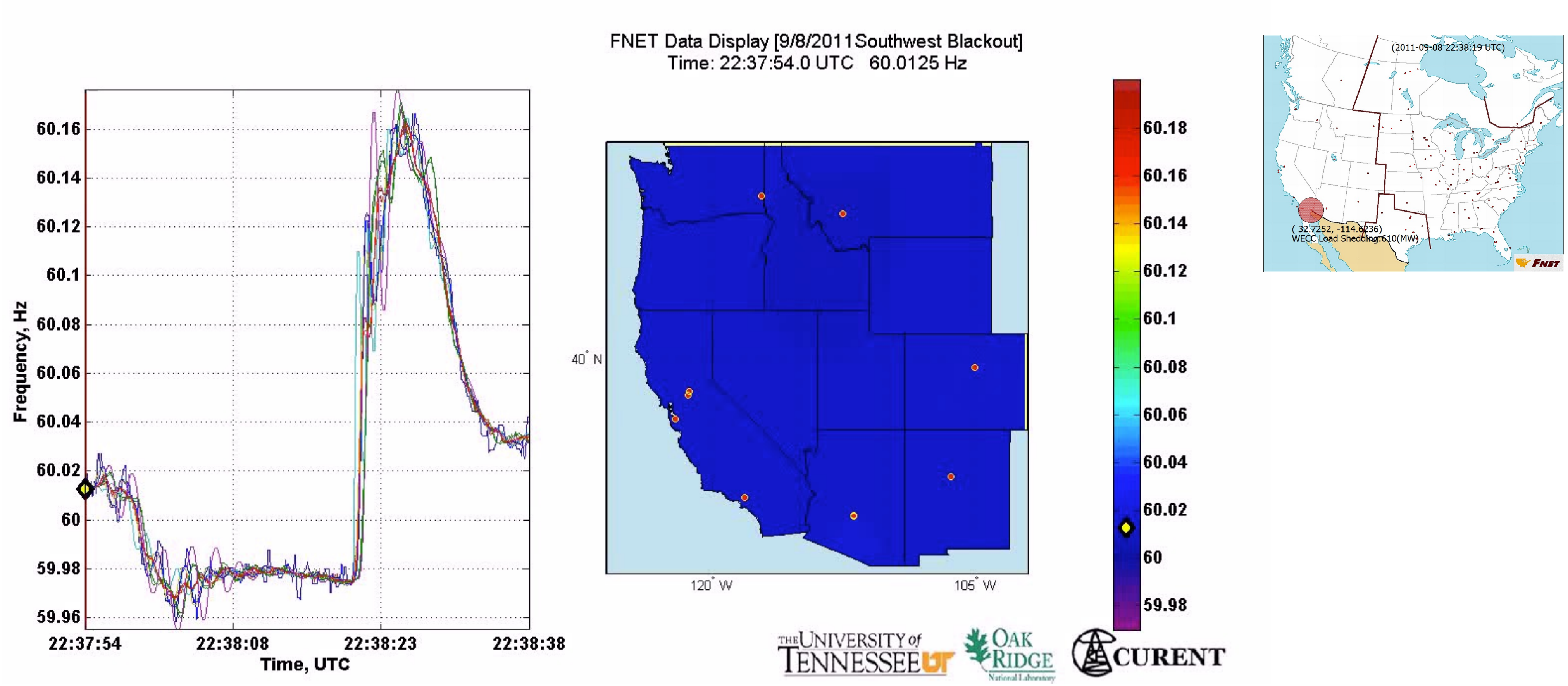
Coherence in Power Networks

- Studied since the 70s
  - Podmore, Price, Chow, Kokotovic, Verghese, Pai, Schweppe,…
- Enables aggregation/model reduction
  - Speed up transient stability analysis
- Many important questions
  - How to identify coherent modes?
  - How to accurately reduce them?
  - What is the cause?
- Many approaches
  - Timescale separations (Chow, Kokotovic,)
  - Krylov subspaces (Chaniotis, Pai '01)
  - Balanced truncation (Liu et al '09)
  - Selective Modal Analysis (Perez-Arriaga, Verghese, Schweppe '82)



Goal: Understand how IBR presence affect classical coherence studies

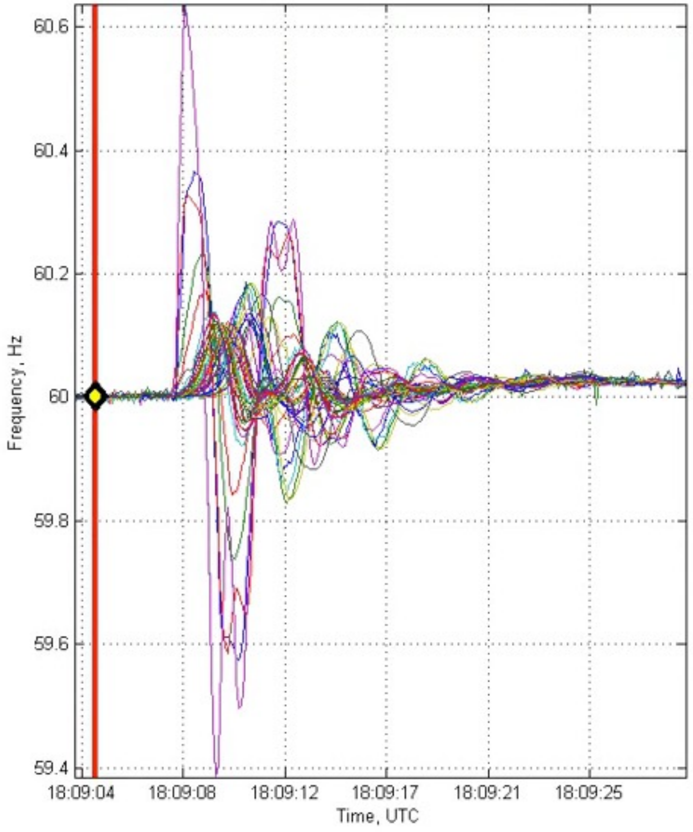
Case Study 1: Network Coherence

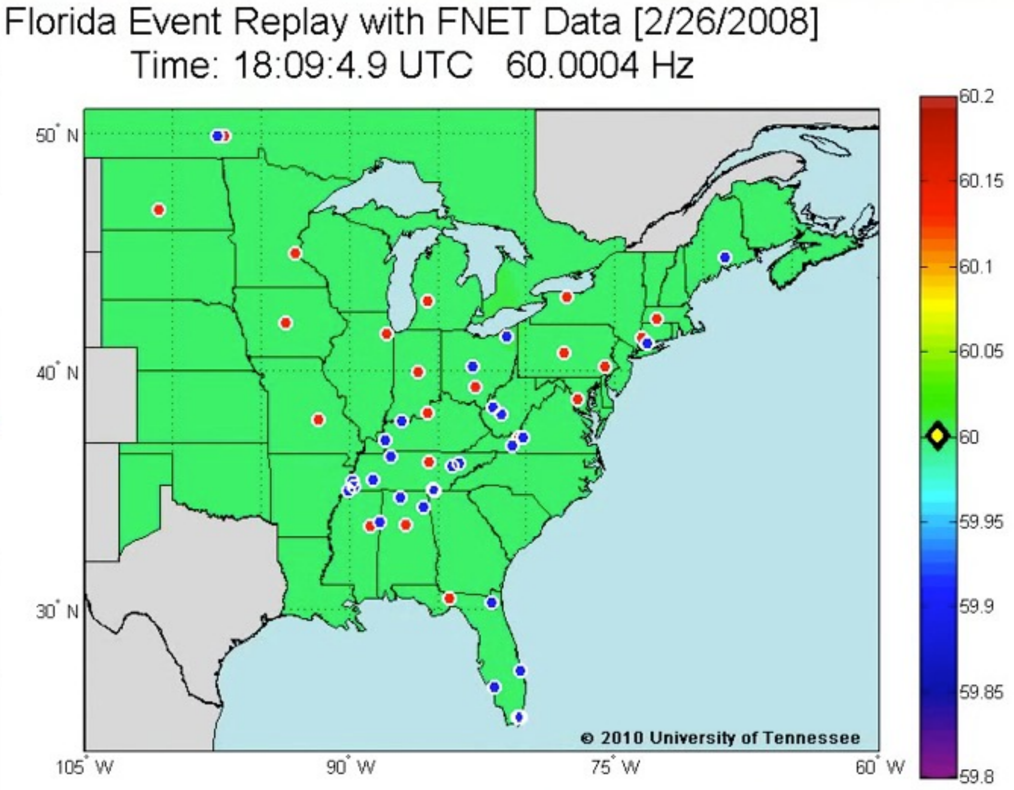


Key Questions:

- How does coherence emerge, and what does it depend on?
- How to characterize the coherent response in the presence of IBRs?

Case Study 2: Coherent Inter-area Modes





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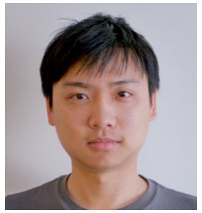
Key Questions:

- How to identify coherent areas?
- Can we model the inter-area oscillations?

Analysis of Coherent Dynamics [CDC 19, Auto 25]



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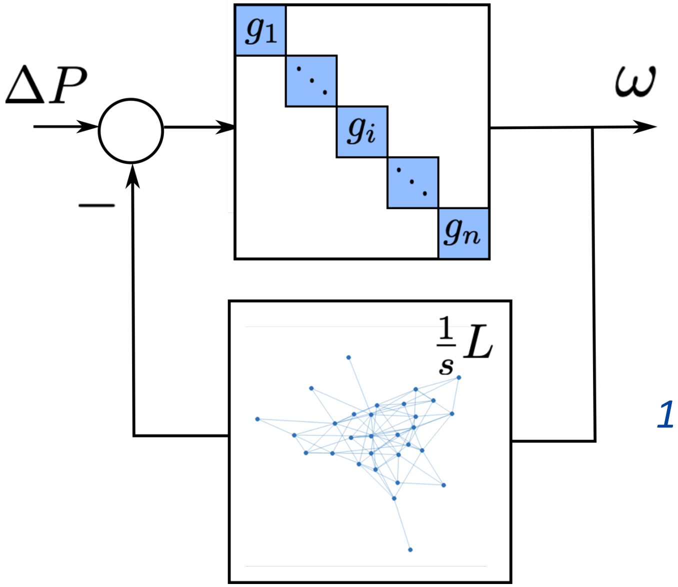

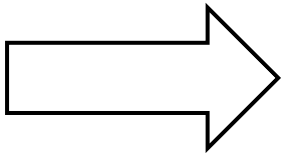

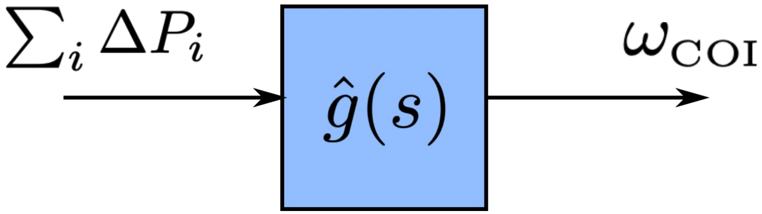

1.When does this network exhibit coherence?

*2.What is the exact **coherent response** of this network?*

- **Problem Setup:**
- Linearized power flows L_{ij}
- Bus i: arbitrary siso tf: $\omega_i = g_i(s)\,\Delta P_i$ (SGs or IBRs)

Example I: SG + Turbine

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{R_i^{-1}}{\tau s + 1}}$$

Example II: IBRs

$$g_i(s) = \frac{1}{\nu_i s + R_i^{-1}}$$

[CDC 19] Min, M. Dynamics concentration of large-scale tightly-connected networks. **Conference on Decision and Control 2019**
[Automatica 25] Min, Pates, M. A frequency domain analysis of slow coherency in networked systems. **Automatica 2025**

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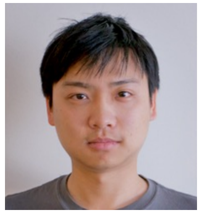

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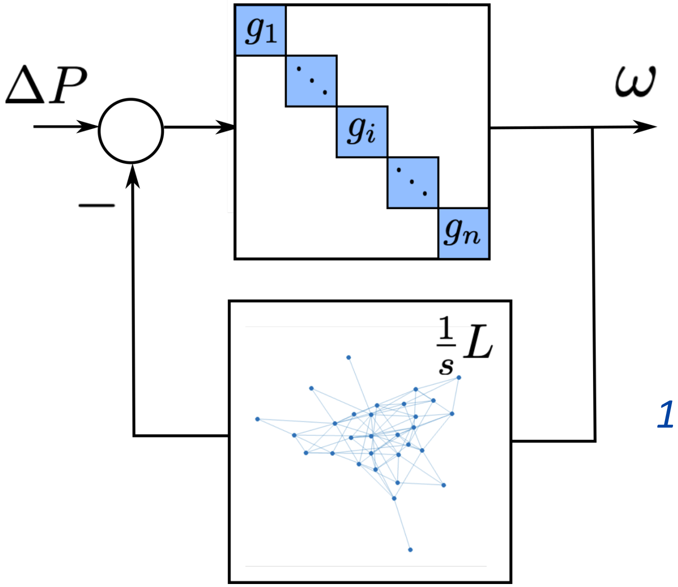

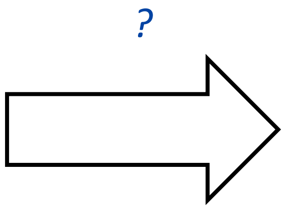

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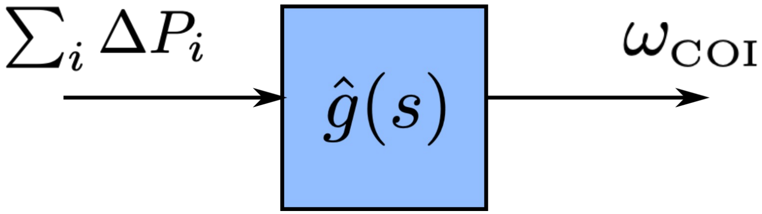

*2. What is the exact **coherent response** of this network?*

1. Coherence can be understood as a **low rank** property the **closed-loop transfer matrix**

2. It emerges as the **effective algebraic connectivity** $\left|\frac{1}{s_0}\lambda_2(L)\right|$ increases

3. The coherent dynamics is given by the **harmonic sum** of bus dynamics

$$\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

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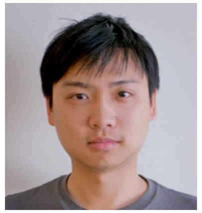
Generalized Center of Inertia [CDC 19, Auto 25]

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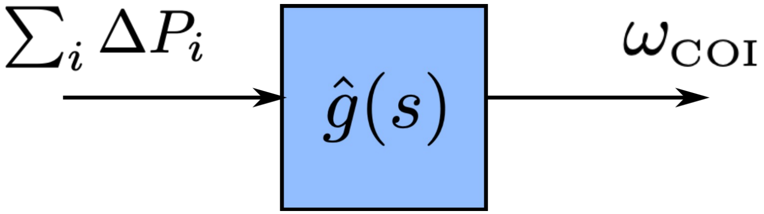
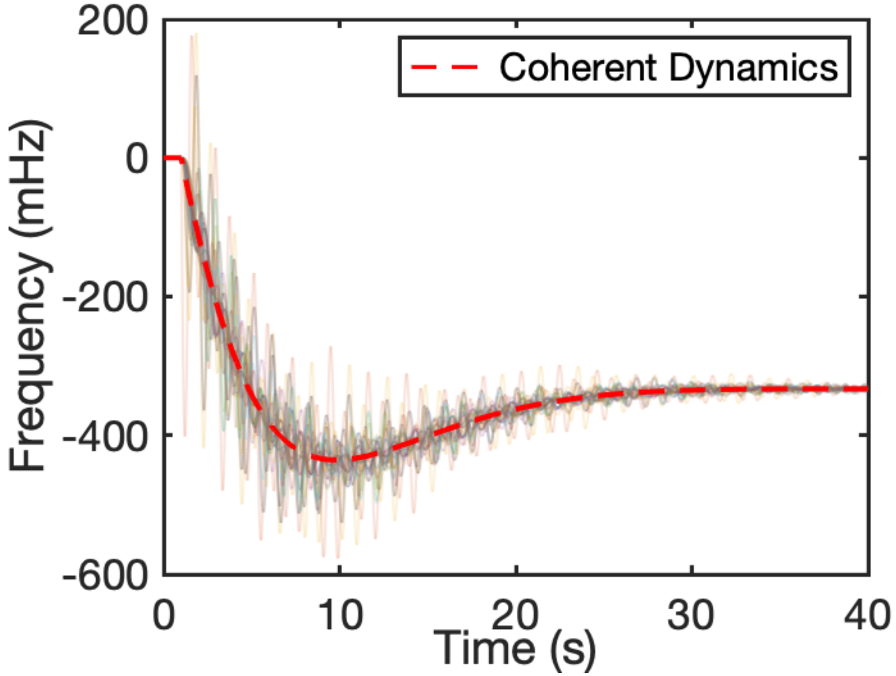


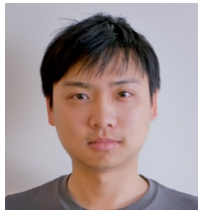


$$\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

- **Coherent Dynamics: $\hat{g}(s)$**
- Representation of aggregate response
- Accuracy of approximation:
  - is frequency dependent
  - increases with network connectivity
- Provides excellent template for reduced order models (via balance-truncations)
- More details [LCSS 20]

[CDC 19] Min, M. Dynamics concentration of large-scale tightly-connected networks. **Conference on Decision and Control 2019**
[LCSS 20] Min, Paganini, M. Accurate reduced-order models for heterogeneous coherent generators. **IEEE LCSS 2020**
[Auto 25] Min, Pates, M. A frequency domain analysis of slow coherency in networked systems. **Automatica 2025**

Weakly-Connected Coherent Networks



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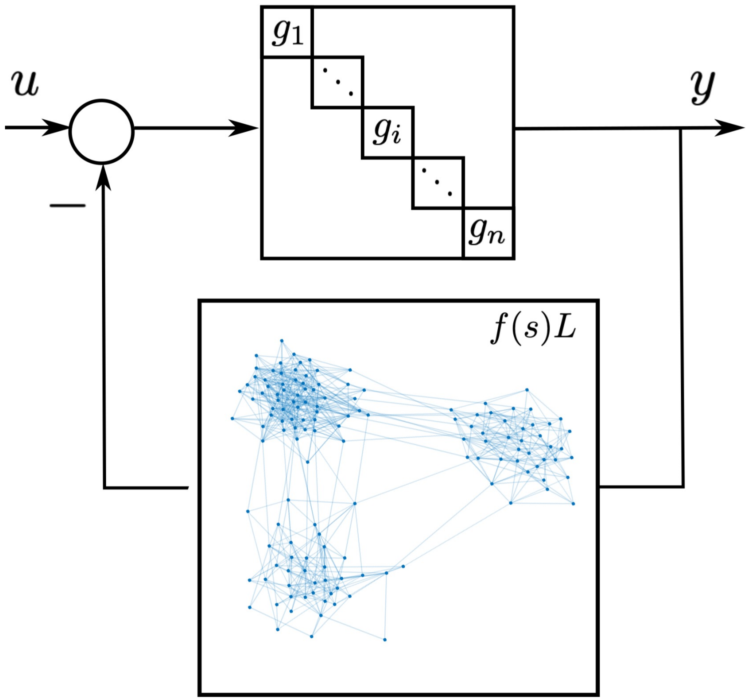
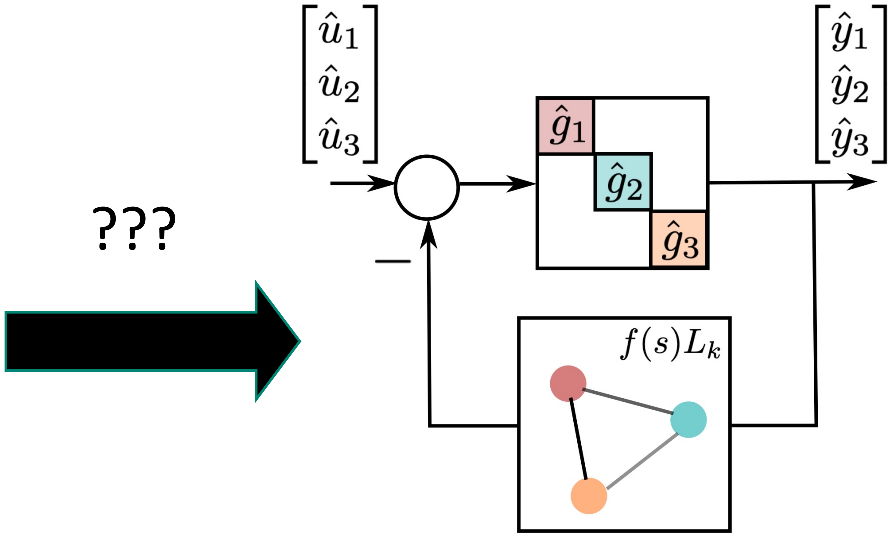
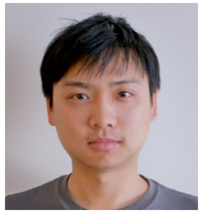




- Aggregate each coherent area
- Inter-area oscillation can be modeled as the interaction among aggregate nodes

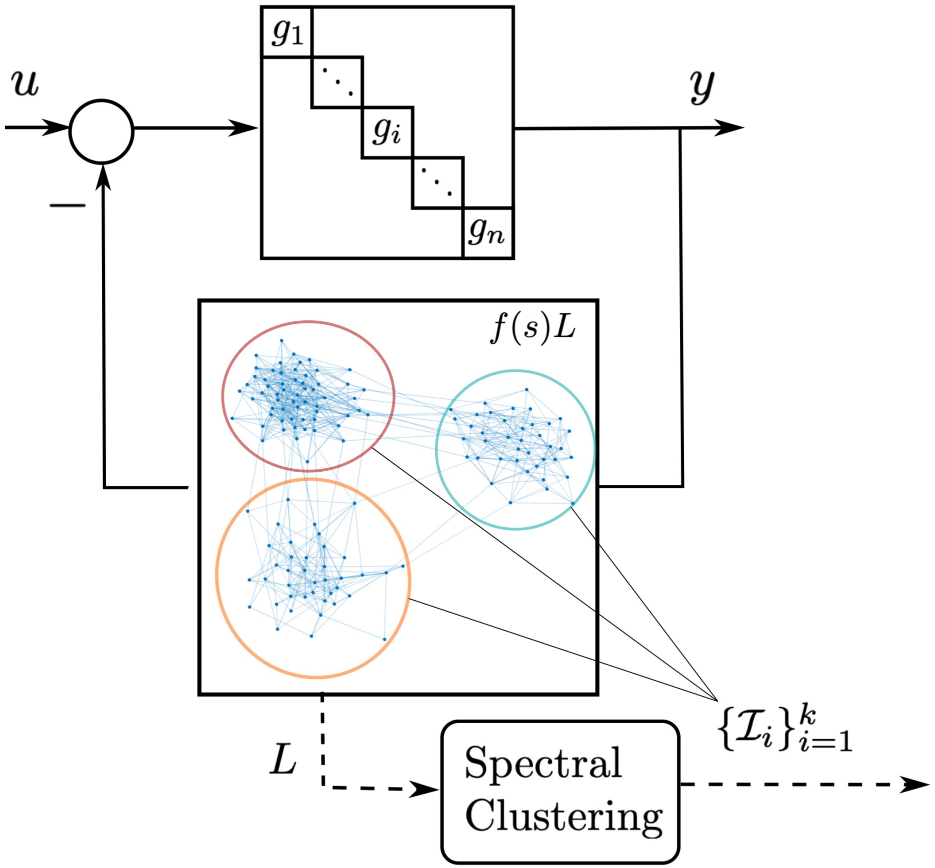
[L4DC 23] Min, M. Learning coherent clusters in weakly-connected network systems. **Leaning for Dynamics and Control 2023**

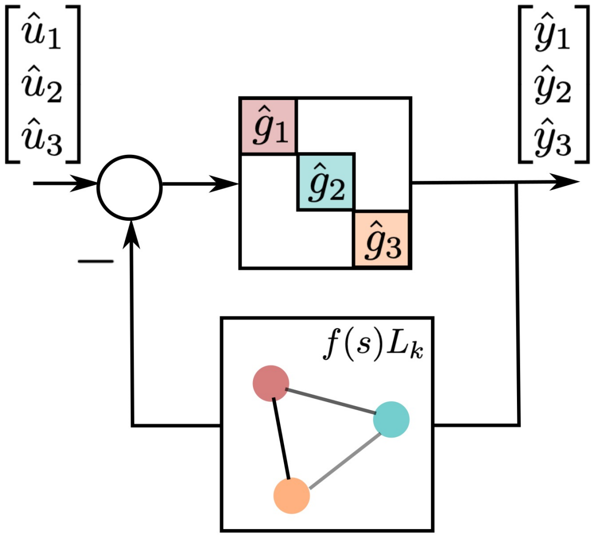
Structure-preserving Network Reduction



Hancheng Min

Step 1: **Identifying** coherent areas





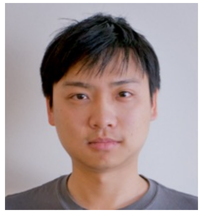
Tightly-connected Networks are coherent

$\Downarrow$

Use **spectral clustering** algorithm to **find** tightly-connected **subnetworks/areas**

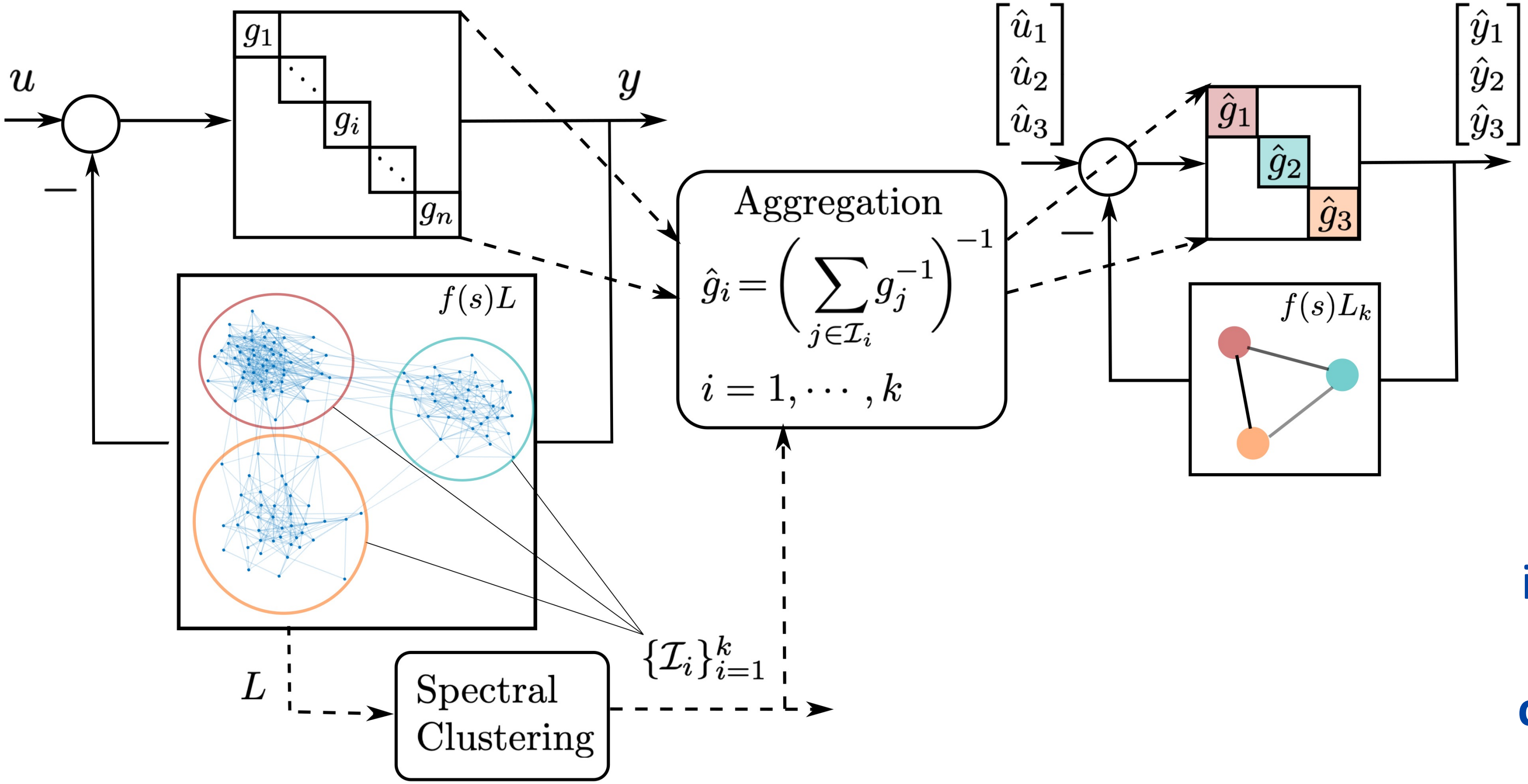
[L4DC 23] Min, M. Learning coherent clusters in weakly-connected network systems. **Leaning for Dynamics and Control 2023**

Structure-preserving Network Reduction



Hancheng Min

Step 2: **Aggregate** coherent areas



Coherent dynamics are given by

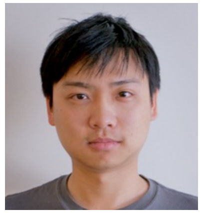
$$\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

$\Downarrow$

Aggregate each identified **coherent area** into its corresponding **coherent dynamics** $\hat{g}(s)$

[L4DC 23] Min, M. Learning coherent clusters in weakly-connected network systems. **Leaning for Dynamics and Control 2023**

Structure-preserving Network Reduction



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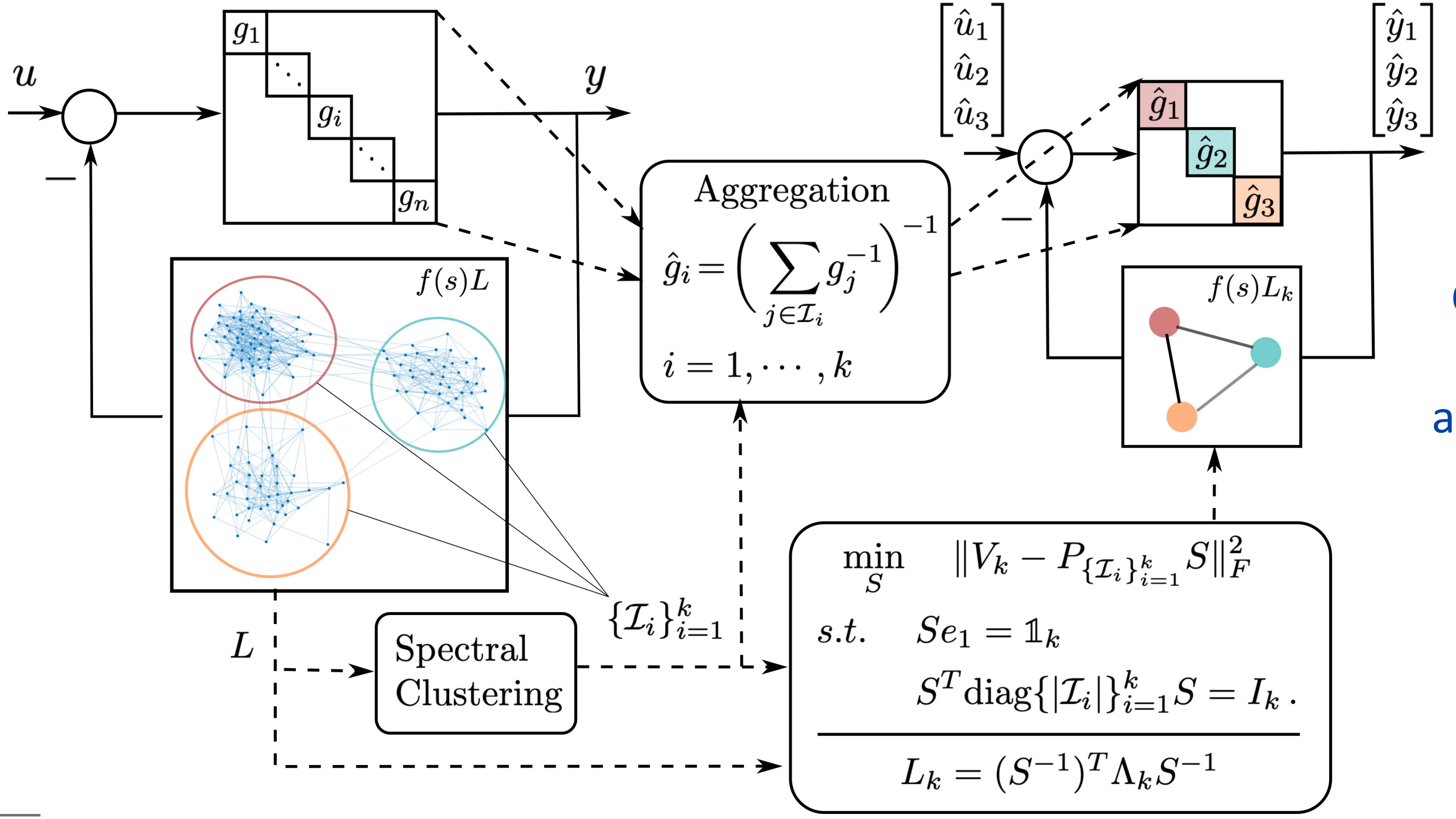
Step 3: Model the **interaction** among aggregate nodes

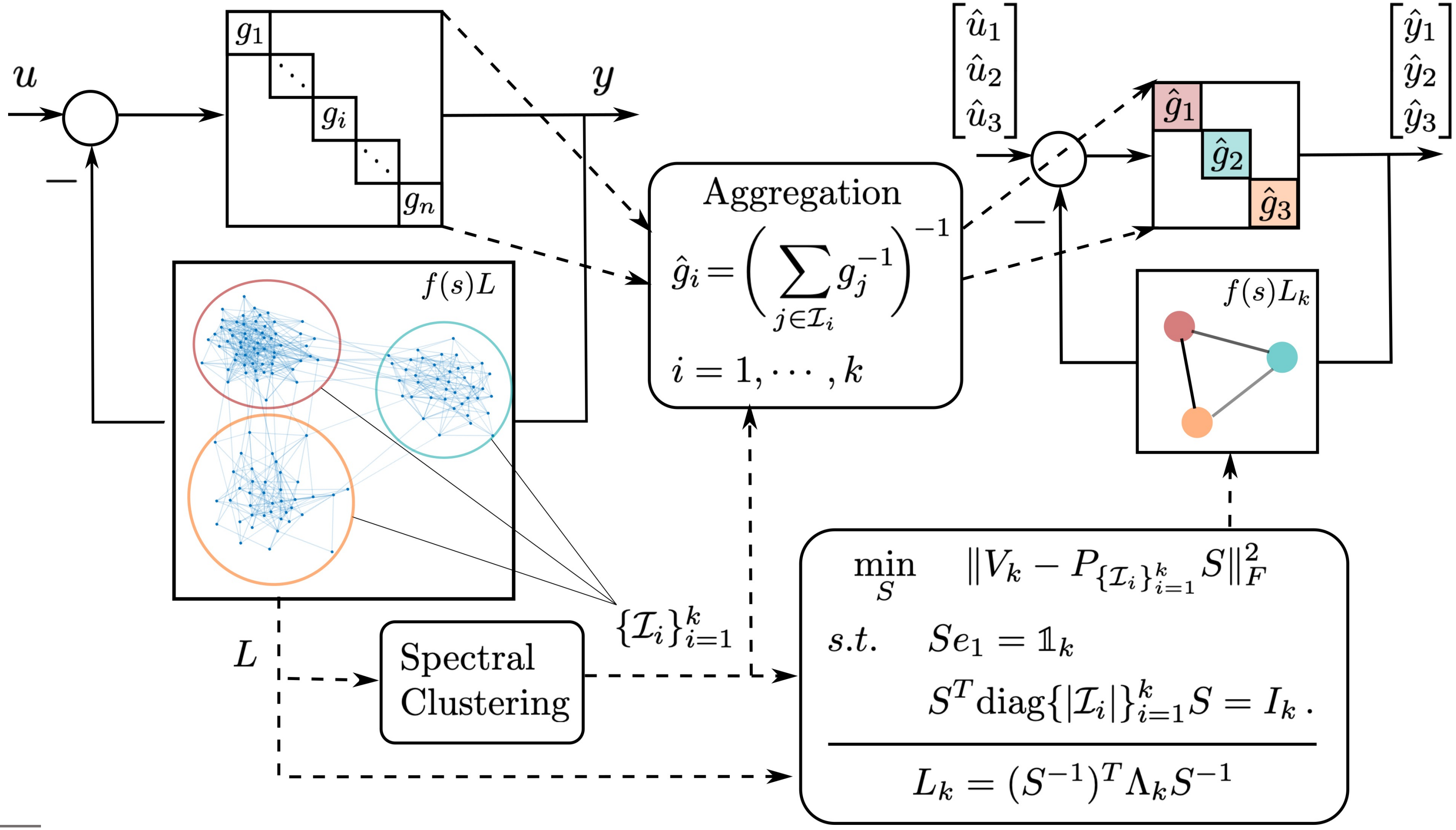


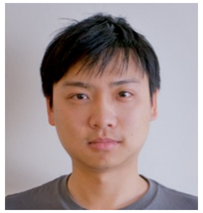
Construct the **reduced network** L_k by solving an optimization problem (it has closed-form solution)

[L4DC 23] Min, M. Learning coherent clusters in weakly-connected network systems. **Leaning for Dynamics and Control 2023**

Approximation Errors

$$\left\|T(s_0) - \hat{T}_k(s_0)\right\|_2 = \mathcal{O}\left(\frac{1}{\lambda_{k+1}(L)}\right) + \mathcal{O}\left(\left\|V_k(L) - P_{\{I_i\}_{i=1}^k} S\right\|_2\right)$$





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Approximation error depends on:

- Whether the network has a multi-cluster structure
- Whether the SC algorithm finds the right clusters
- How well one model the interaction

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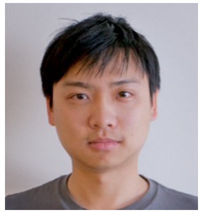
Numerical validation – RTS 96 test case



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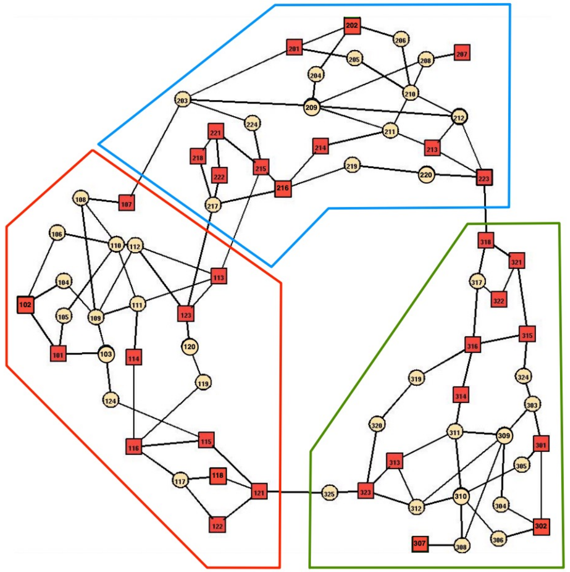

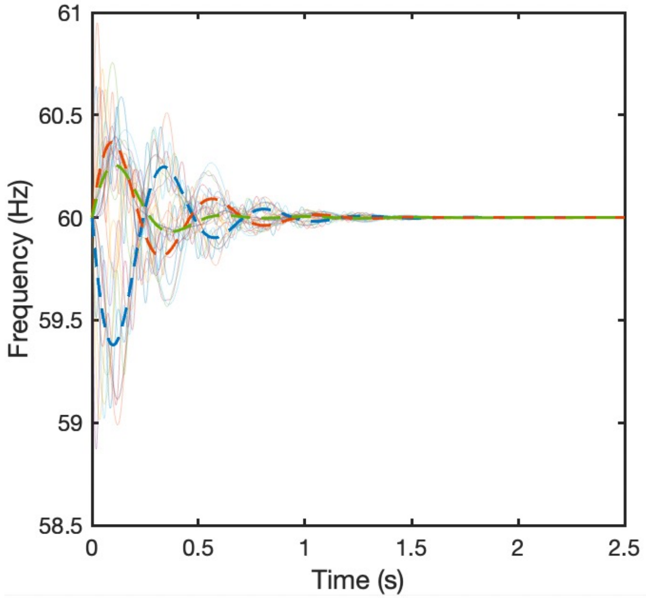

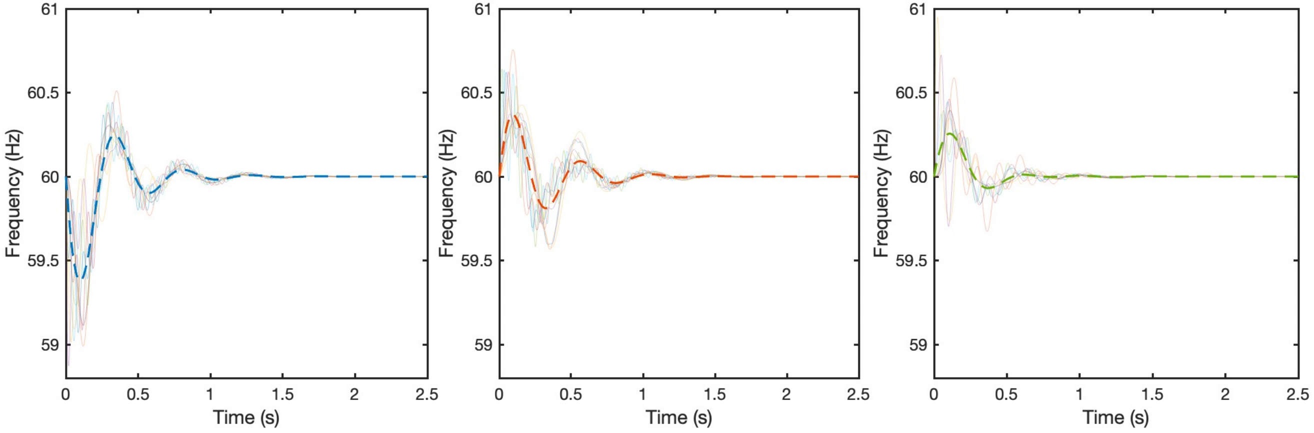

- The IEEE reliability test system: 1996
- 3 areas, 33 generators in total
- Different rotor angles across each area at initialization
- Solid lines: actual frequency response
  Dashed lines: reduced model

[L4DC 23] Min, M. Learning coherent clusters in weakly-connected network systems. **Leaning for Dynamics and Control 2023**

Outline

- **Merits and trade-offs of low inertia**
  - Control Perspective: Lighter systems are easier to control!
- **Scale-free Stability Analysis of Grids**
  - Generalizes passivity notions using network information
- **Analysis of Weakly-Connected Coherent Networks**
  - Generalized Center of Inertia captures IBR dynamics
  - Identification of coherent modes via spectral clustering
- **Grid Shaping Control**
  - Grid-following/forming control framework for controlling future girds

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Grid Shaping Control

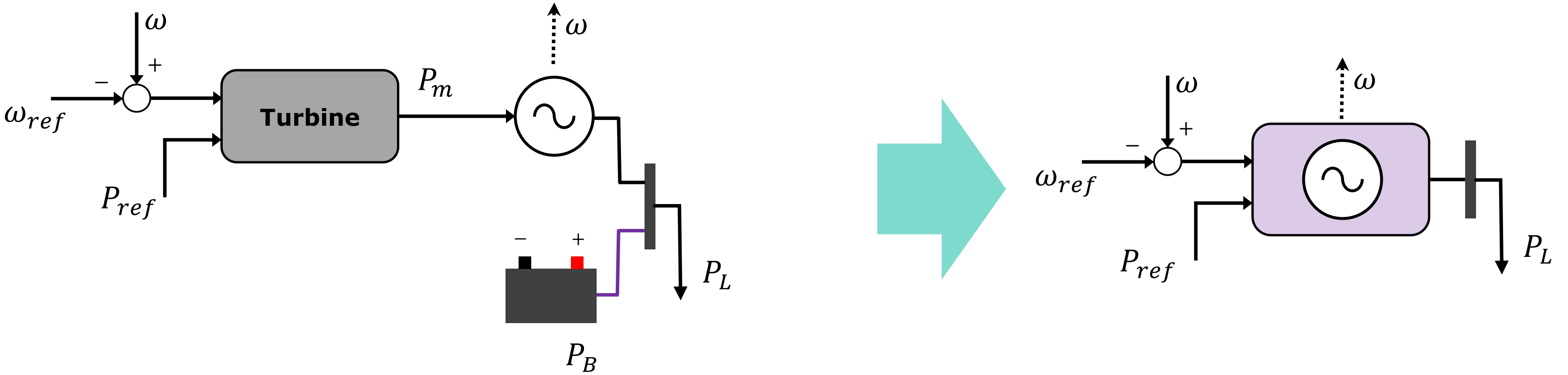
Use model matching control to shape system response

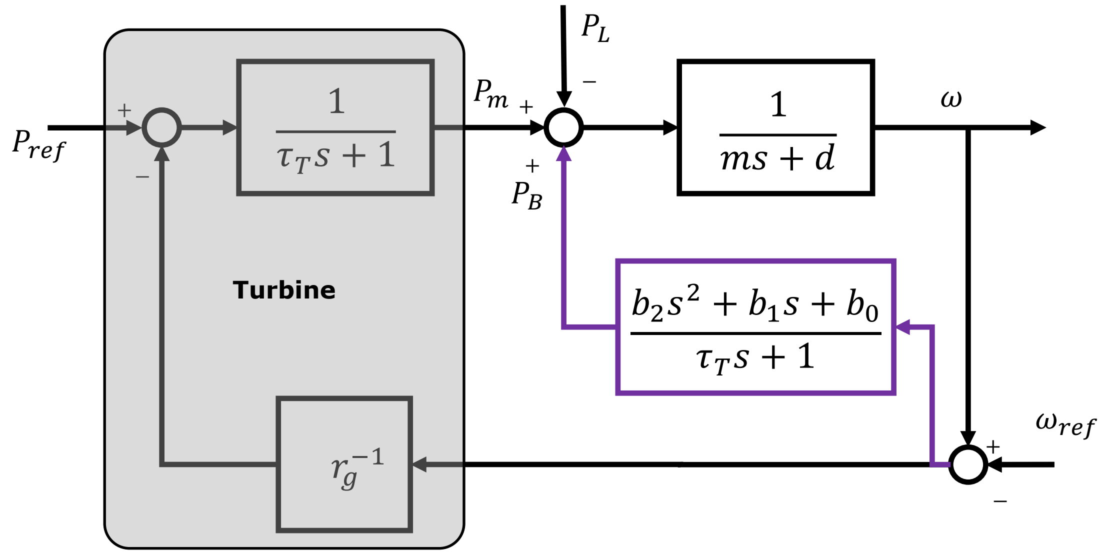
Grid-following IBRs

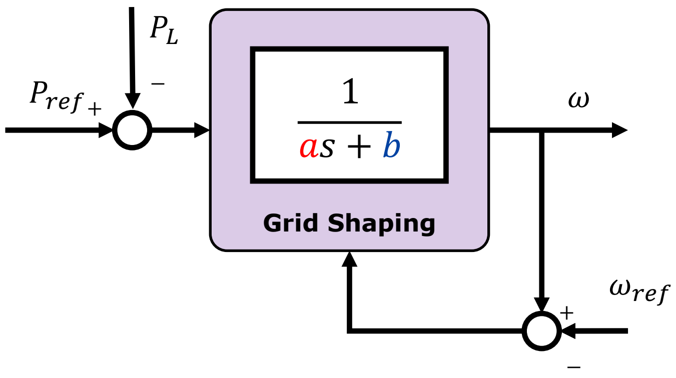
Grid-forming IBRs

Grid-shaping with GFL IBRs [TPS 21]

Yan Jiang Eliza Cohn Petr Vorobev







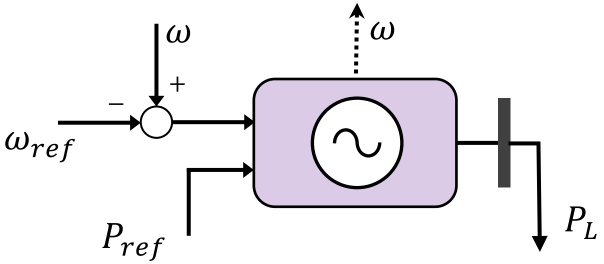
Tunable Performance:

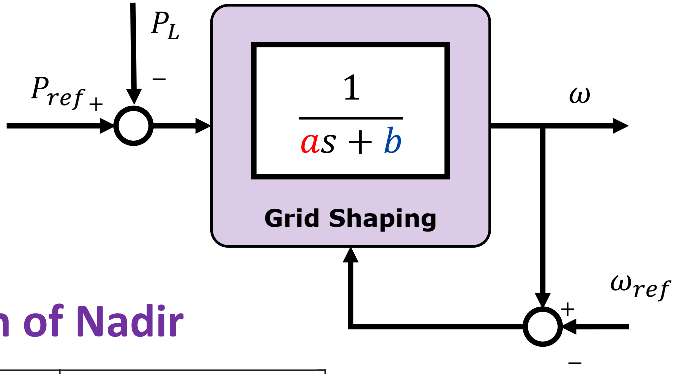
$$\text{RoCoF} = \frac{1}{a}\Delta P, \ \ \Delta\omega = \frac{1}{b}\Delta P$$

[TPS 21] Jiang, Cohn, Vorobev, M. Storage-based frequency shaping control **Transactions on Power Systems 2021**

Grid-shaping with GFL IBRs [TPS 21]



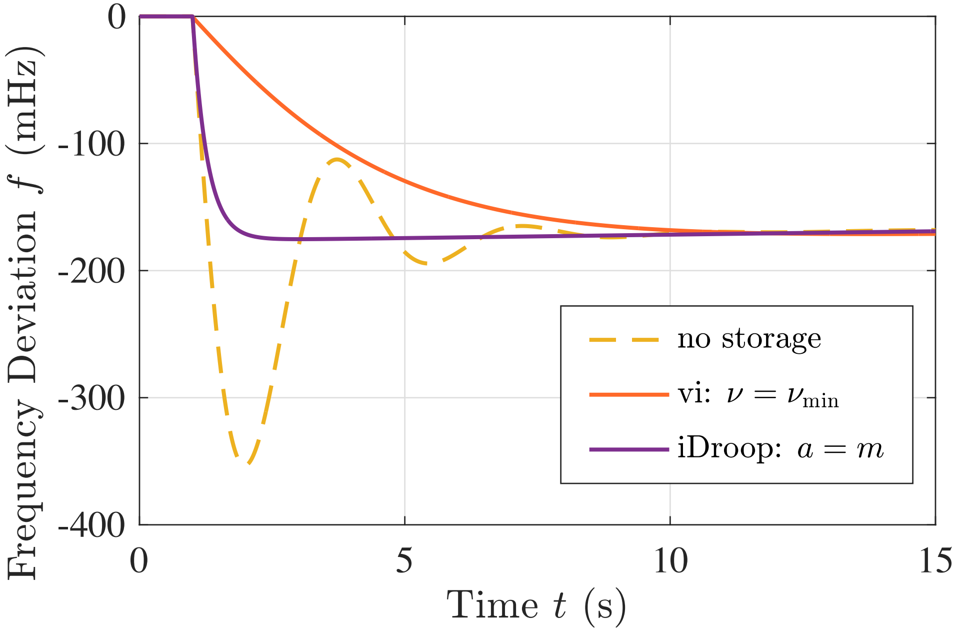


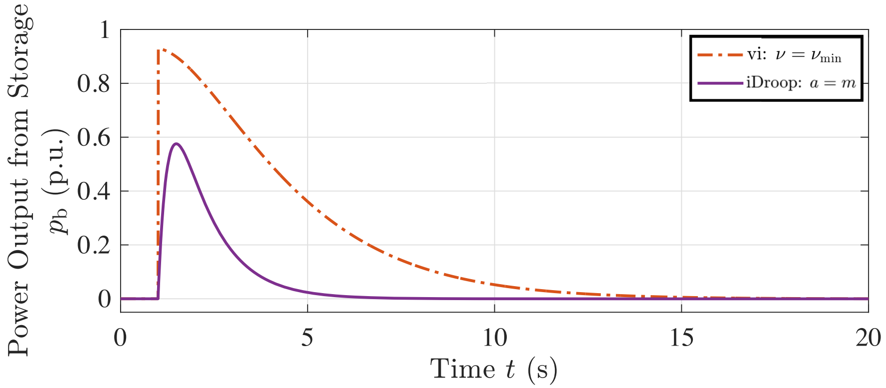


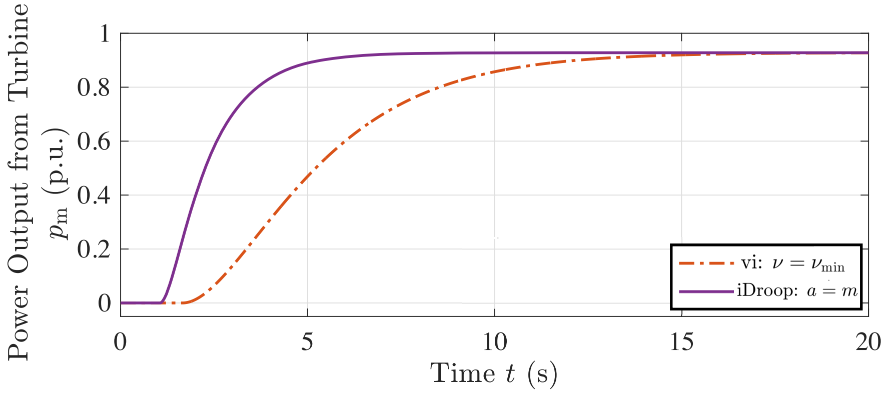
Tunable Performance:

$$\text{RoCoF} = \frac{1}{a}\Delta P, \ \Delta\omega = \frac{1}{b}\Delta P$$

Example: Efficient Elimination of Nadir

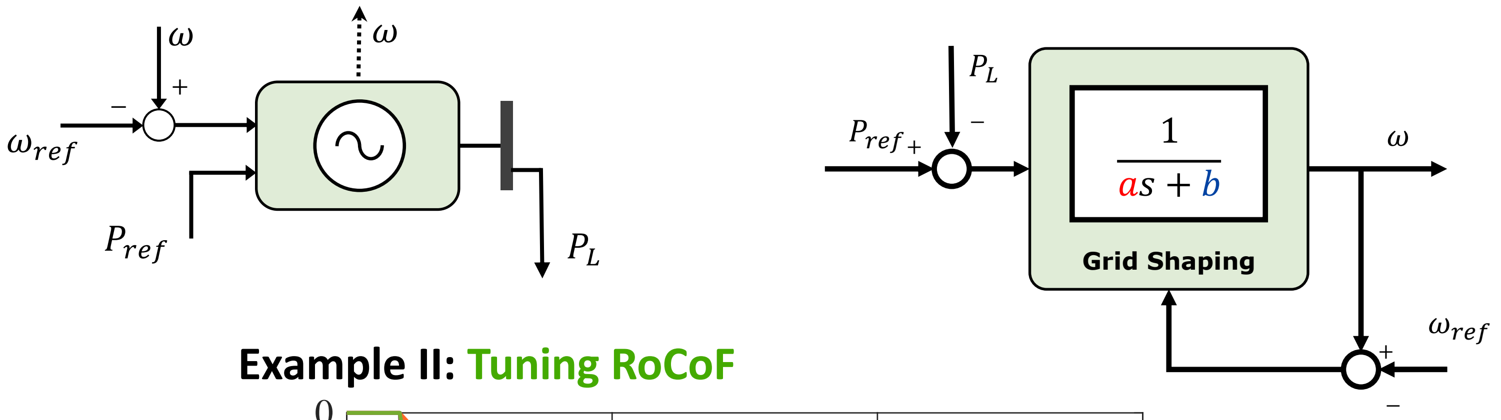






[TPS 21] Jiang, Cohn, Vorobev, M. Storage-based frequency shaping control **Transactions on Power Systems 2021**

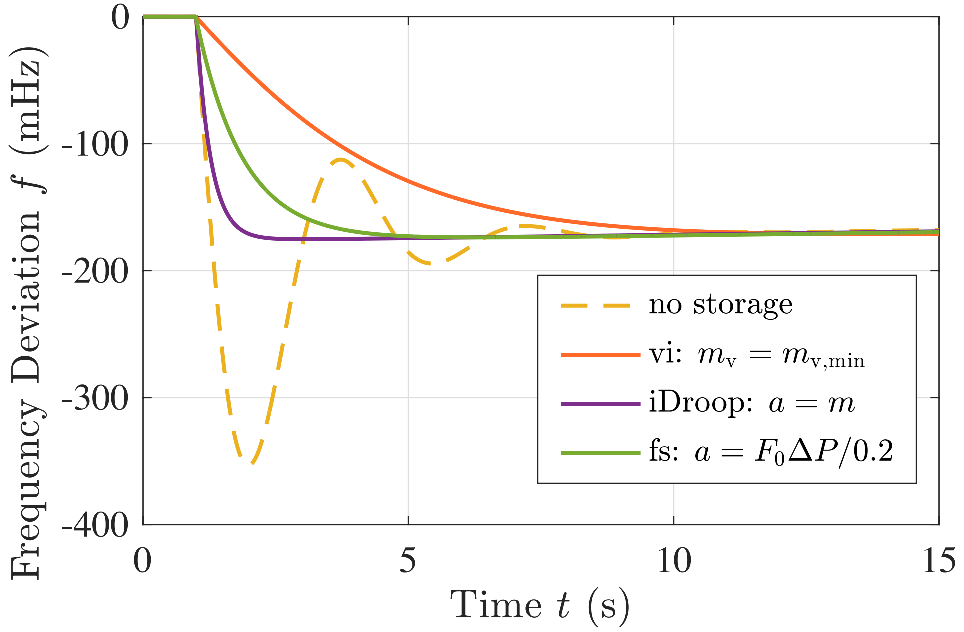
Grid-shaping with GFL IBRs [TPS 21]



Tunable Performance:

$$\text{RoCoF} = \frac{1}{a}\Delta P,\ \ \Delta\omega = \frac{1}{b}\Delta P$$

Example II: Tuning RoCoF

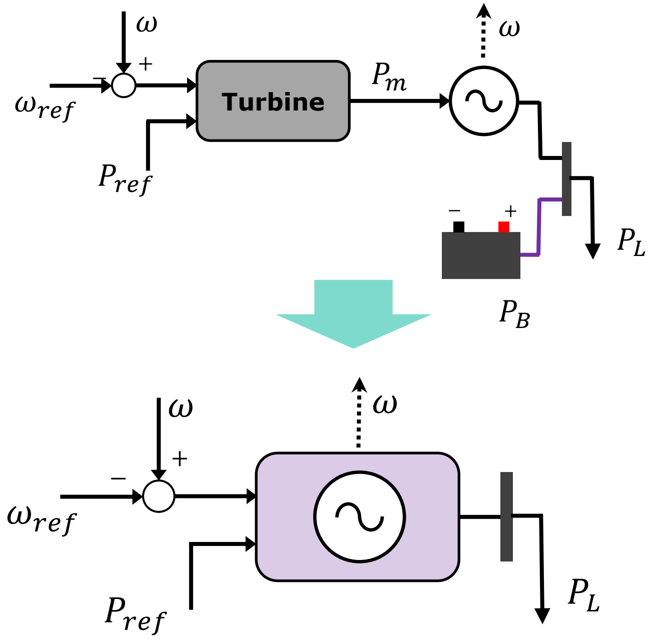


[TPS 21] Jiang, Cohn, Vorobev, M. Storage-based frequency shaping control **Transactions on Power Systems 2021**

Grid Shaping Control

Use model matching control to shape system response

Grid-following IBRs



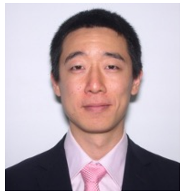
Tunable Performance:

$$\text{RoCoF} = \frac{1}{a}\Delta P,\ \ \Delta\omega = \frac{1}{b}\Delta P$$

Grid-forming IBRs

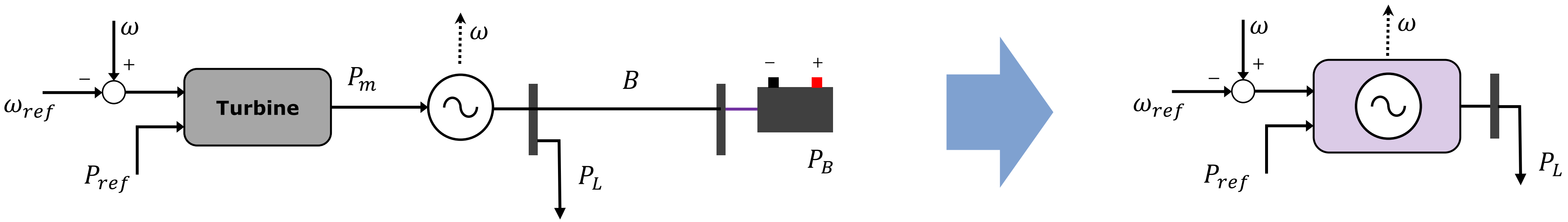
GFM Grid-shaping Through Lines [LCSS 23]

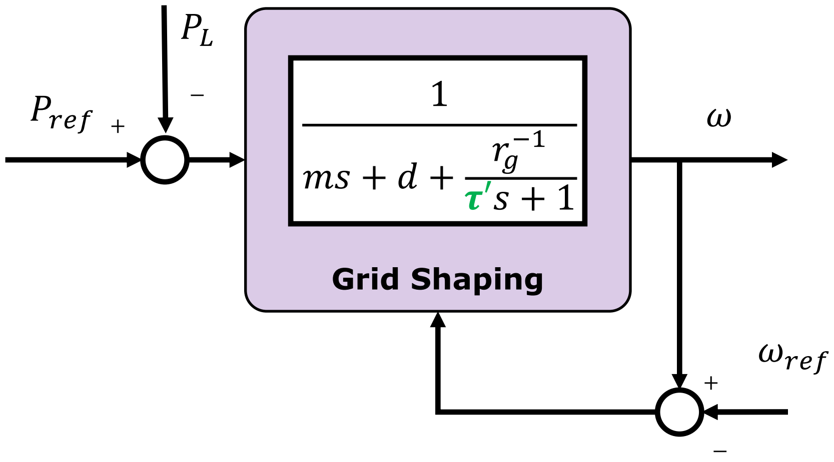

B. K. Poolla

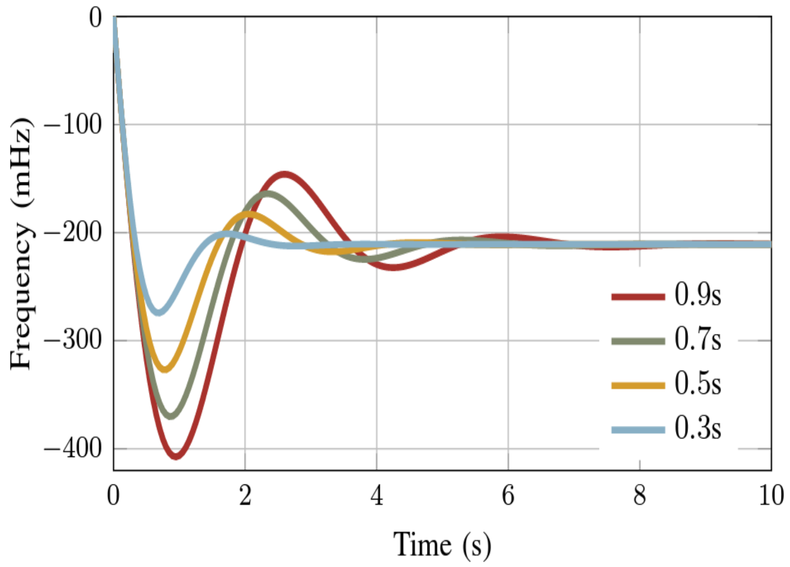

Y. Lin


A. Bernstein

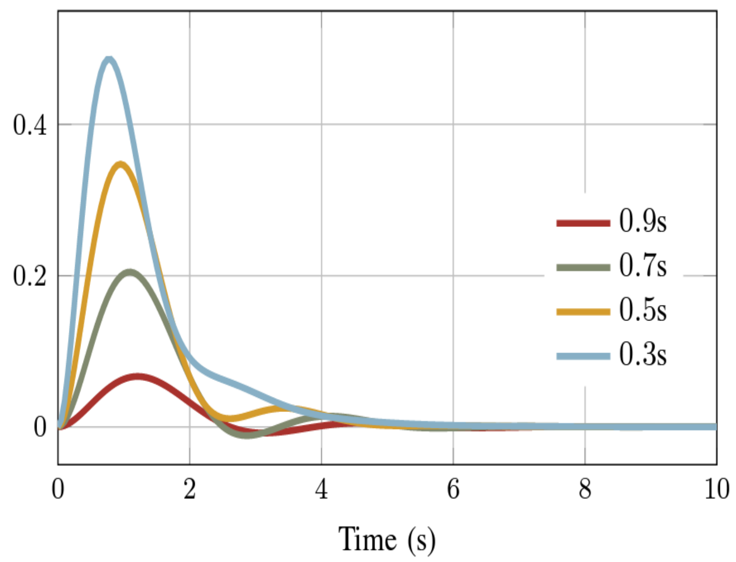

D. Groß







Frequency response for a 1 p.u. load step



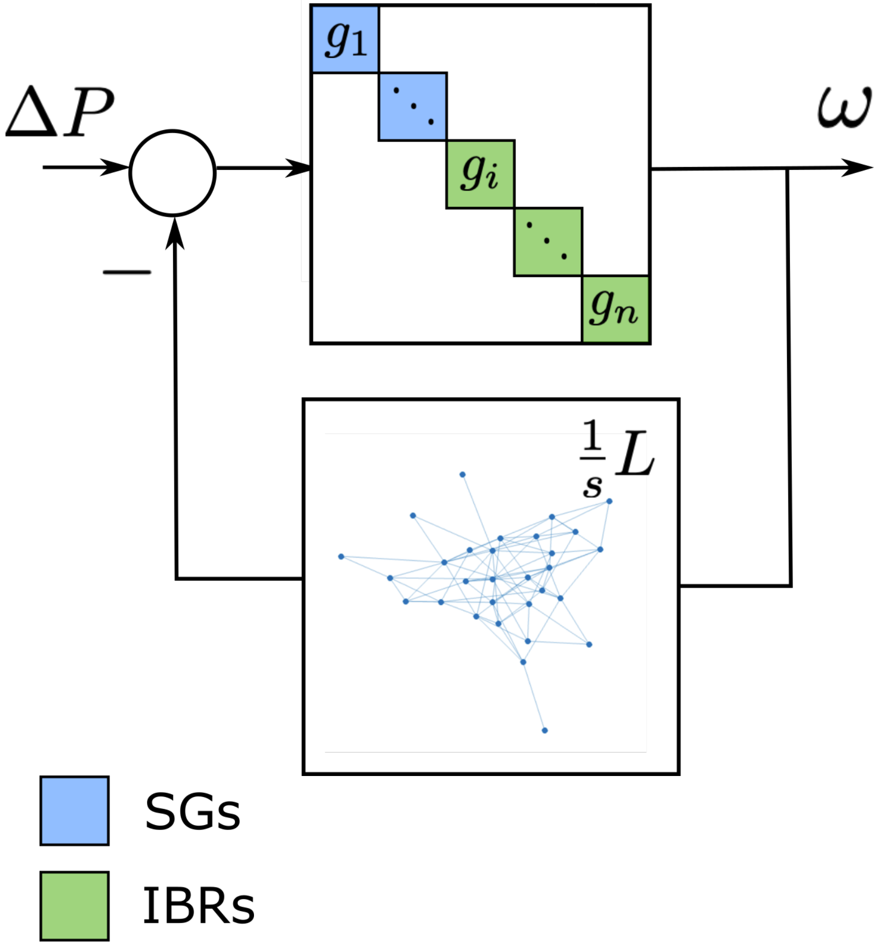
IBR power injection for a 1 p.u. load step

Tunable Performance:

E.g.: Turbine Time Constant $= \tau'$

[LCSS 23] Poolla, Lin, Bernstein, M, Groß. Frequency shaping control for weakly-coupled grid-forming IBRs **IEEE Control Systems Letters 2023**

GFM System-wide Grid-shaping [LCSS 20]



$$\sum_i \Delta P_i \rightarrow \left(\sum_{i\in \mathrm{SG}} g_i^{-1}(s) + \sum_{i \in \mathrm{IBR}} g_i^{-1}(s) \right)^{-1} \rightarrow \omega_{\mathrm{COI}}$$

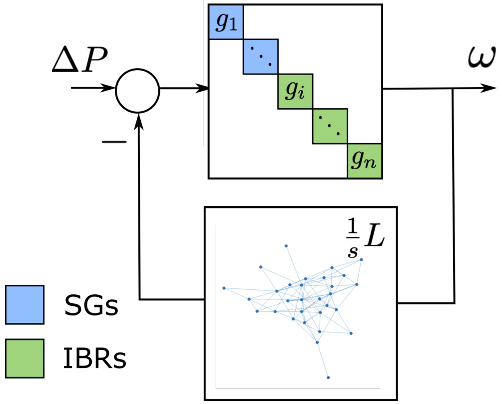
$$\sum_i \Delta P_i \rightarrow \frac{1}{as+b} \rightarrow \omega_{\mathrm{COI}}$$

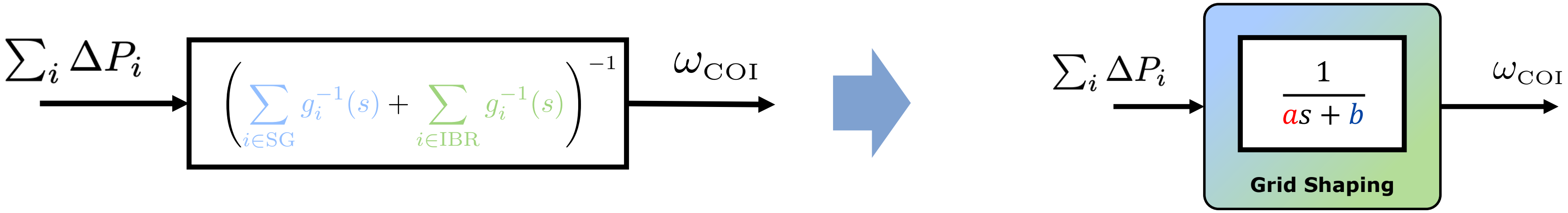
Grid Shaping

Tunable Performance: RoCoF $= \frac{1}{a}\Delta P,\ \Delta\omega = \frac{1}{b}\Delta P$

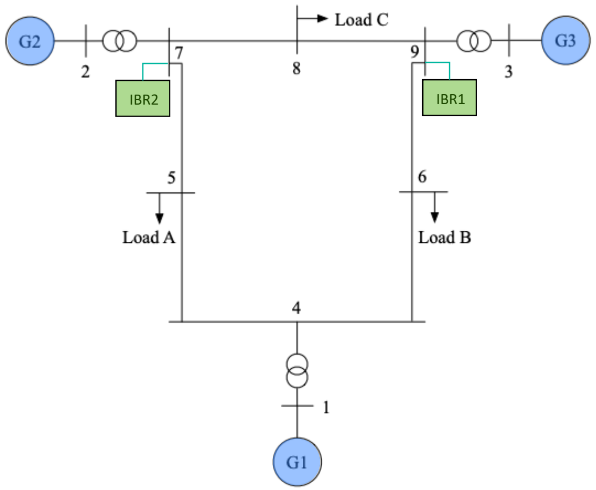
[LCSS 20] Jiang, Bernstein, Vorobev, M. Grid-forming frequency shaping control for low-inertia power systems **IEEE Control Systems Letters 2020**

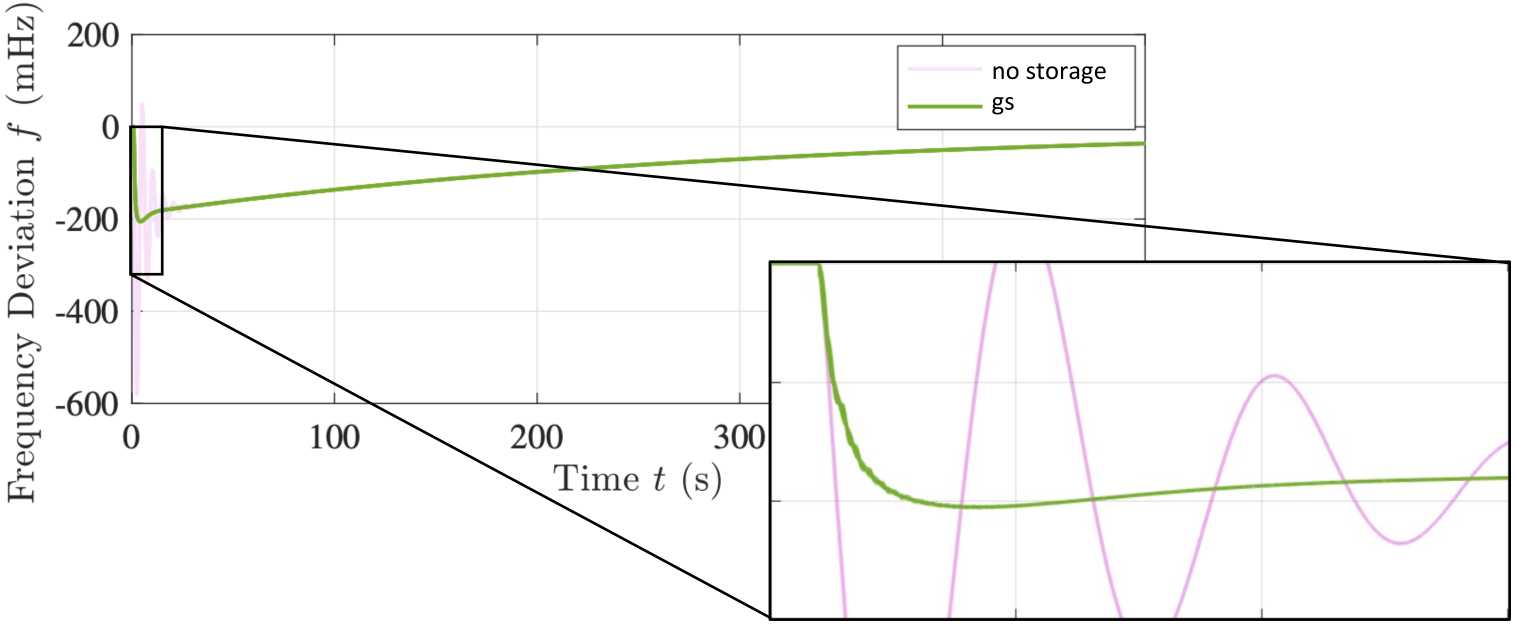
GFM System-wide Grid-shaping [LCSS 20]





Tunable Performance: RoCoF $= \frac{1}{a}\Delta P$, $\Delta\omega = \frac{1}{b}\Delta P$



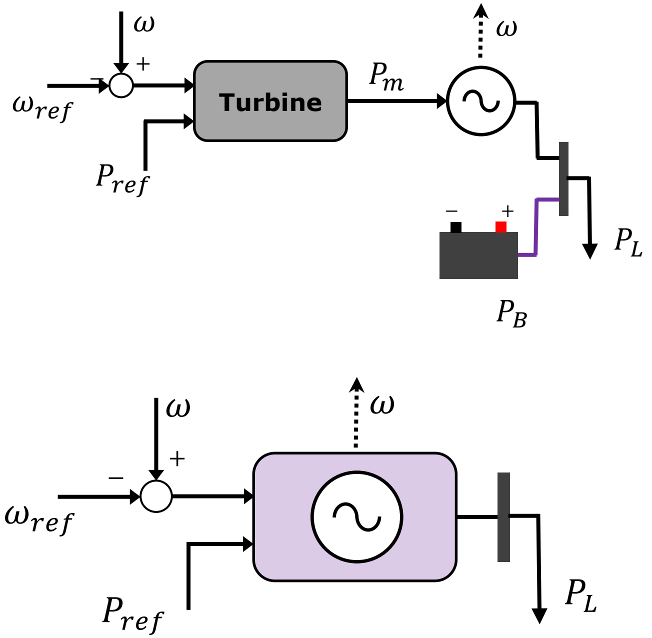


[LCSS 20] Jiang, Bernstein, Vorobev, M. Grid-forming frequency shaping control for low-inertia power systems **IEEE Control Systems Letters 2020**

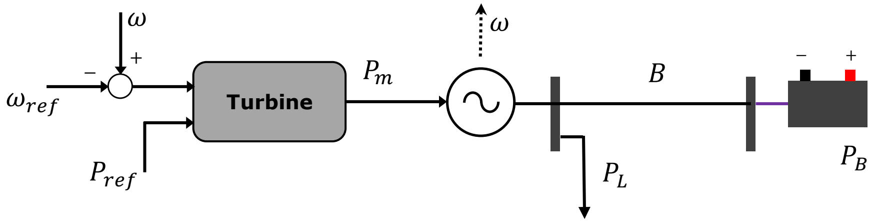
Grid Shaping Control

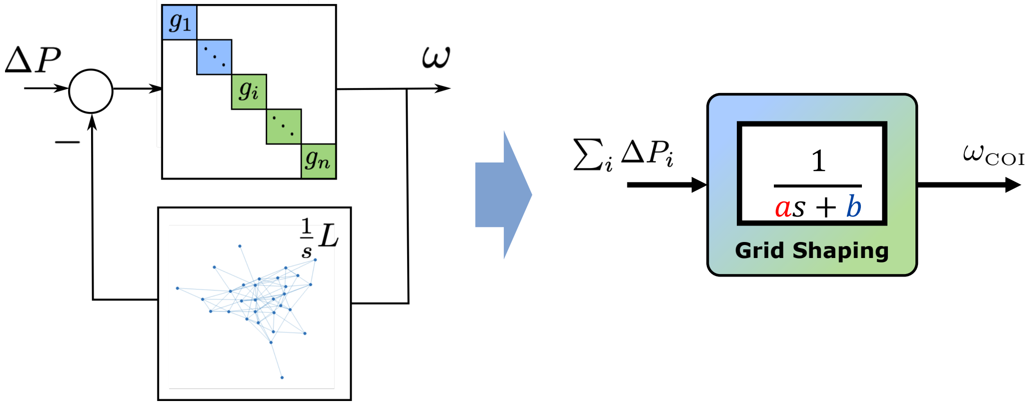
Use model matching control to shape system response

Grid-following IBRs



Grid-forming IBRs





Tunable Performance: RoCoF $= \frac{1}{a}\Delta P,\ \Delta\omega = \frac{1}{b}\Delta P,\ \tau', \ldots$

Summary

- **Merits and trade-offs of low inertia**
  - Control Perspective: Lighter systems are easier to control!
  - Smarter controller can provide multiple benefits in Nadir, RoCoF, inter-area oscillations, and disturbance rejection, with less effort

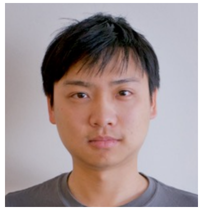
- **Scale-free Stability Analysis of Grids**
  - Generalizes passivity notions using network information
  - Decentralized test based on local models
  - Compatible with H_∞-synthesis methods

- **Analysis of Weakly-Connected Coherent Networks**
  - Generalized Center of Inertia captures IBR dynamics
  - Provide a new tunable target to meet system specs
  - Coherent modes identified via spectral clustering

- **Grid Shaping Control**
  - Grid-following/forming control framework for future girds
  - Leverages IBRs to *shape* the coherent response

Thanks!


Yan Jiang


Hancheng Min

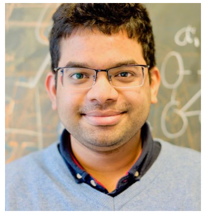

Eliza Cohn

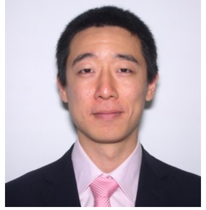

Petr Vorobev


Richard Pates


Fernando Paganini


Dominic Groß


Bala K. Poolla


Yashen Lin


Andrey Bernstein

Merits and trade-offs of low inertia

[TAC 21] Jiang, Pates, M, Dynamic droop control in low inertia power systems. **Transactions on Automatic Control, 2021**

Analysis of Weakly-Connected Coherent Networks

[CDC 19] Min, M. Dynamics concentration of large-scale tightly-connected networks. **Conference on Decision and Control 2019**

[LCSS 20] Min, Paganini, M. Accurate reduced-order models for heterogeneous coherent generators. **IEEE Control Systems Letters 2020**

[L4DC 23] Min, M. Learning coherent clusters in weakly-connected network systems. **Leaning for Dynamics and Control 2023**

[Auto 25] Min, Pates, M. A frequency domain analysis of slow coherency in networked systems. **Automatica 2025**

Scale-free Stability Analysis

[TCNS 19] Pates, M, Robust scale-free synthesis for frequency control in power systems. **Transactions on Control of Network Systems, 2019**

[GM 24] Siahaan, M, Geng. Decentralized Stability Criteria for Grid-Forming Control in Inverter-Based Power Systems. **IEEE PES GM 2024**

Grid Shaping Control

[LCSS 20] Jiang, Bernstein, Vorobev, M. Grid-forming frequency shaping control for low-inertia power systems. **Control Systems Letters 2020**

[TPS 21] Jiang, Cohn, Vorobev, M. Storage-based frequency shaping control. **Transactions on Power Systems 2021**

[LCSS 23] Poolla, Lin, Bernstein, M, Groß. Frequency shaping control for weakly-coupled grid-forming IBRs. **IEEE Control Systems Letters 2023**

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Backup Slides

Network Coherence: Heterogeneous Case

$$T(s) = \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T + \boxed{T(s) - \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T} \qquad \bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

The effect of non-coherent dynamics vanishes as:

- For almost any $s_0 \in \mathbb{C}$

$$\lim_{\lambda_2(L)\to+\infty} \left\| T(s_0) - \frac{1}{n}\bar{g}(s_0)\mathbb{1}\mathbb{1}^T \right\| = 0$$

- For $s_0 \in \mathbb{C}$, a pole of $f(s)$

$$\lim_{s\to s_0} \left\| T(s) - \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T \right\| = 0$$

- Excluding zeros: the limit holds at zero, but by different convergence result
- We can further prove **uniform convergence** over a compact subset of complex plane, if it doesn't contain any zero nor pole of $\bar{g}(s)$
- Extensions for random network ensembles, $g_i(s) \coloneqq g(s, w_i)$ (w_i random), then $\bar{g}(s) = (E_w[\,g^{-1}(s,w)])^{-1}$
- Convergence of transfer matrix is **related to time-domain response** by Inverse Laplace Transform

Connection to Time Domain

If $\bar{g}(s)$ and $T(s)$ stable ($\left||\bar{g}|\right|_\infty, \left||T|\right|_\infty \leq \gamma$), then there is $\bar{\lambda} = O(\gamma/\varepsilon)$ such that:

- **ε-approximation,** for any network L, with $\lambda_2(L) \geq \bar{\lambda}$

$$\sup_{t>0} |y_i(t) - \bar{y}(t)| \leq \varepsilon$$

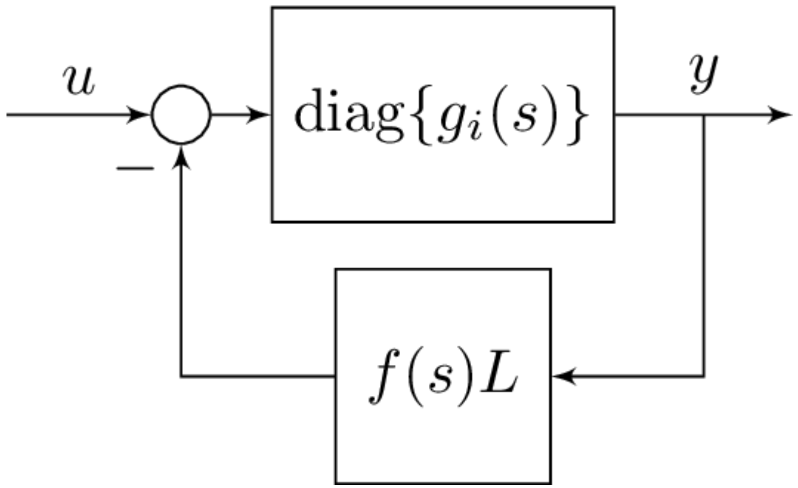
where $\bar{y}(t)$ is the coherence dynamics response: $y(s) = \bar{g}(s)\frac{1}{n}\sum_{i=1}^{n} u_i(s)$

- **element-wise coherence**, for any pair of nodes i and j

$$\sup_{t>0} |y_i(t) - y_j(t)| \leq 2\varepsilon$$

Example: Icelandic Power Grid

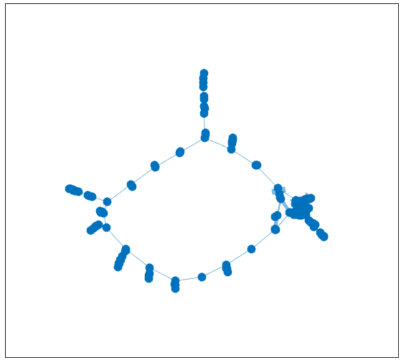
- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)



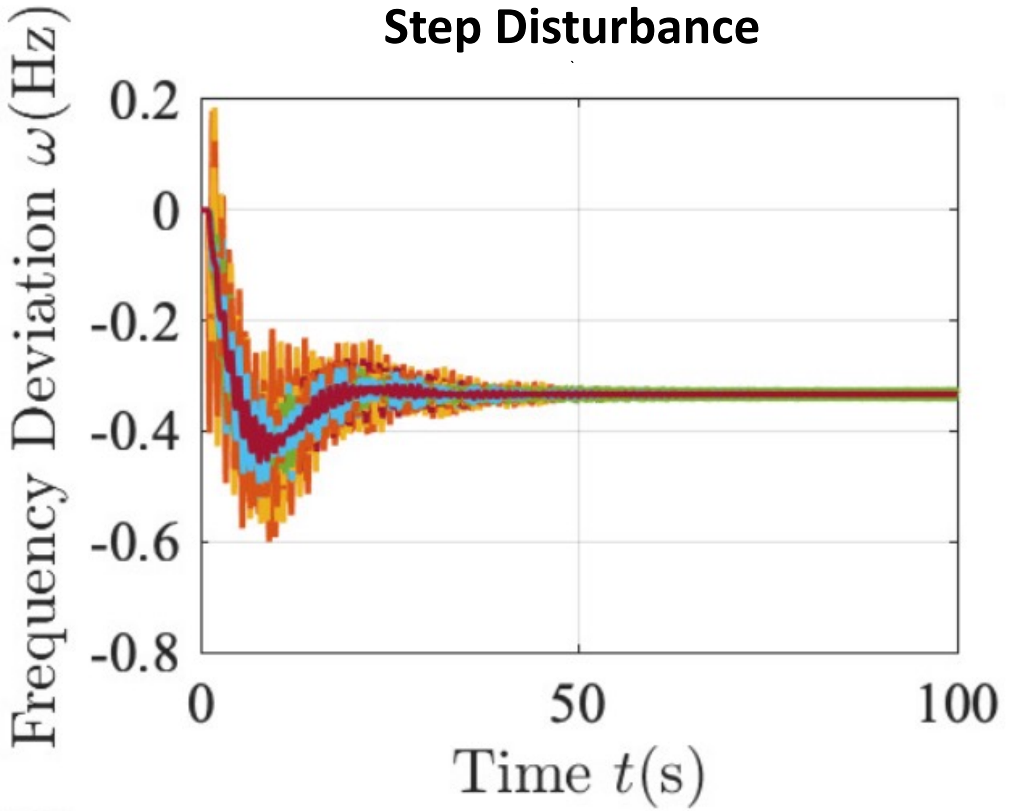
$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s+1}}$$

$$f(s) = \frac{1}{s}$$

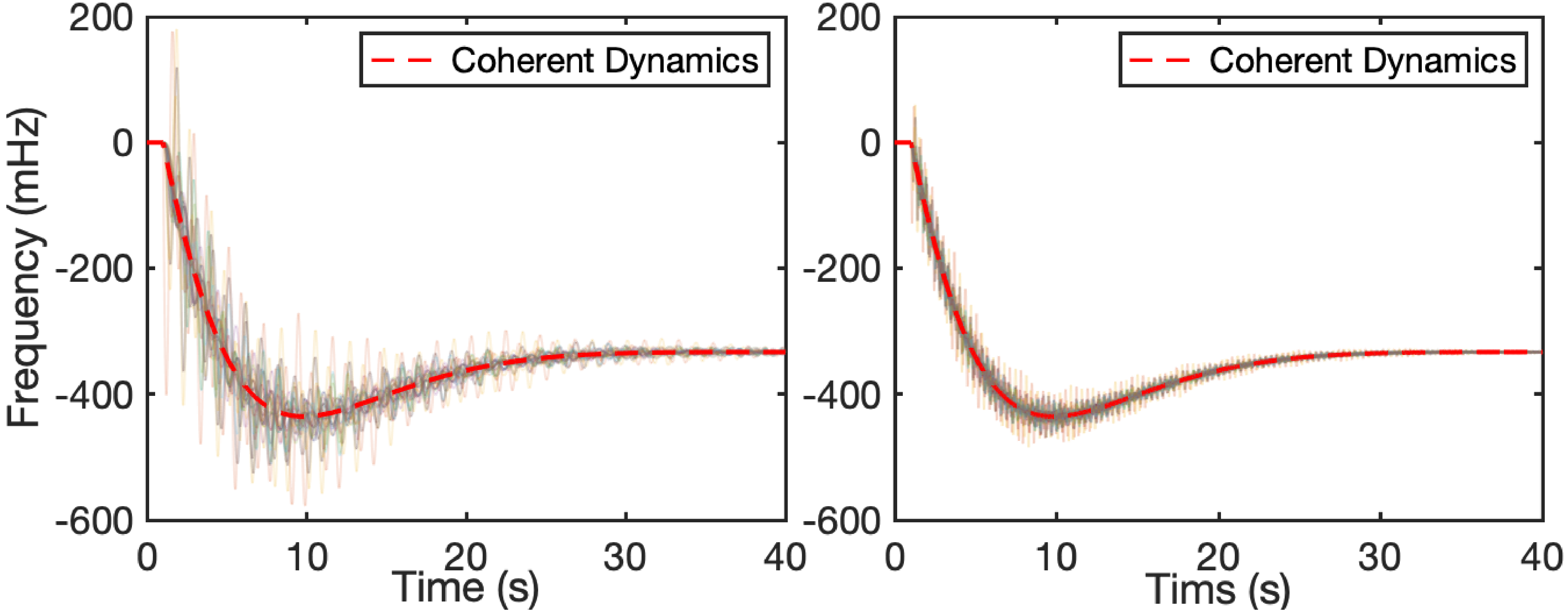
Icelandic Grid



Step Disturbance



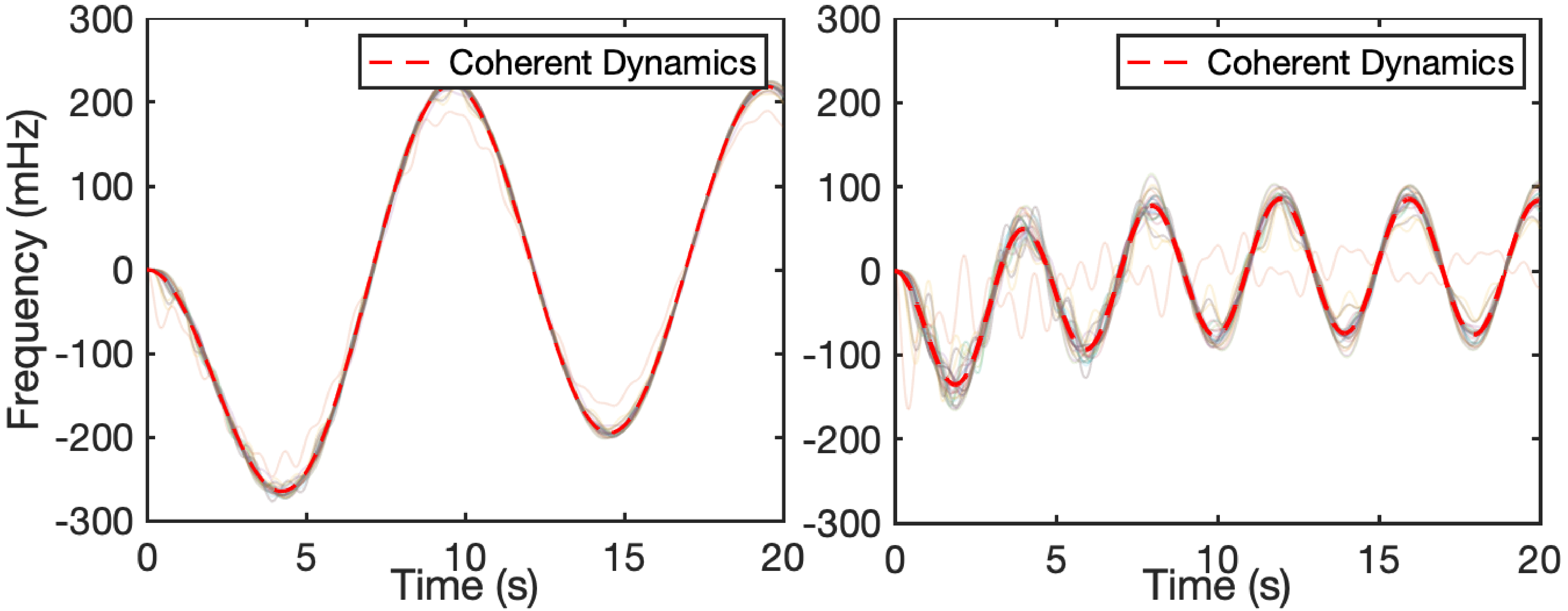
Example: Effect of Network Algebraic Connectivity $\lambda_2(L) \uparrow$



Coherent dynamics acts as a more accurate version of the Center of Inertia (CoI)

Example: Sinusoidal Disturbances: $\sin(\boldsymbol{\omega_d t})$

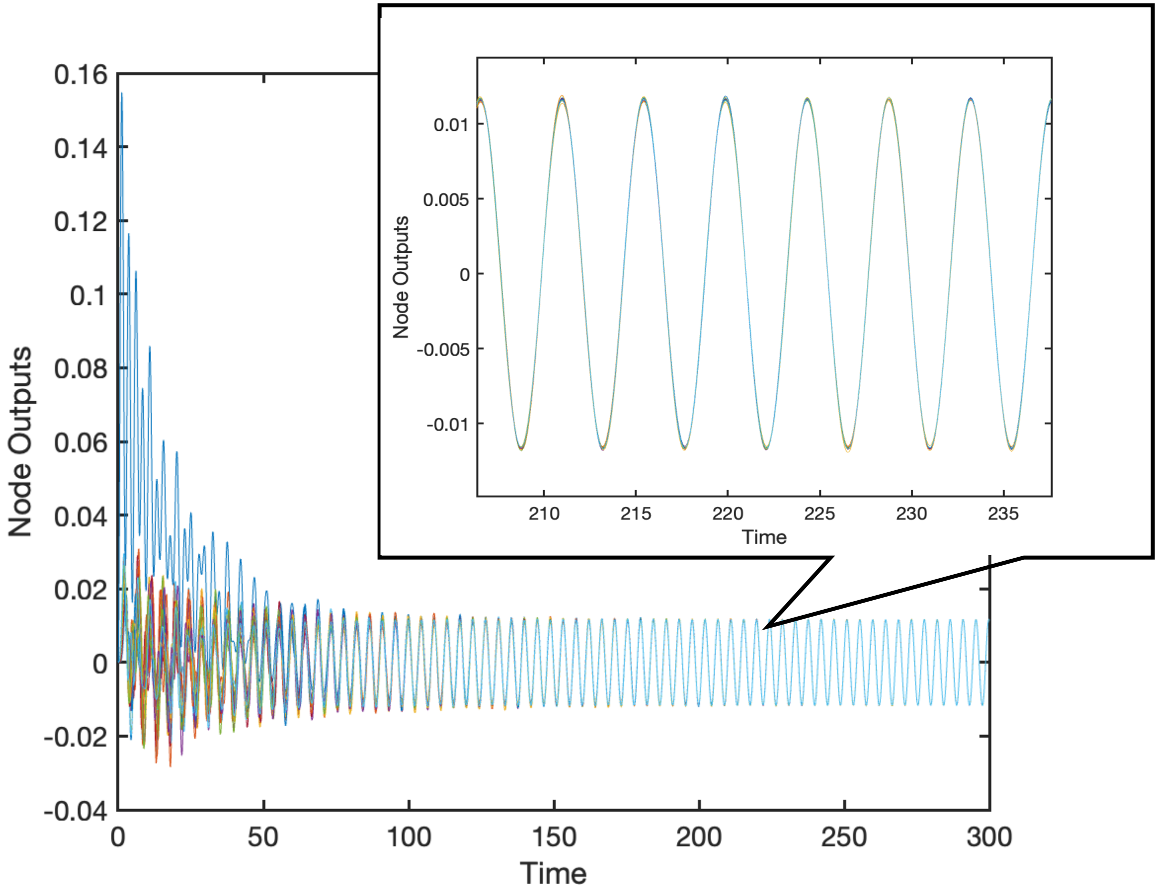
$\omega_d \uparrow$



Frequency-dependent Coherence from Coupling Dynamics

- <u>Frequency dependent coherence</u>:
  A stable network responds coherently when subject to signal with its frequency component concentrated around pole of $f(s)$

- <u>An Artificial Example</u>:
  A stable heterogenous network with $f(s) = \frac{s}{s^2+\omega_0^2}$ is "synchronized" by external sinusoidal input $\sin \omega_0 t$
  (Such coherence is robust to small changes in input frequency)



First order nodal dynamics $g_i(s) = \frac{1}{m_i s + d_i}$

20 nodes with $m_i \sim Unif(1,5), d_i \sim Unif(0.1, 0.5)$

12-regular graph with unit weights

Sin input to the first node(shown in blue) only