

Embracing Low Inertia for Power System Frequency Control

A Frequency Shaping Approach

Enrique Mallada



JOHNS HOPKINS
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University of Michigan

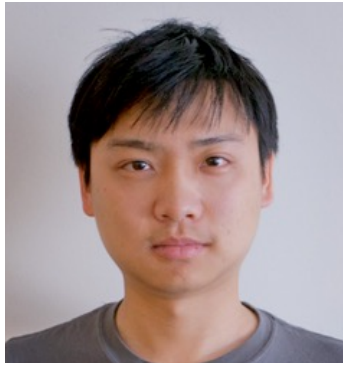
ECE Seminar

April 11, 2022

Acknowledgements



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Richard Pates



LUND UNIVERSITY



Fernando Paganini



Decarbonization of electricity is key to mitigate climate change

California lifts renewable energy target to 73% by 2032

The California Public Utilities Commission raised renewable energy procurement targets, plans for a more aggressive decarbonization plan, and includes increased reliability provisions.

FEBRUARY 14, 2022 **RYAN KENNEDY**

Vermont House passes 75% by 2032 renewable energy mandate

Published March 11, 2015

ENVIRONMENT

Maryland bill mandating 50% renewable energy by 2030 to become law, but without Gov. Larry Hogan's signature

By Scott Dance
Baltimore Sun • May 22, 2019 at 6:40 pm

New York mandates 70% renewable energy by 2030

By Kelsey Misbrener | October 15, 2020

Oregon bill targets 100% clean power by 2040, with labor and environmental justice on board

After Democratic cap-and-trade bills faltered in the face of GOP revolts, an electricity-focused, consensus-driven bill gains ground in Oregon.

23 June 2021

Virginia becomes the first state in the South to target 100% clean power

The state's Democratic majority is doing what Democratic majorities do.

By David Roberts | @drvols | Updated Apr 13, 2020, 2:56pm EDT

Decarbonization of electricity is key to mitigate climate change

California lifts renewable energy target to 73% by 2032

The California Public Utilities Commission targets, plans for a more aggressive decarbonization, and reliability provisions.

FEBRUARY 14, 2022 RYAN KENNEDY

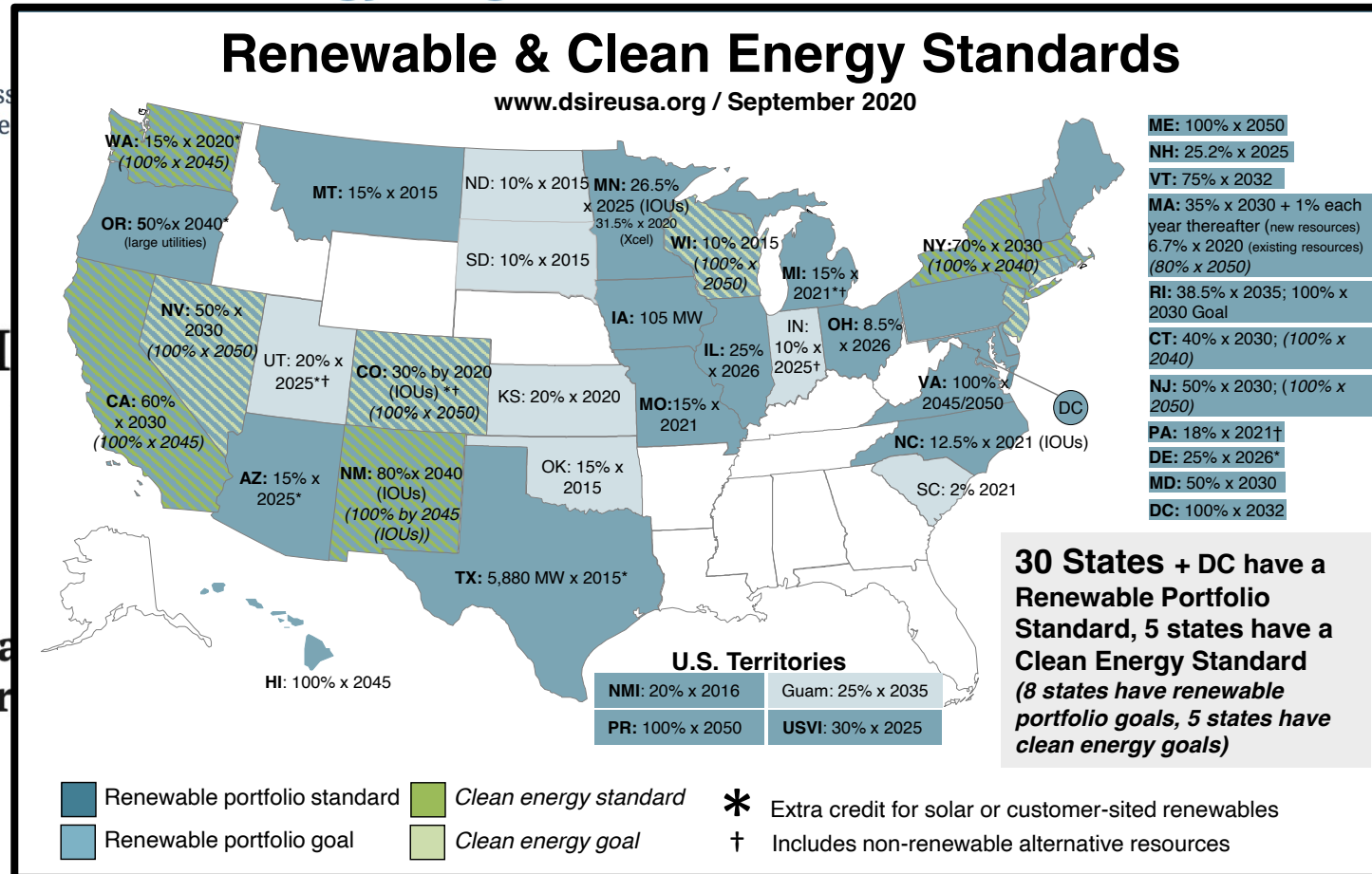
Vermont Hikes renewable energy targets

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ENVIRONMENT

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trade bills faltered in the face of GOP opposition, consensus-driven bill gains ground in

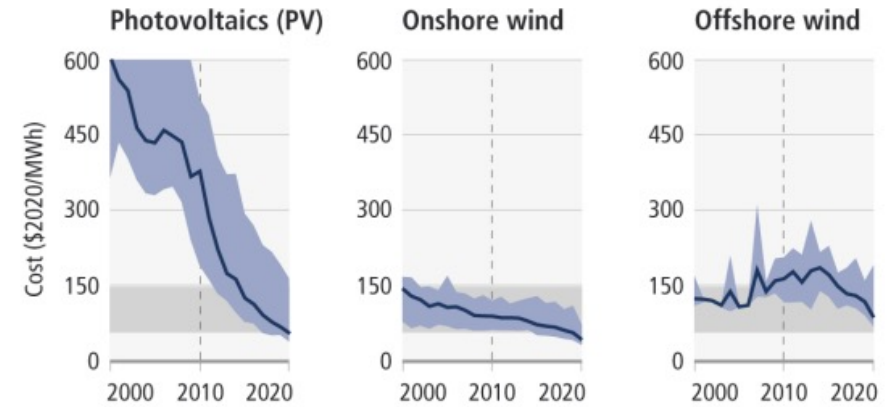
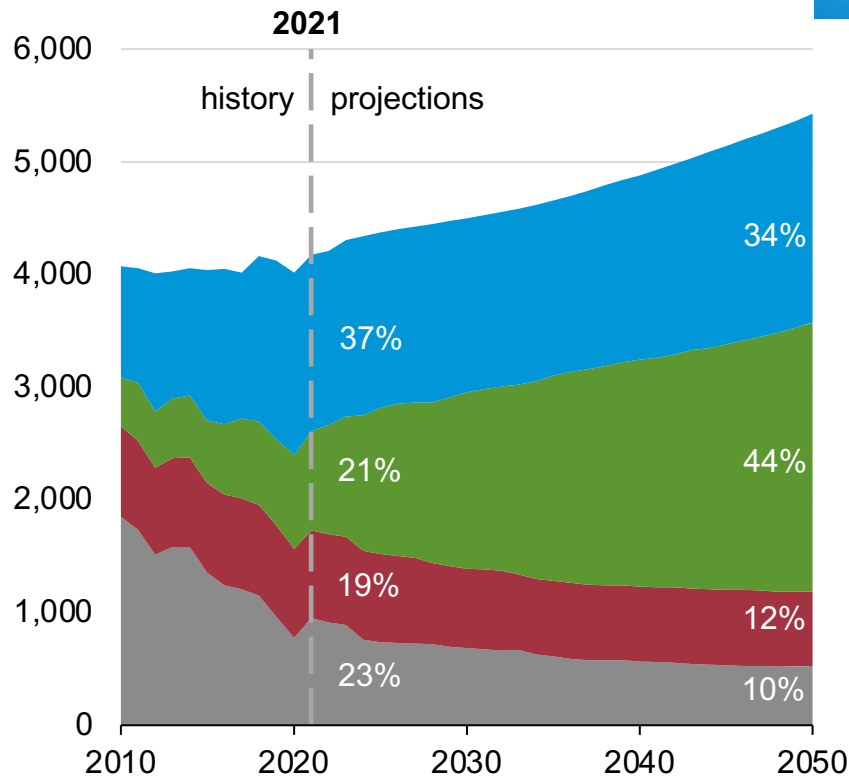
ate in the South to

The state's Democratic majority is doing what Democratic majorities do.

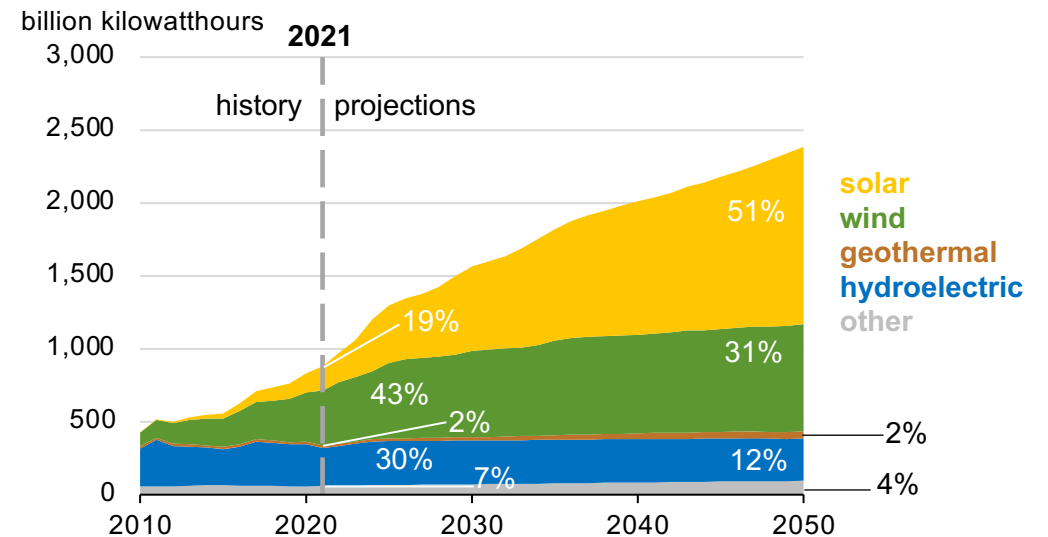
By David Roberts | @drvolts | Updated Apr 13, 2020, 2:56pm EDT

Decarbonization of electricity is key to mitigate climate change

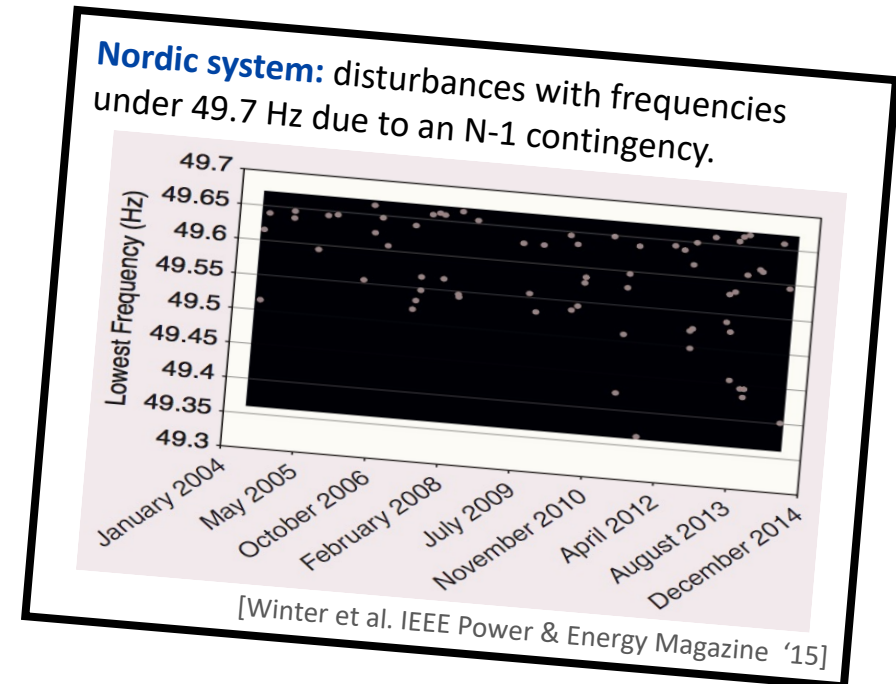
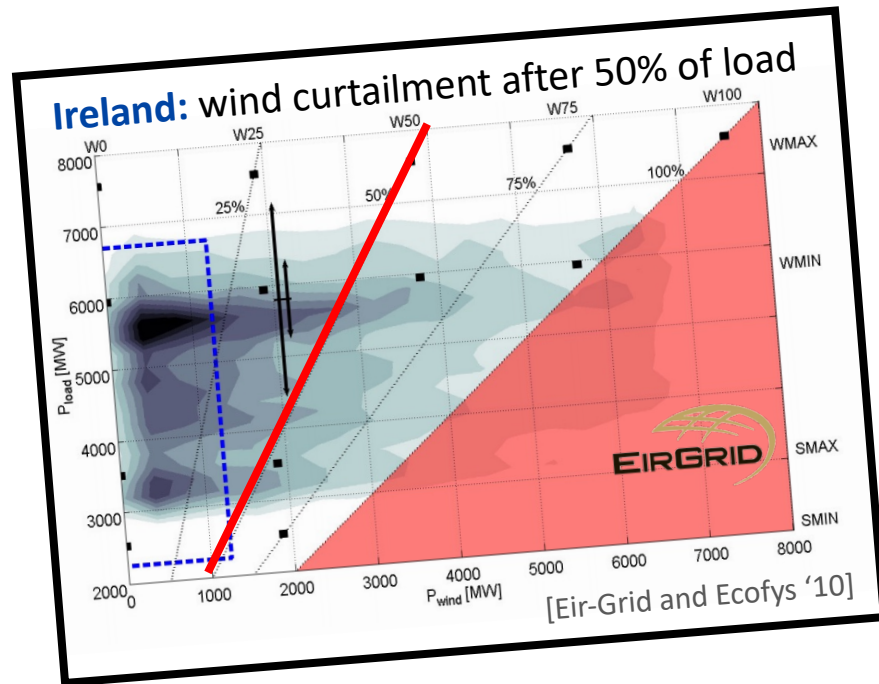
U.S. electricity generation from selected fuels
AEO2022 Reference case
 billion kilowatthours



U.S. renewable electricity generation, including end use
AEO2022 Reference case
 billion kilowatthours



Dynamic Degradation



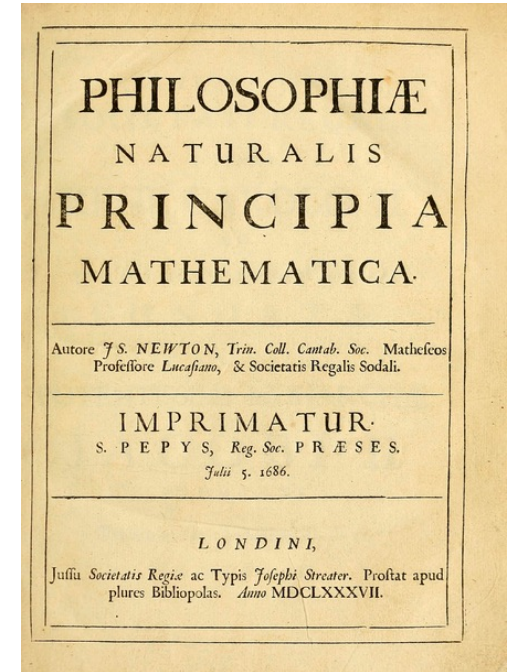
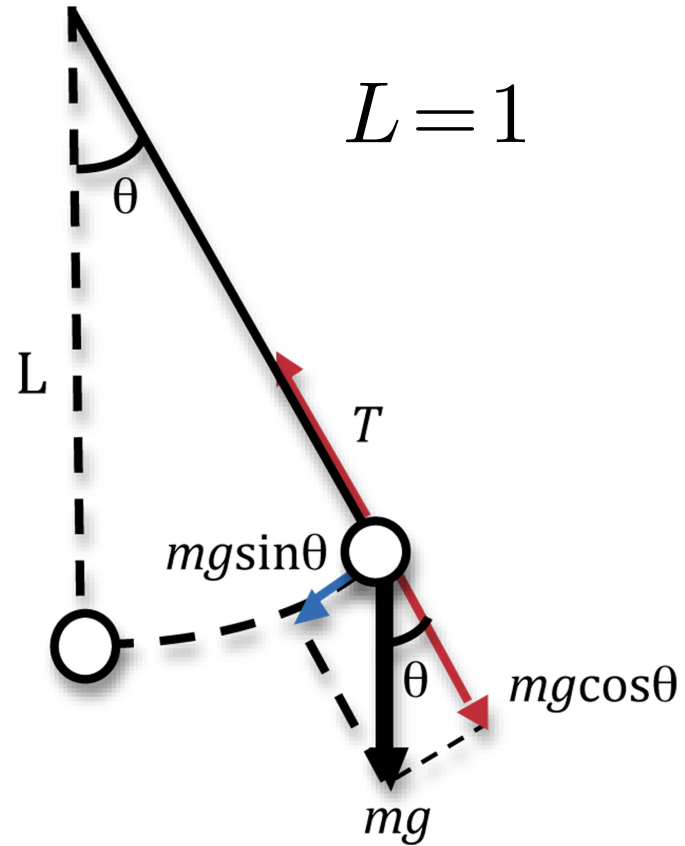
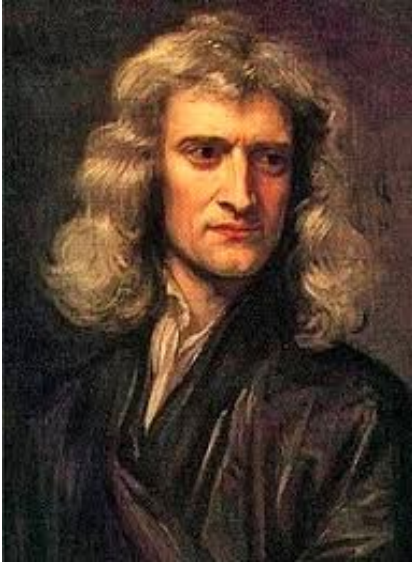
In the United States:



“While the three [contiguous] U.S. interconnections currently exhibit adequate frequency response performance above their interconnection frequency response obligations, there has been a significant decline in the frequency response performance of the Western and Eastern Interconnections,” FERC said.

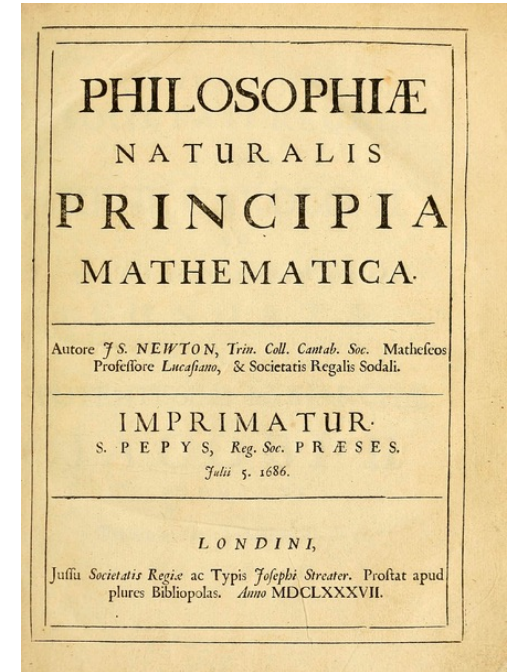
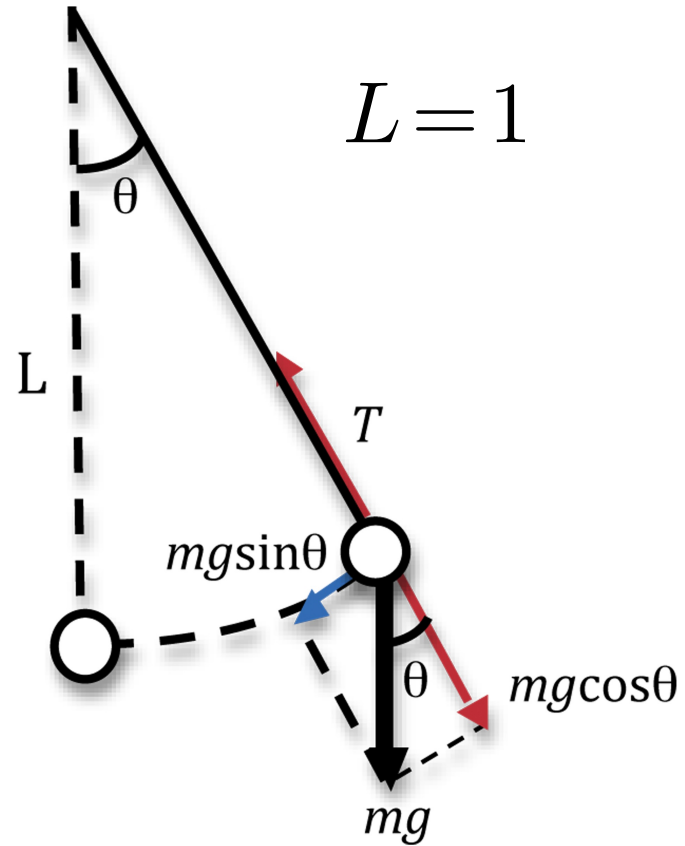
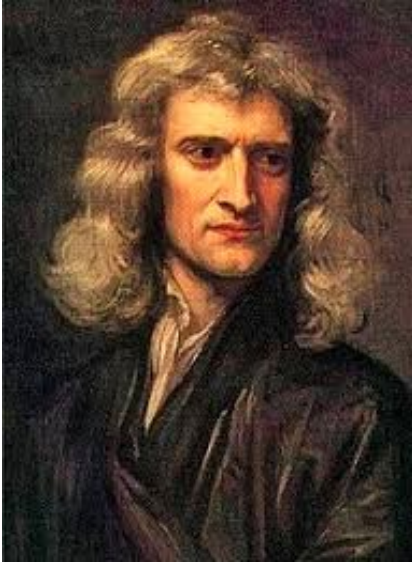
[FERC, Nov. 16]

Merits and Trade-offs of Inertia



$$m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f$$

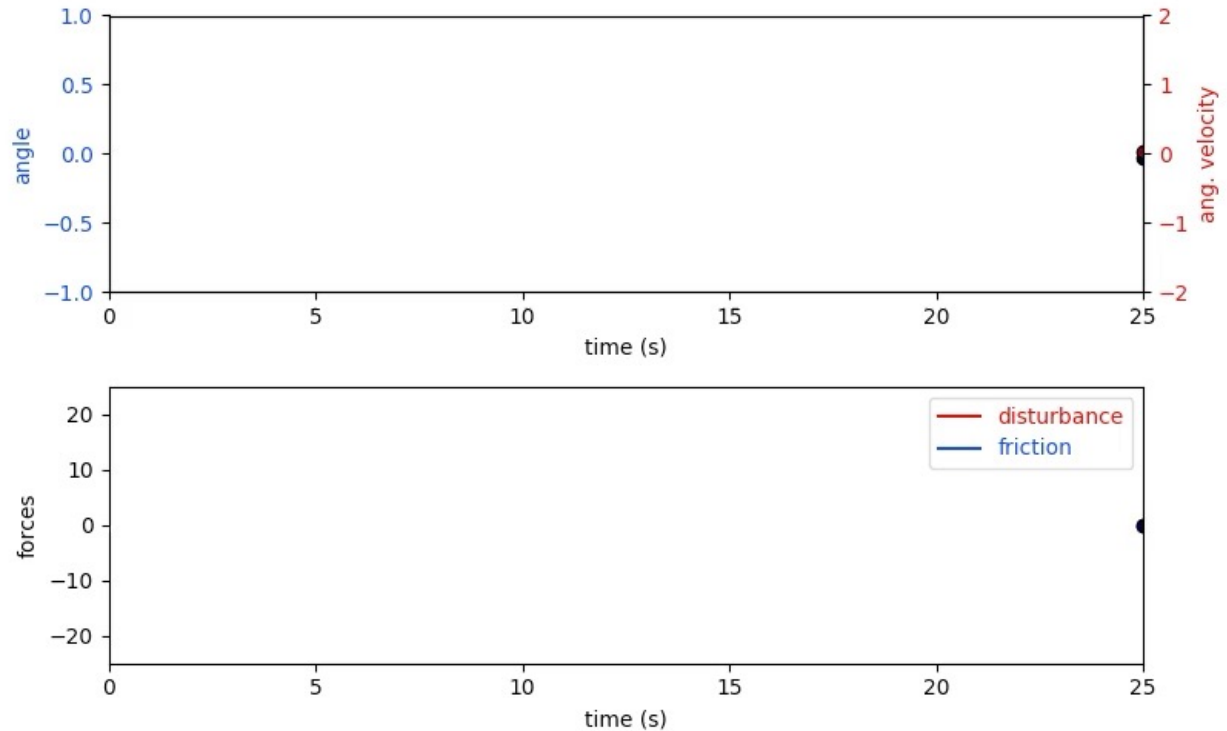
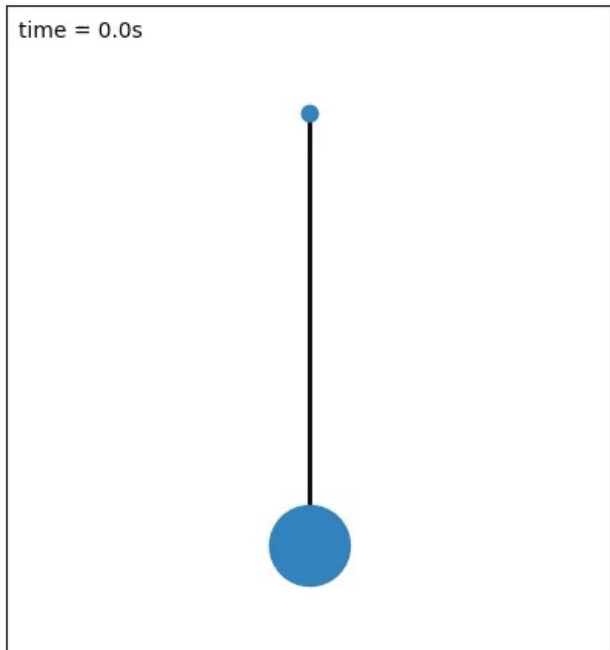
Merits and Trade-offs of Inertia



$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g \sin \theta + \frac{f}{m}$$

Merits and Trade-offs of Inertia

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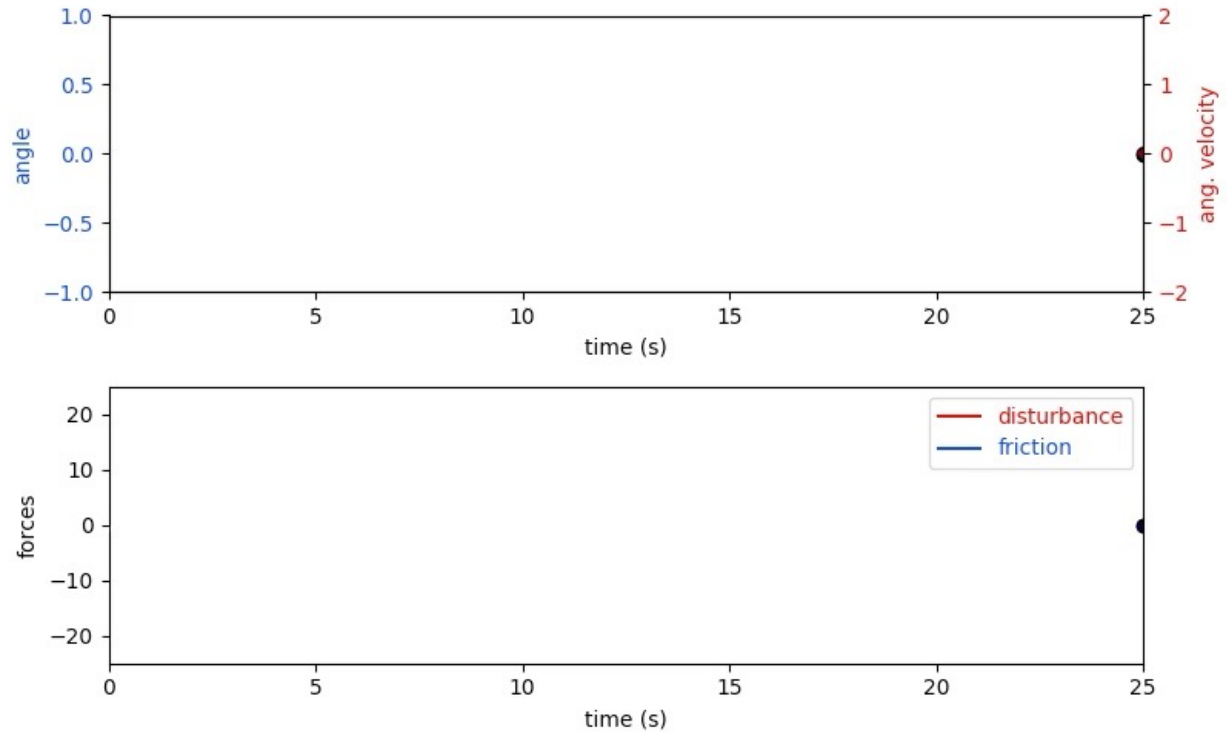
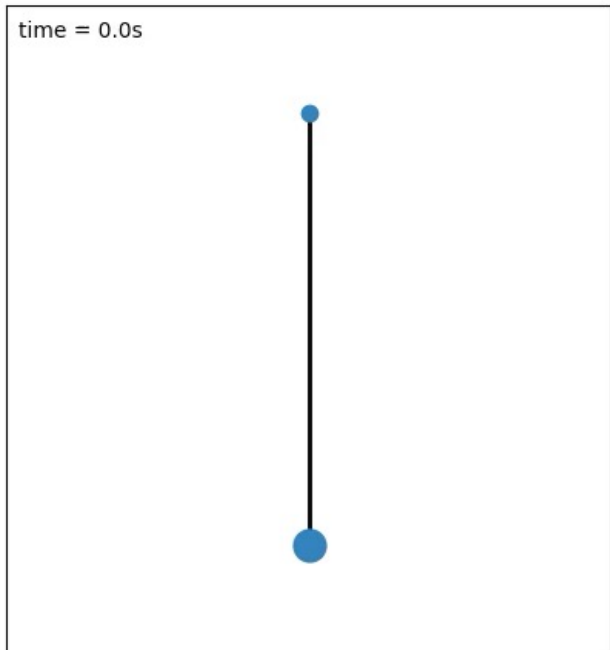


Pros: Provides natural disturbance rejection

Cons: Hard to regain steady-state

Merits and Trade-offs of **Low Inertia**

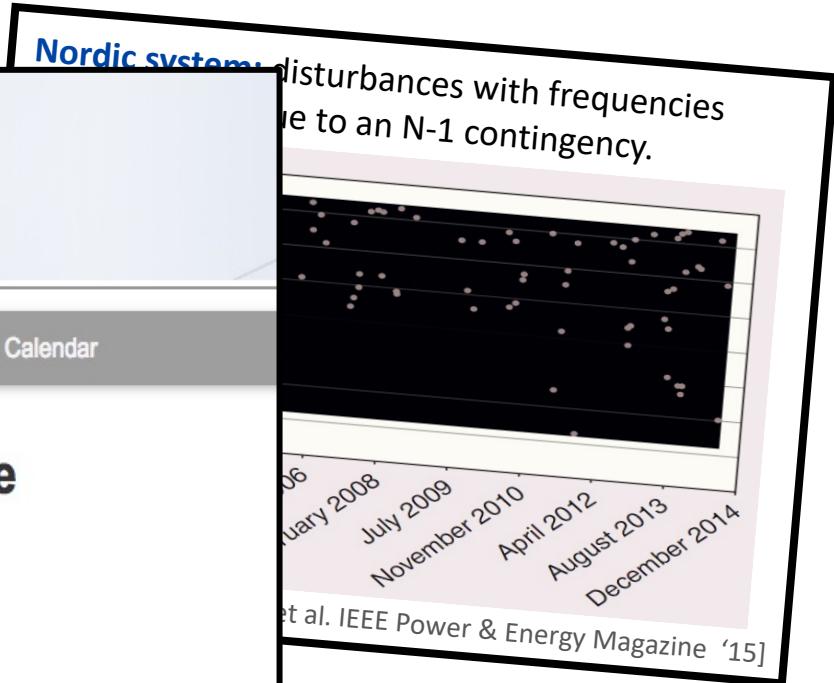
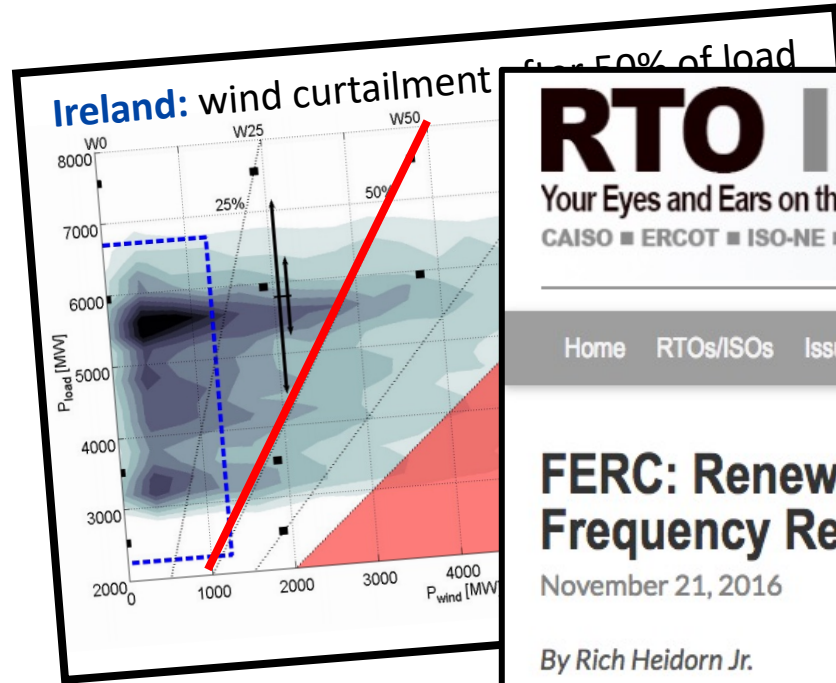
$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g \sin \theta + \frac{f}{m}$$



Cons: Susceptible to disturbances

Pros: Regains steady-state faster

Dynamic Degradation



RTO Insider
 Your Eyes and Ears on the Organized Electric Markets
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FERC: Renewables Must Provide Frequency Response
 November 21, 2016

By Rich Heidorn Jr.

In a rulemaking reflecting both reliability concerns and the technological advances of renewable generators, FERC on Thursday proposed revising the *pro forma* Large Generator Interconnection Agreement (LGIA) and Small Generator Interconnection Agreement (SGIA) to require all newly interconnecting facilities to install and enable primary frequency response capability (RM16-6).

In the U.S. ...

“While the frequency response obligations, there has been a significant decline in the frequency response performance of the Western and Eastern Interconnections,” FERC said.

...atory Commission ... adequate frequency ...

[FERC, Nov. 16]

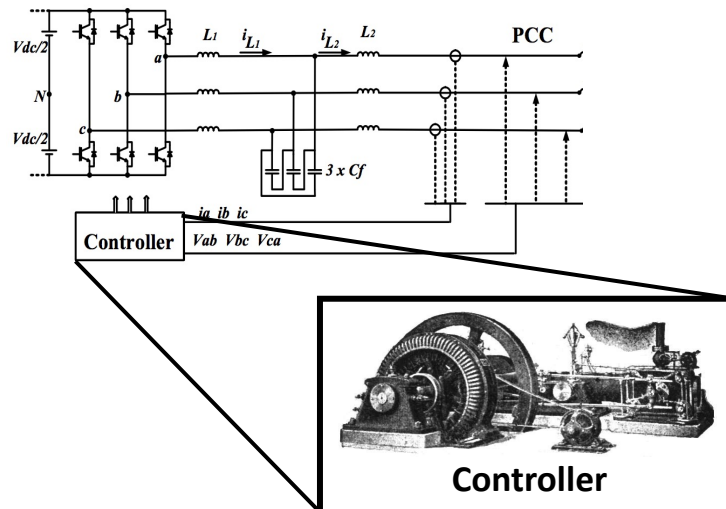
Inverter-based Control

Challenges

- Measurements with noise and delays
- Stability + robustness (plug & play)
- Lack of incentives

Current approach: Use inverter-based control to mimic generators response

Virtual Synchronous Generator



Telecom Analogy



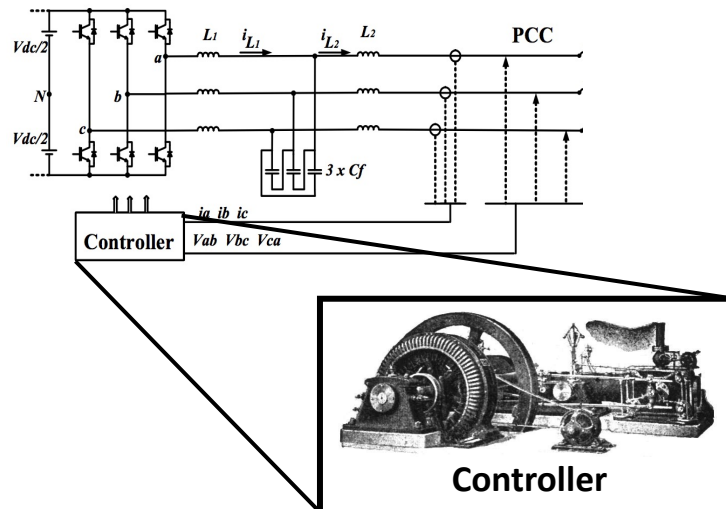
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Telecom Analogy



It works, but perhaps there is something better...

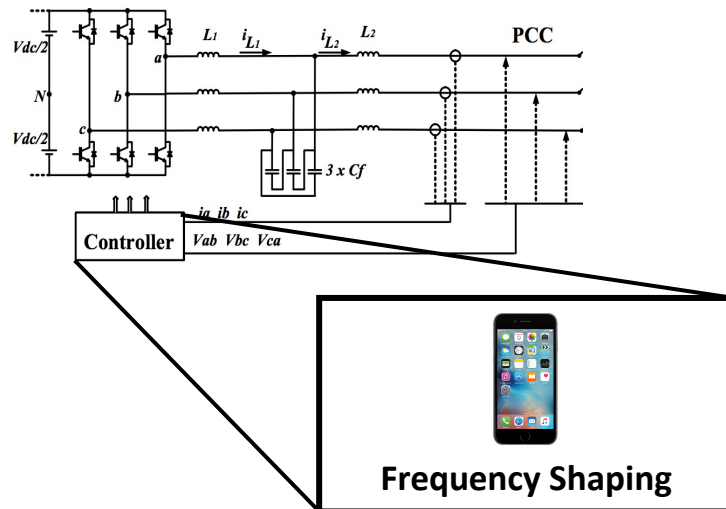
Inverter-based Control

Challenges

- Measurements with noise and delays
- Stability + robustness (plug & play)
- Lack of incentives

Our approach: Design and tune of controllers rooted on **sound control principles**

Frequency Shaping Control



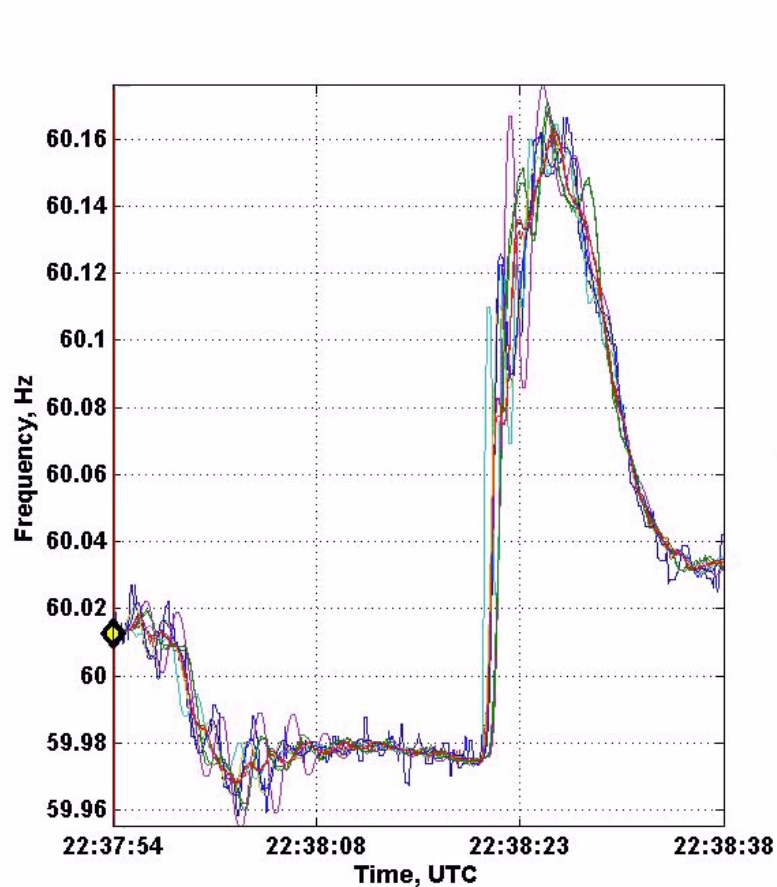
Design Objectives:

- Exploit power electronics capabilities
- Improve Dynamic Performance
- Minimize control effort
- Stability and Robustness

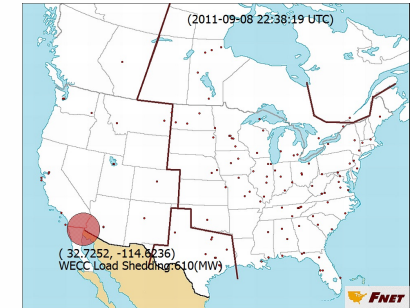
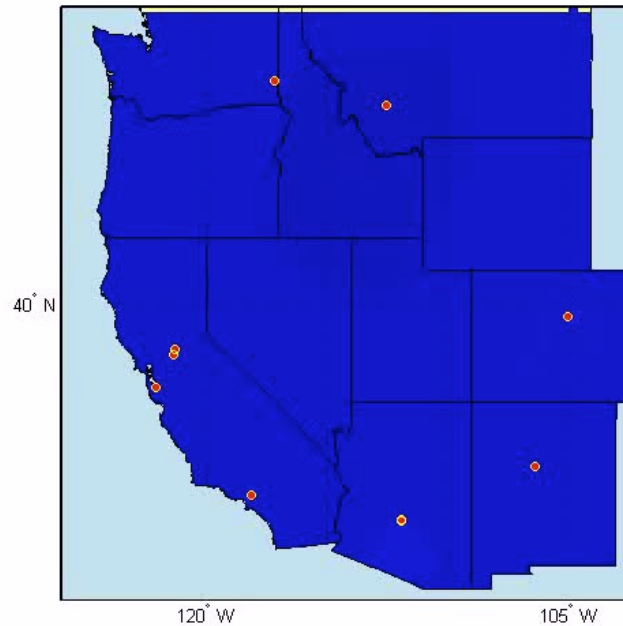
Roadmap to Low Inertia Frequency Control

- Network Performance Analysis
- Limits of Virtual Inertia and Droop Control
- Control Design: Frequency Shaping

Power System Performance



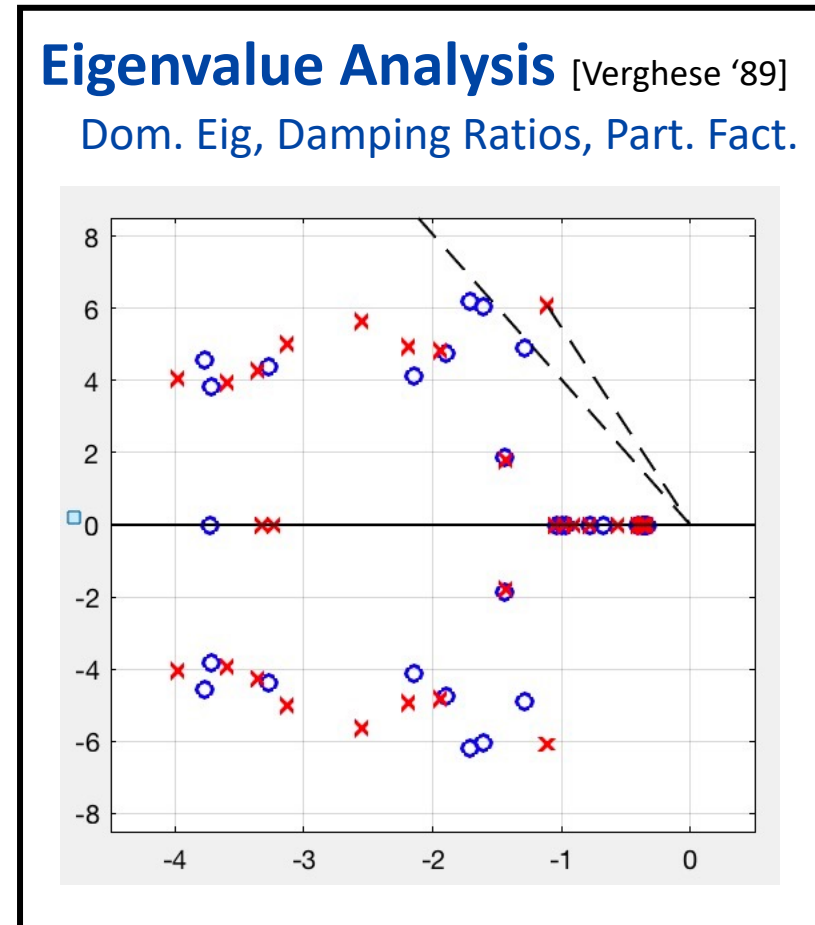
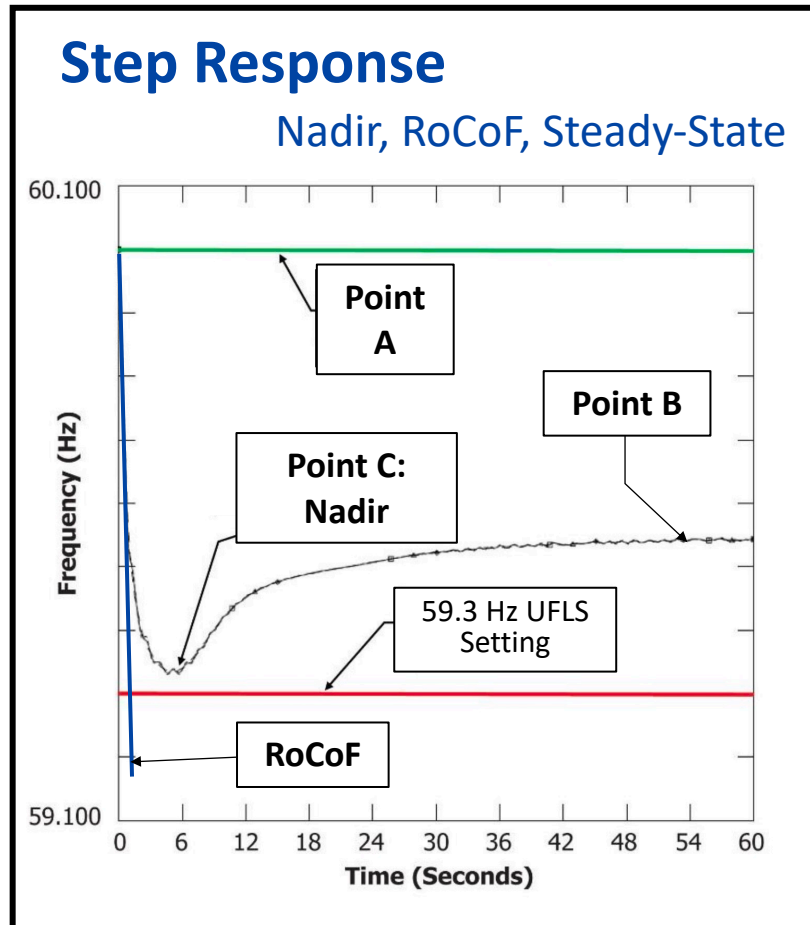
FNET Data Display [9/8/2011 Southwest Blackout]
Time: 22:37:54.0 UTC 60.0125 Hz



Depends on several factors: generators, network, disturbance
good performance metric must identify the source of the degradation!

Power Engineering Metrics

based on classical control theory...

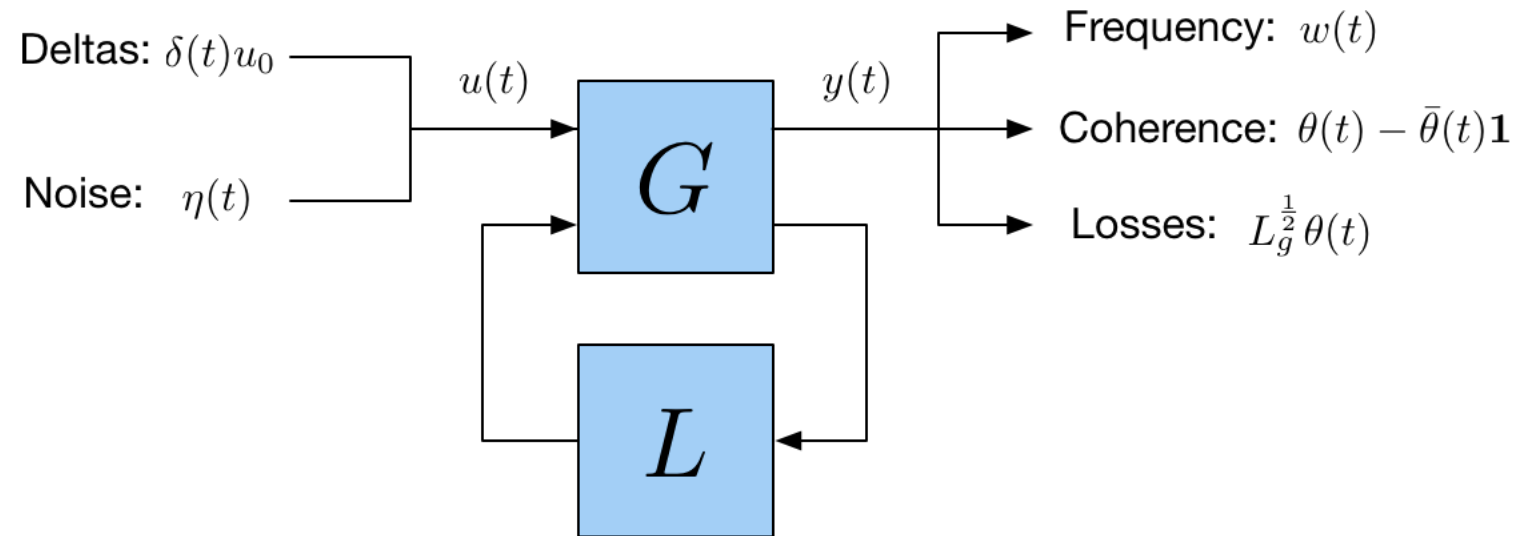


+ Domain specific, capture system degradation

- Relation with cause? Eigenvalue sensitivity? More than one "dominant" eig.?

System Theoretic Metrics

[Tegling... '15, Poolla... '15, Grunberg... '16, Simpson-Porco... '16, Wu et al '16, Adreasson '17, Coletta '17...]



\mathcal{L}_∞ -norm:

$$\|y\|_\infty := \sup_{t \geq 0} \max_i |y_i(t)|$$

\mathcal{L}_2 -norm:

$$\|y\|_2 := \left(\int_0^\infty y(t)^T y(t) dt \right)^{\frac{1}{2}}$$

+ Close form solutions, qualitative analysis, computational methods

- Restrictive assumptions, not direct connection with RoCoF, Nadir, step disturbances

Performance Specification

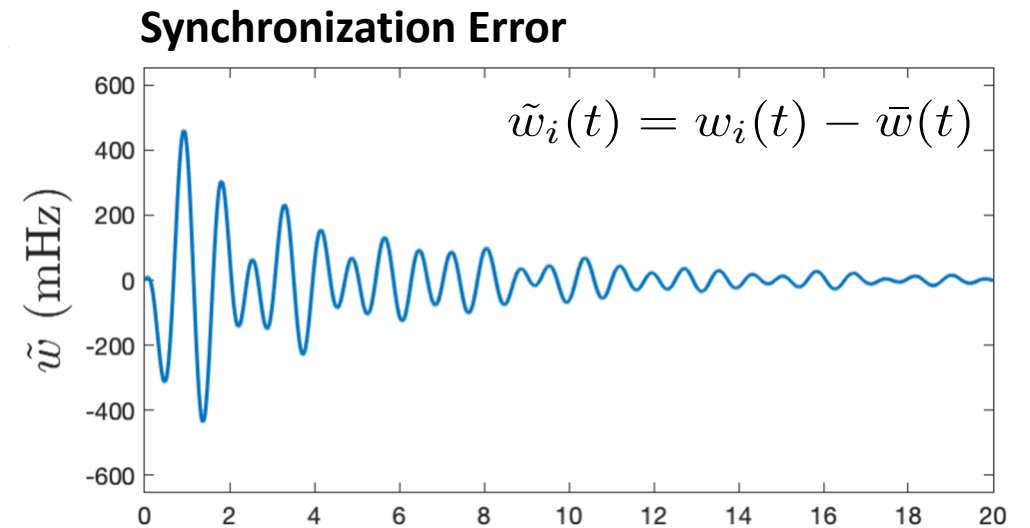
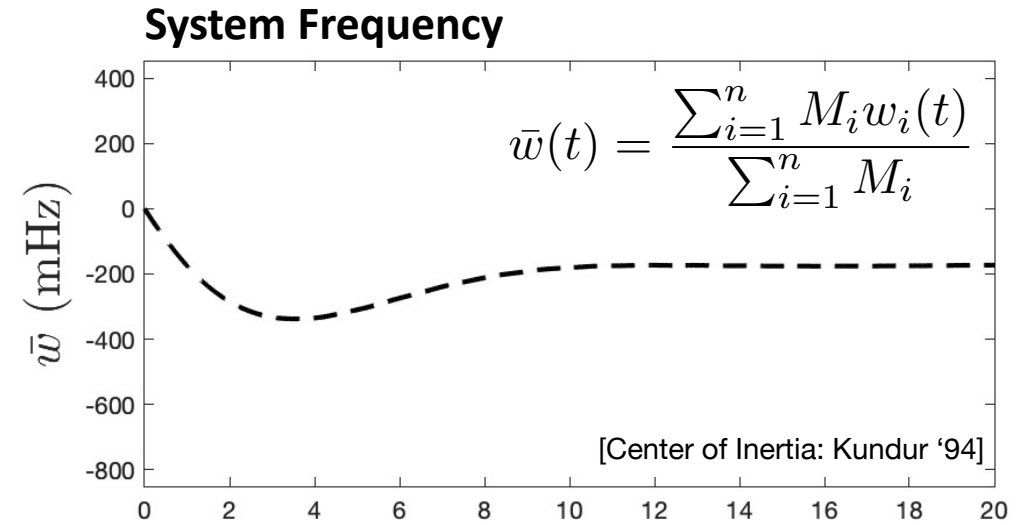
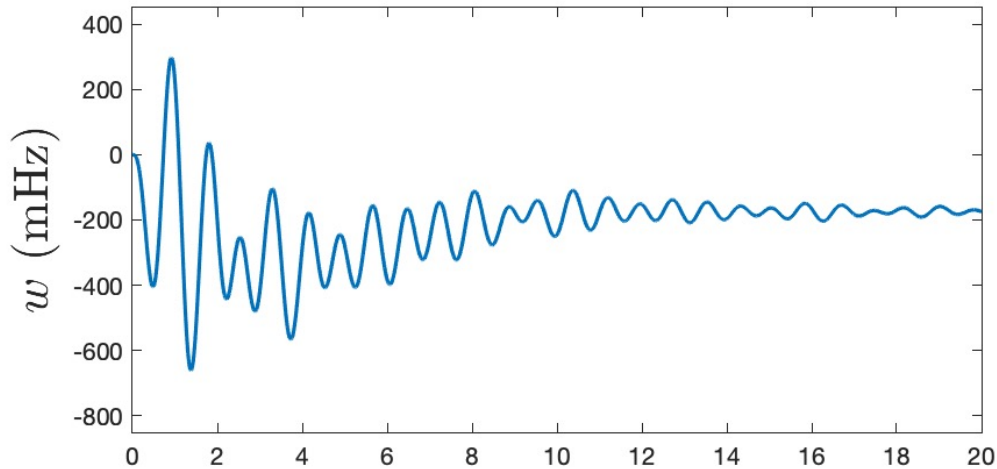
Frequency Response

Control Effort

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, **IEEE Transactions on Automatic Control**, 2020

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control**, 2021

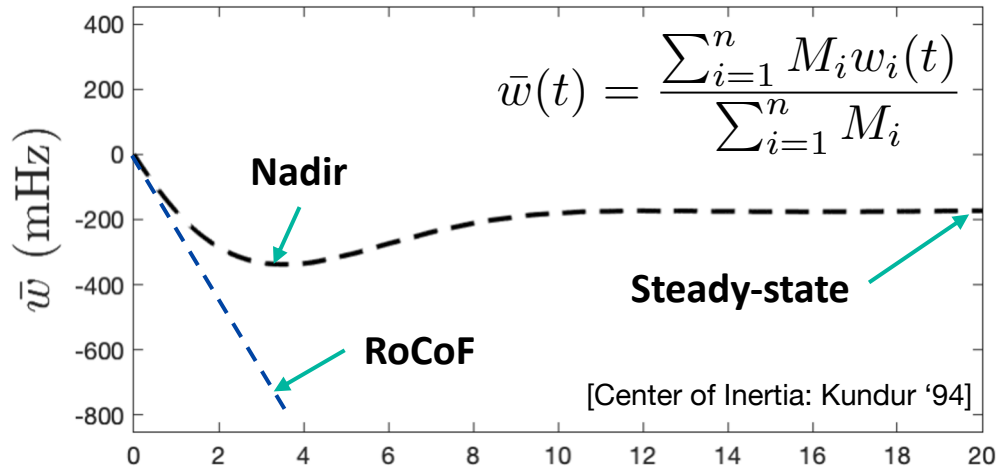
Decomposition of Step Response



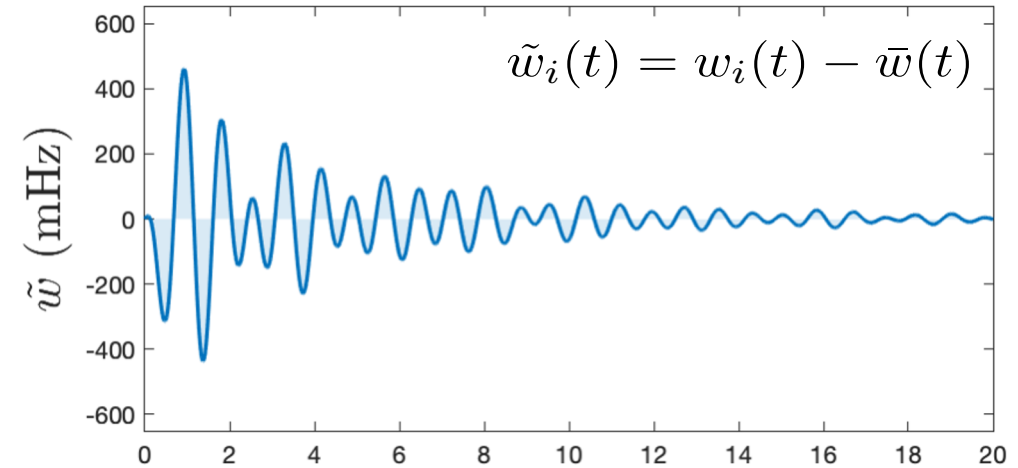
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 [TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control**, 2021

Step Disturbance Performance

System Frequency



Deviation from Mean



Nadir

$$\|\bar{w}\|_{\infty} := \sup_{t \geq 0} |\bar{w}(t)|$$

RoCoF

$$\|\dot{\bar{w}}\|_{\infty} := \sup_{t \geq 0} |\dot{\bar{w}}(t)|$$

Steady-state

$$\|\dot{\bar{w}}\|_{\infty} := \sup_{t \geq 0} |\dot{\bar{w}}(t)|$$

Synchronization Cost

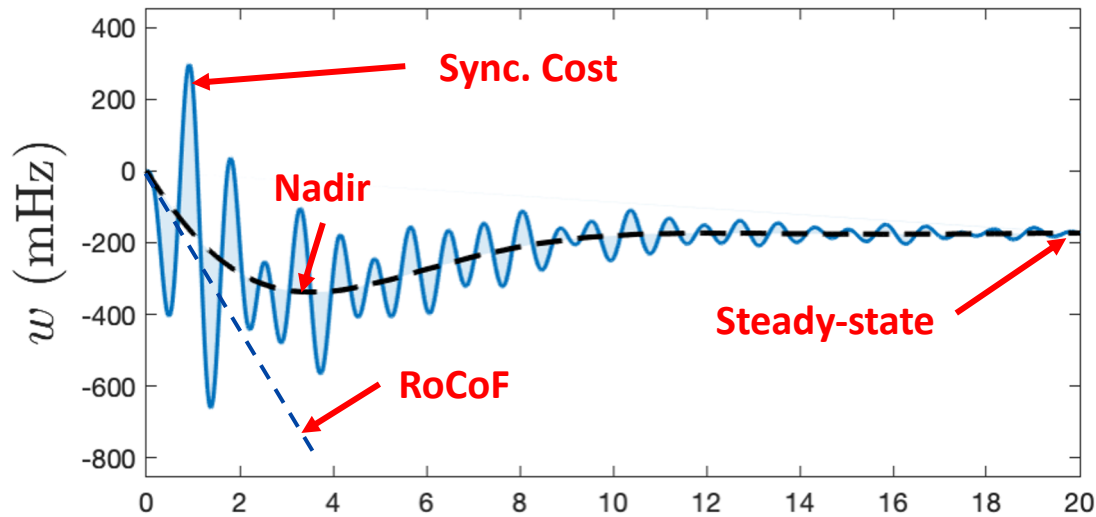
$$\|\tilde{w}\|_2 = \left(\int_0^{+\infty} \sum_{i=1}^n \tilde{w}_i^2(t) dt \right)^{\frac{1}{2}}$$

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, IEEE Transactions on Automatic Control, 2020

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Performance Specification

Frequency Response



Control Effort

System Freq. :
$$\bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

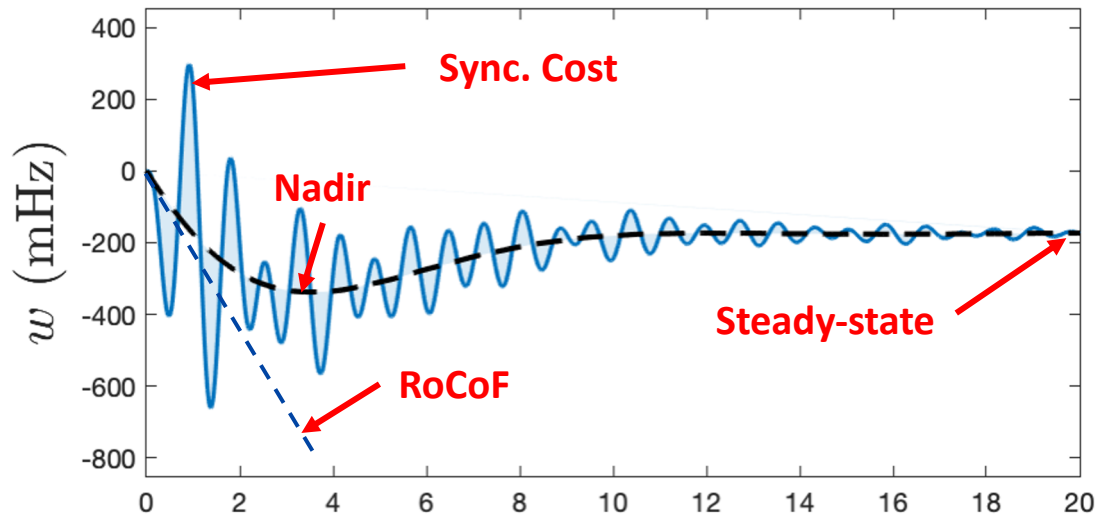
Sync. Error :
$$\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

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Performance Specification

Frequency Response



Control Effort

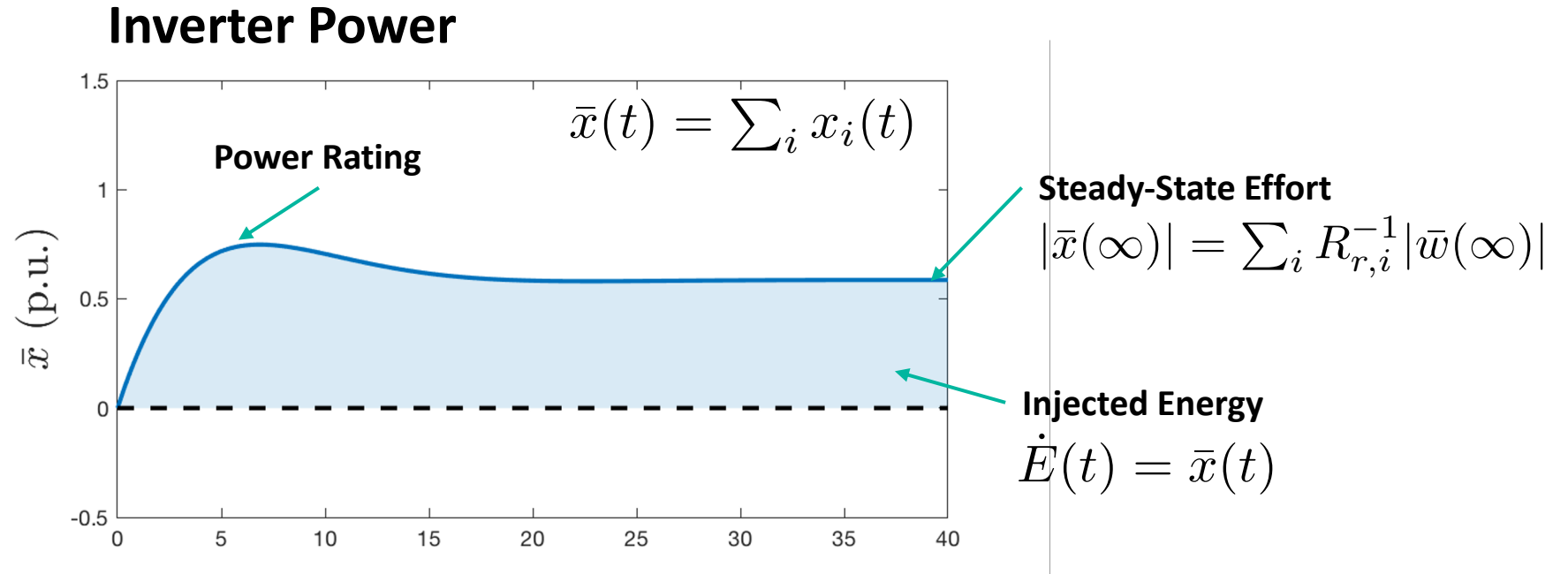
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Control Effort



Power Rating

$$\|\bar{x}\|_{\infty} := \sup_{t \geq 0} |\bar{x}(t)|$$

Max Energy

$$\|E\|_{\infty} := \sup_{t \geq 0} |E(t)|$$

Steady-State Effort

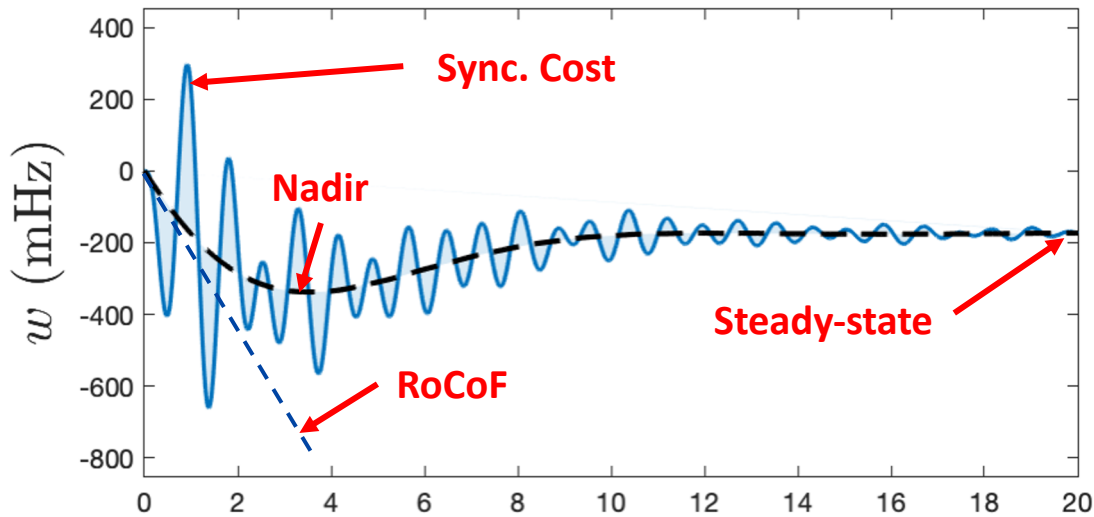
$$|\bar{x}(\infty)| = \sum_i R_{r,i}^{-1} |\bar{w}(\infty)|$$

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Performance Specification

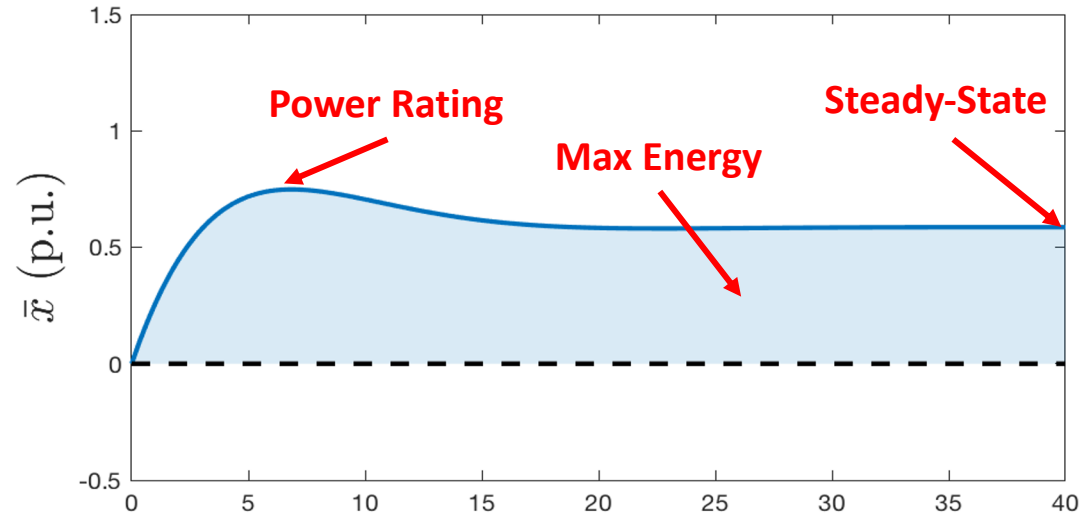
Frequency Response



$$\text{System Freq. : } \bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

$$\text{Sync. Error : } \tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Control Effort



$$\text{Injected Power: } \bar{x}(t) = \sum_i x_i(t)$$

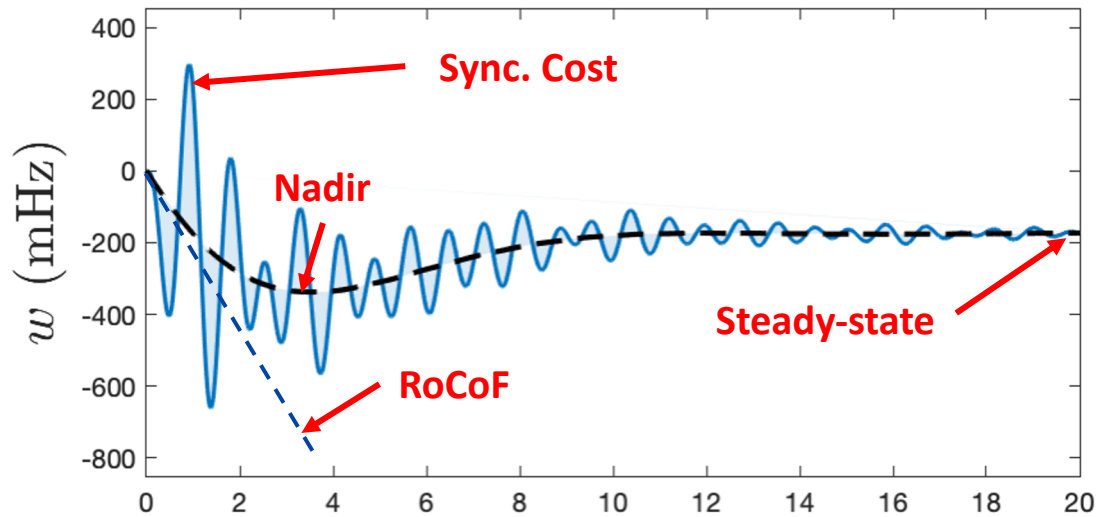
$$\text{Injected Energy: } \dot{E}(t) = \bar{x}(t)$$

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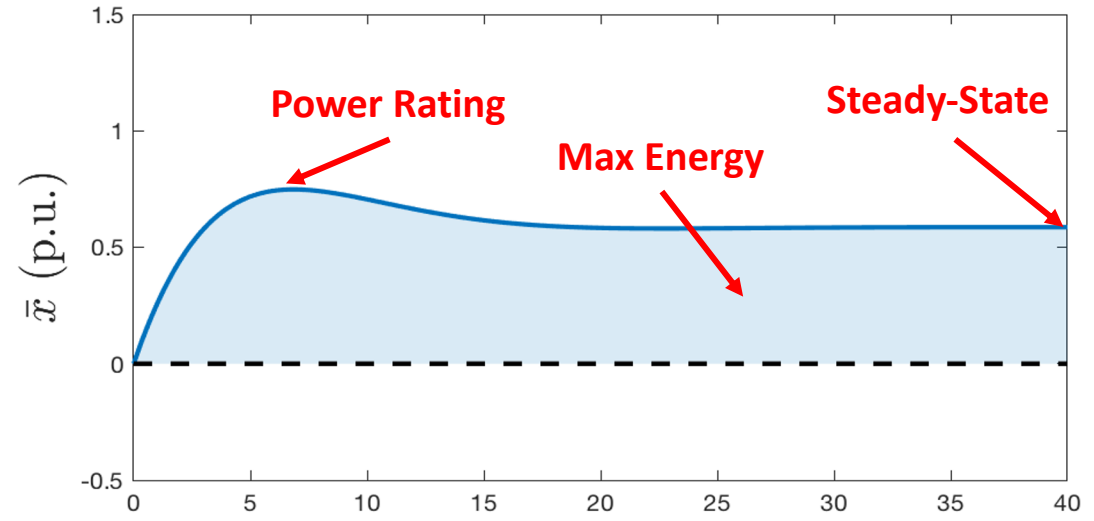
Frequency Response



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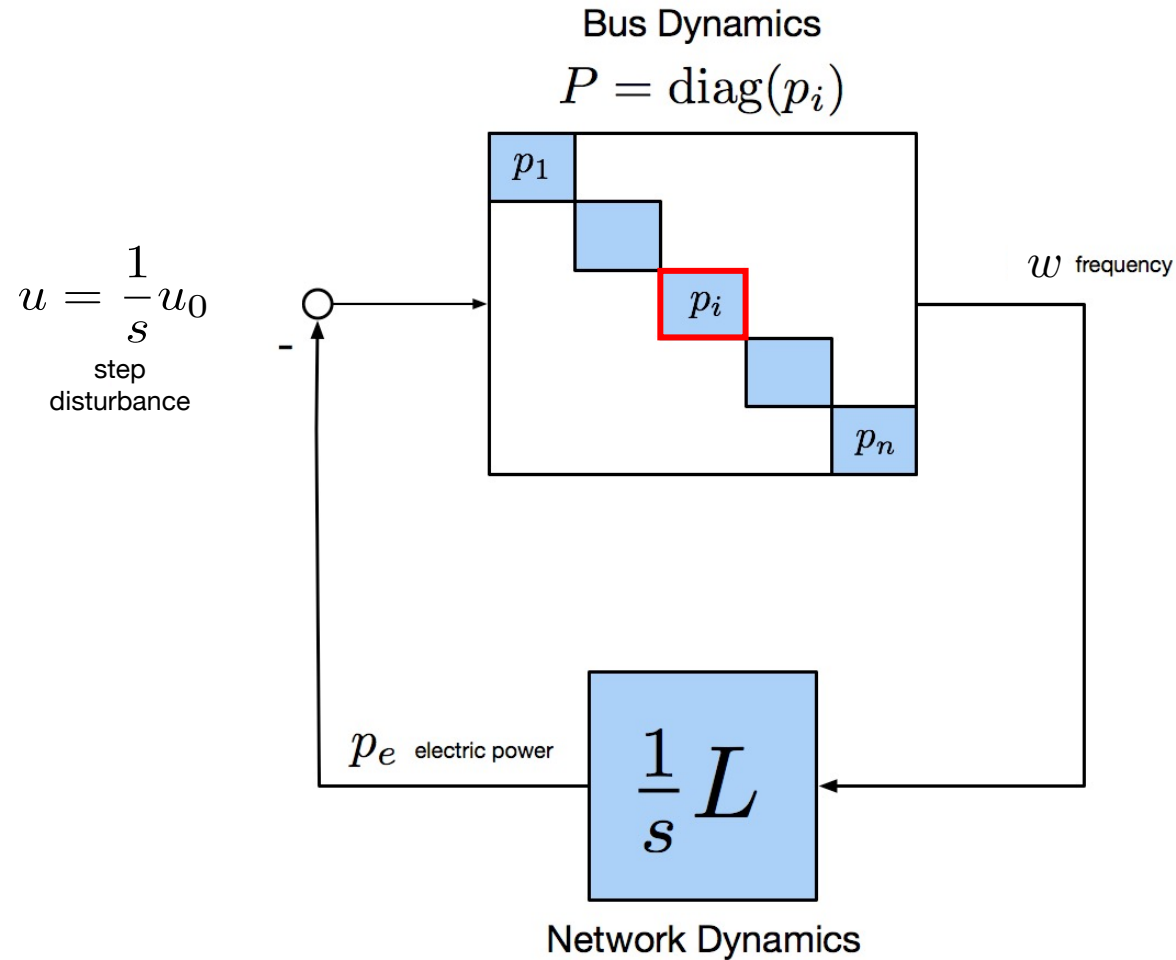


$$\text{Injected Power: } \bar{x}(t) = \sum_i x_i(t)$$

$$\text{Injected Energy: } \dot{E}(t) = \bar{x}(t)$$

Benchmark: Quantify control ability to eliminate overshoot in the Nadir

Power Network Model



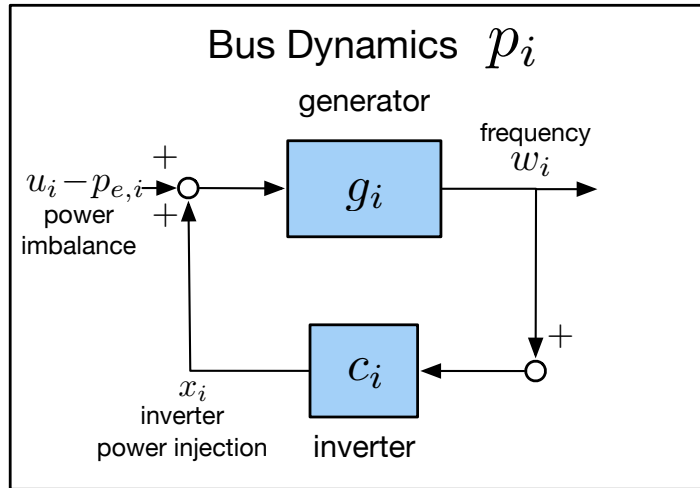
Step: $u = \frac{1}{s} u_0$

Laplacian Matrix

$$L_{ij} = \begin{cases} -B_{ij} & \text{if } ij \in E \\ \sum_k B_{ik} & \text{if } i = j \\ 0 & \text{o.w.} \end{cases}$$

[Bergen Hill '81]

Bus Dynamics

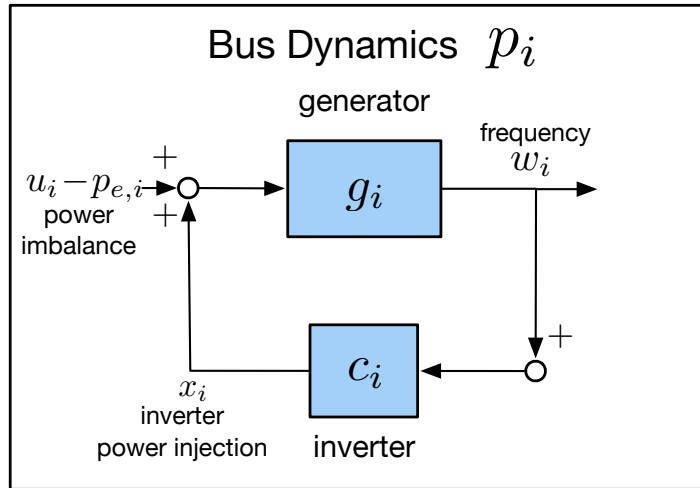


Generator: $g_i : (u_i - p_{e,i} + x_i) \mapsto w_i$

Model: Swing Equations + Turbine

$$g_i : \begin{cases} \dot{\theta}_i = w_i \\ M_i \dot{w}_i = -D_i w_i + q_i + (u_i - p_{e,i} + x_i) \\ \tau_i \dot{q}_i = -R_{g,i}^{-1} w_i - q_i \end{cases}$$

Bus Dynamics



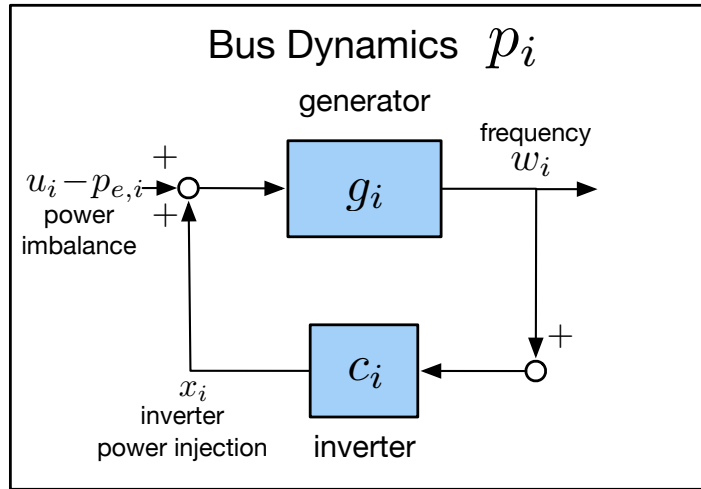
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$$g_i(s) = \frac{\tau_i s + 1}{M_i \tau_i s^2 + (M_i + D_i \tau_i) s + D_i + r_{g,i}^{-1}}$$

Bus Dynamics



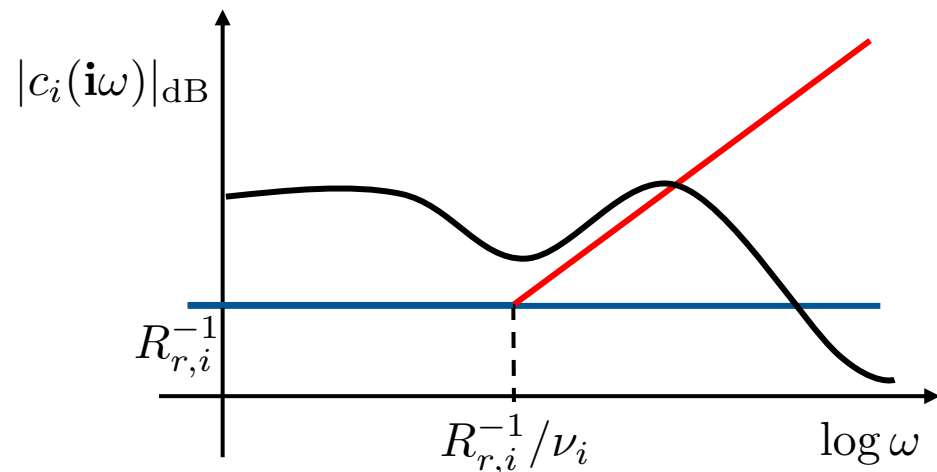
Inverter: $c_i : w_i \mapsto x_i$

Droop Control and Virtual Inertia:

$$c_i : \begin{cases} x_i = -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), \\ c_i(s) = -(\nu_i s + R_{r,i}^{-1}) \end{cases}$$

Closed-loop Bus Dynamics:

$$p_i : \begin{cases} \dot{\theta}_i = w_i \\ (M_i + \nu_i) \dot{w}_i = -(D_i + R_{r,i}^{-1}) w_i + q_i + (u_i - p_{e,i}) \\ \tau_i \dot{q}_i = -q_i - R_{g,i}^{-1} w_i \end{cases}$$



Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i be the *normalized rating*, $f_i = \frac{S_i}{S_{\text{base}}}$, and assume

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

Swing Equations + Turbine

$$g_0(s) = \frac{\tau s + 1}{m\tau s^2 + (m + d\tau)s + d + r^{-1}}$$

Virtual Inertia

$$c_0(s) = -(\nu s + r_r^{-1})$$

$$M_i = f_i m, \quad D_i = f_i d, \quad R_{g,i} = \frac{1}{f_i} r_g, \quad \tau_i = \tau$$

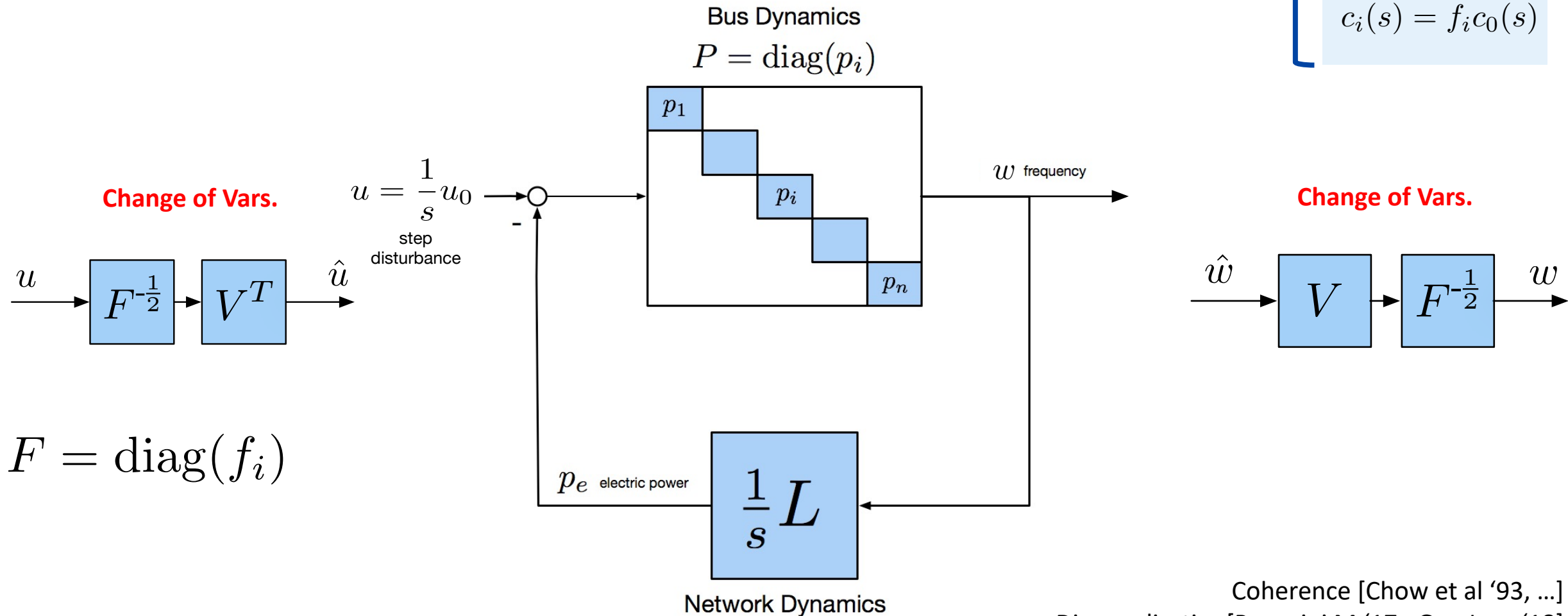
$$\nu_i = f_i \nu, \quad R_{r,i} = \frac{1}{f_i} r_r$$

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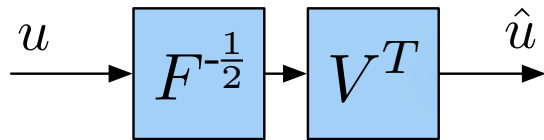
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$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

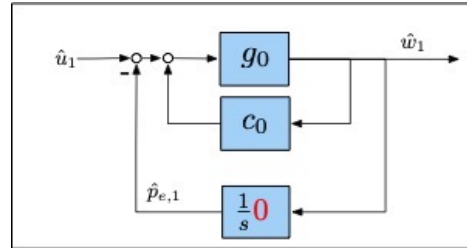
Change of Vars.



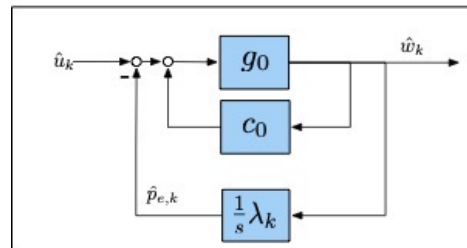
$$F = \text{diag}(f_i)$$

Eigenvalues of: $L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}$

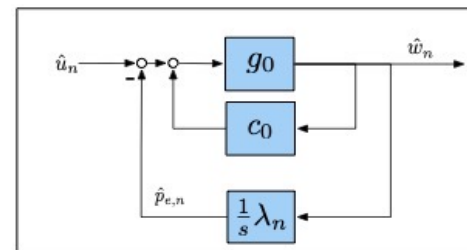
$$0 = \lambda_0 \leq \lambda_1 \leq \dots \leq \lambda_{n-1}$$



⋮



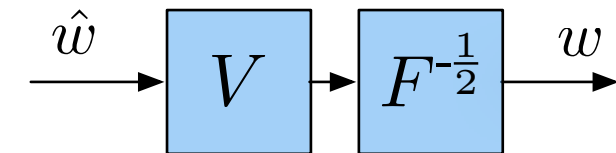
⋮



System Frequency

$$\bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

Change of Vars.



Sync Error

$$\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Coherence [Chow et al '93, ...]

Diagonalization [Paganini M '17, Guo Low '18]

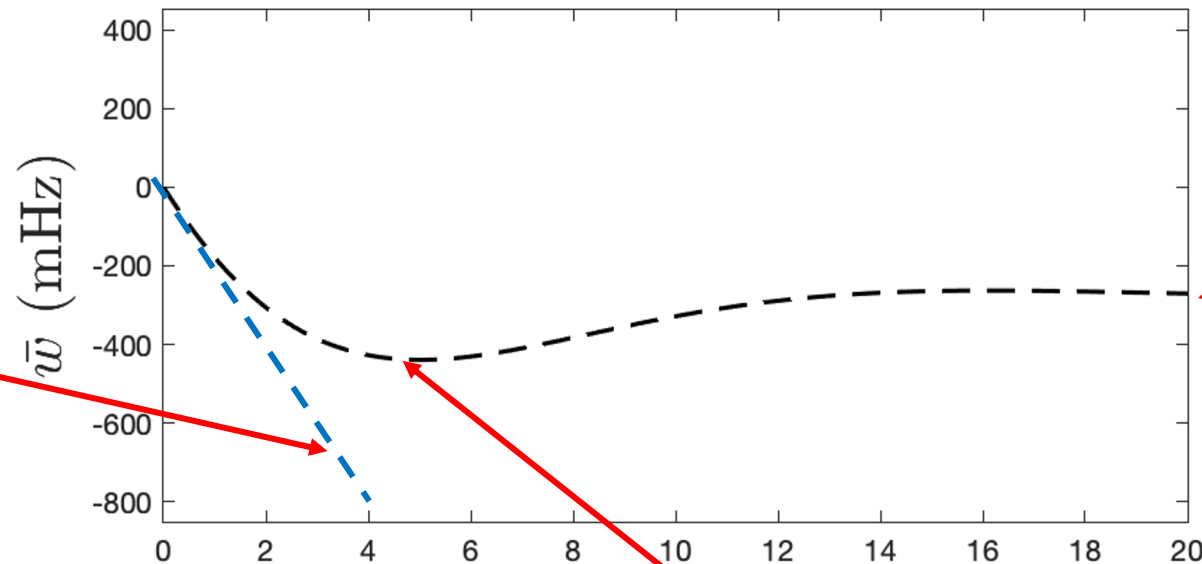
System Frequency – Performance Analysis w/o Inverters

System frequency \bar{w} response:

$$\bar{w}(t) = \frac{\sum_i u_{0i}}{\sum_i f_i} \frac{1}{d + r_g^{-1}} \left[1 - e^{-\eta t} \left(\cos(\omega_d t) - \frac{\gamma - \eta}{\omega_d} \sin(\omega_d t) \right) \right]$$

Maximal RoCoF:
initial response.
Inertia appears directly

$$\|\bar{w}\|_\infty = \frac{|\sum_i u_{0,i}|}{\sum_i f_i} \frac{1}{m}$$



Steady-state.
No dependence on inertia

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i} \frac{1}{d + r_g^{-1}}$$

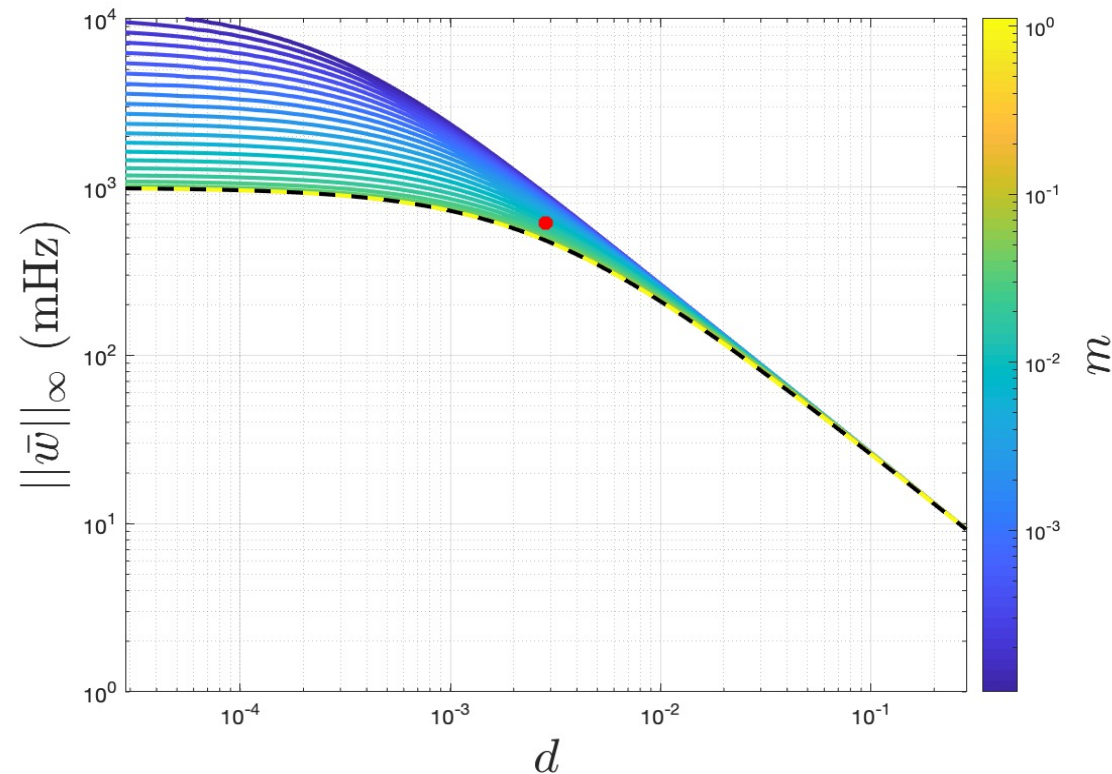
Nadir at overshoot.
Decreases (mildly)
with inertia

$$\|\bar{w}\|_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{d + r_g^{-1}} \left(1 + \sqrt{\frac{\tau r_g^{-1}}{m}} e^{-\frac{\eta}{\omega_d} (\phi + \frac{\pi}{2})} \right)$$

System Frequency – Nadir Sensitivity w/o Inverters

Nadir:

$$\|\bar{w}\|_{\infty} = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{d + r_g^{-1}} \left(1 + \sqrt{\frac{\tau r_g^{-1}}{m}} e^{-\frac{\eta}{\omega_d} (\phi + \frac{\pi}{2})} \right)$$

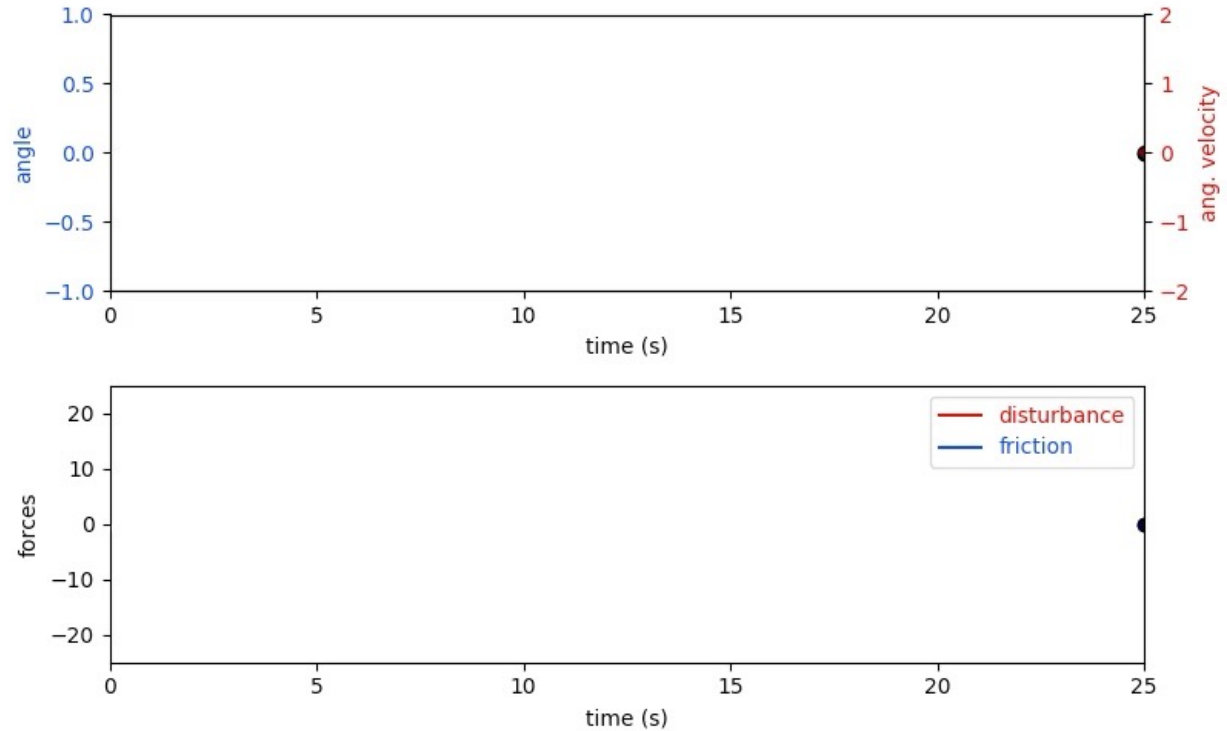
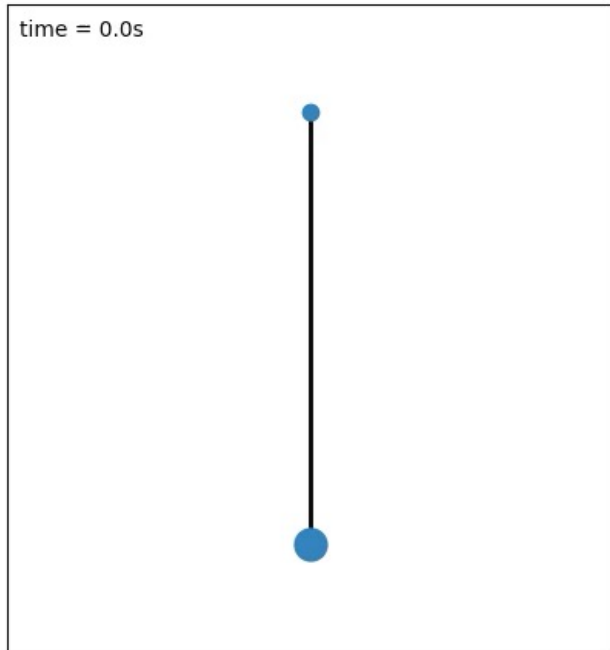


Roadmap to Low Inertia Frequency Control

- Network Performance Analysis
- Limits of Virtual Inertia and Droop Control
- Control Design: Frequency Shaping

Control of **Low** Inertia Pendulum

$$m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f + u$$

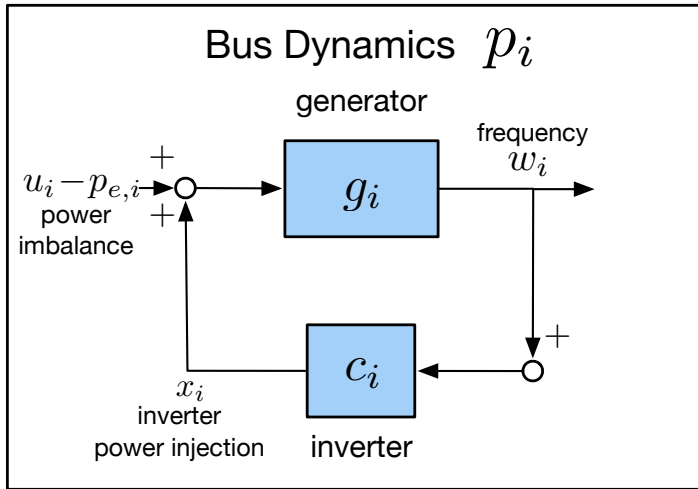


Cons: Susceptible to disturbances

Pros: Regains steady-state faster

Bus Dynamics /w Virtual Inertia and Droop Control

Inverter: $C_i : w_i \mapsto x_i$

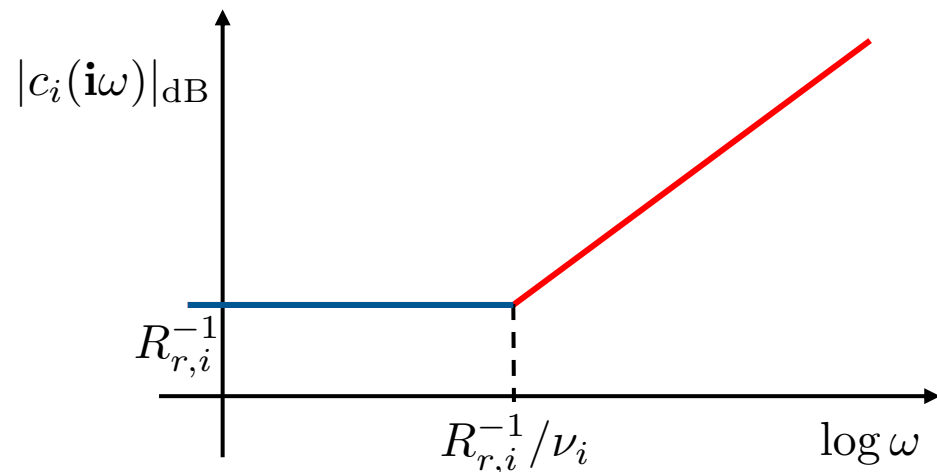


Droop Control and Virtual Inertia:

$$C_i : \left\{ \begin{aligned} x_i &= -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), \\ C_i(s) &= -(\nu_i s + R_{r,i}^{-1}) \end{aligned} \right.$$

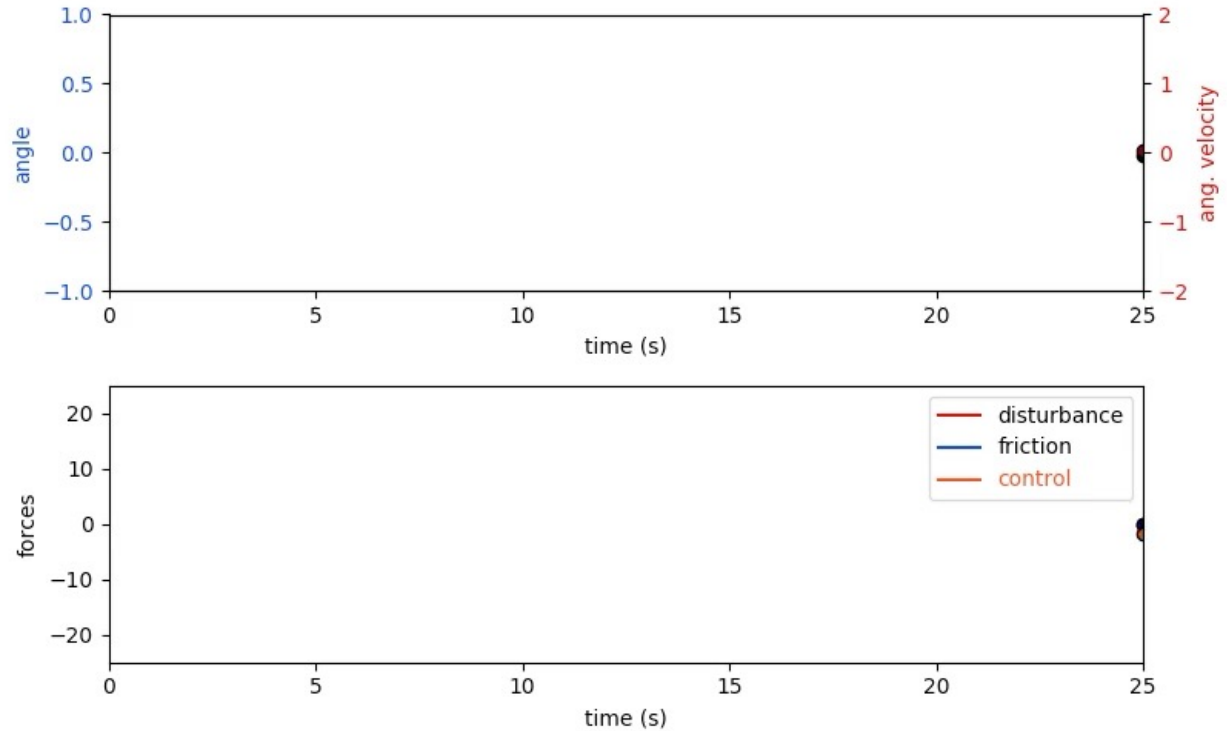
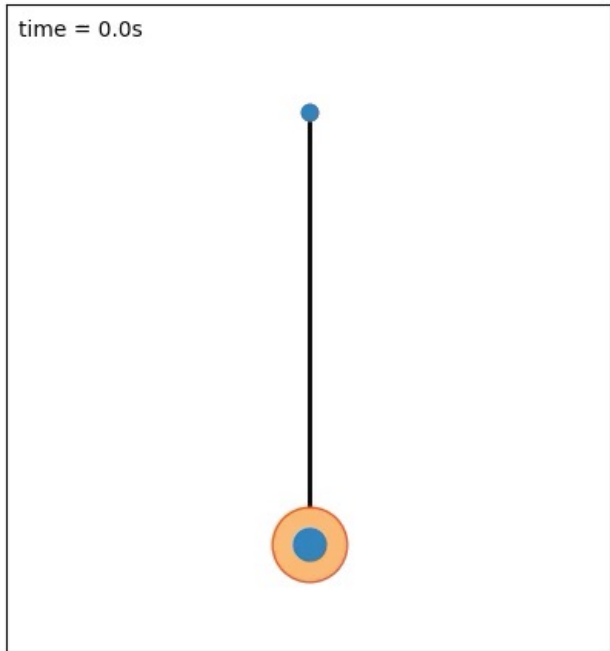
Closed-loop Bus Dynamics:

$$p_i : \left\{ \begin{aligned} \dot{\theta}_i &= w_i \\ (M_i + \nu_i) \dot{w}_i &= -(D_i + R_{r,i}^{-1}) w_i + q_i + (u_i - p_{e,i}) \\ \tau_i \dot{q}_i &= -q_i - R_{g,i}^{-1} w_i \end{aligned} \right.$$



Control of **Low** Inertia Pendulum

Virtual **Mass** Control: $m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f - \nu\ddot{\theta}$



Pros:

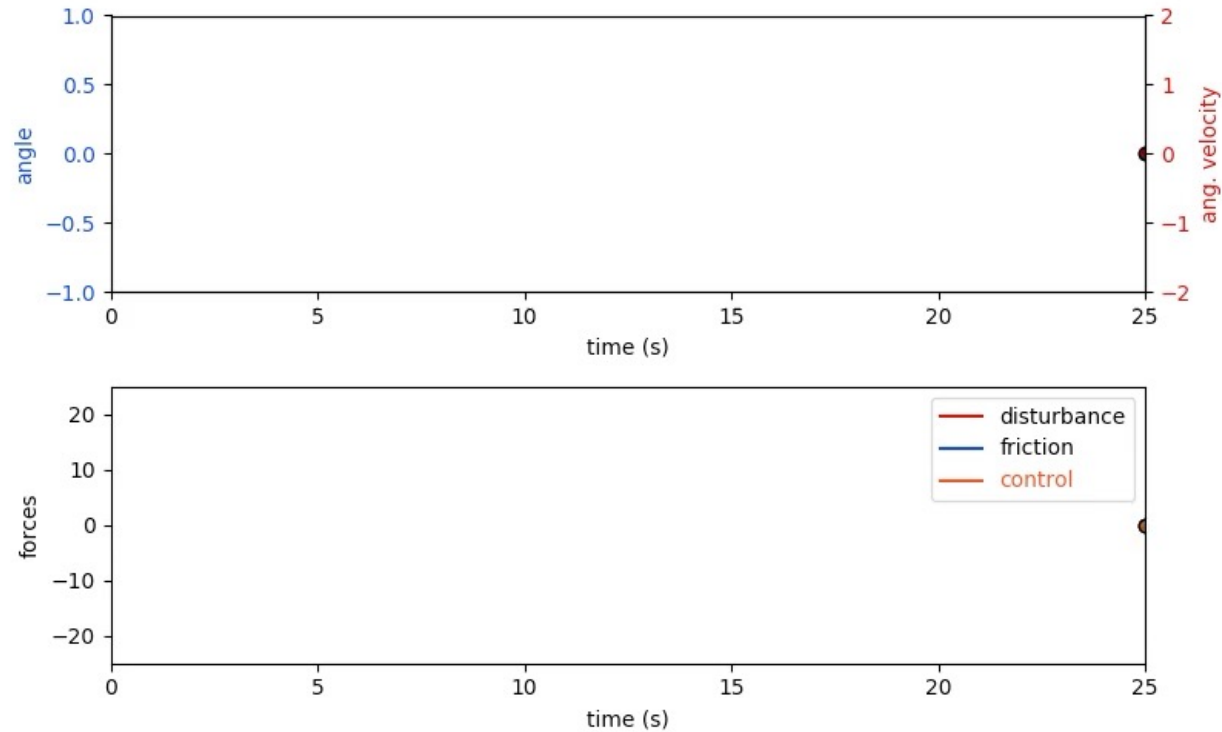
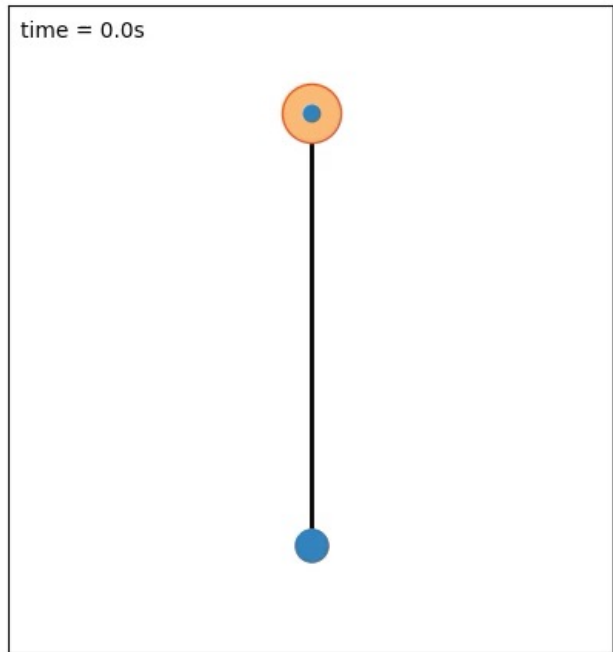
Provides disturbance rejection

Cons:

Hard to regain steady-state + **excessive control effort**

Control of **Low** Inertia Pendulum

Virtual **Friction** Control: $m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f - r^{-1}\dot{\theta}$



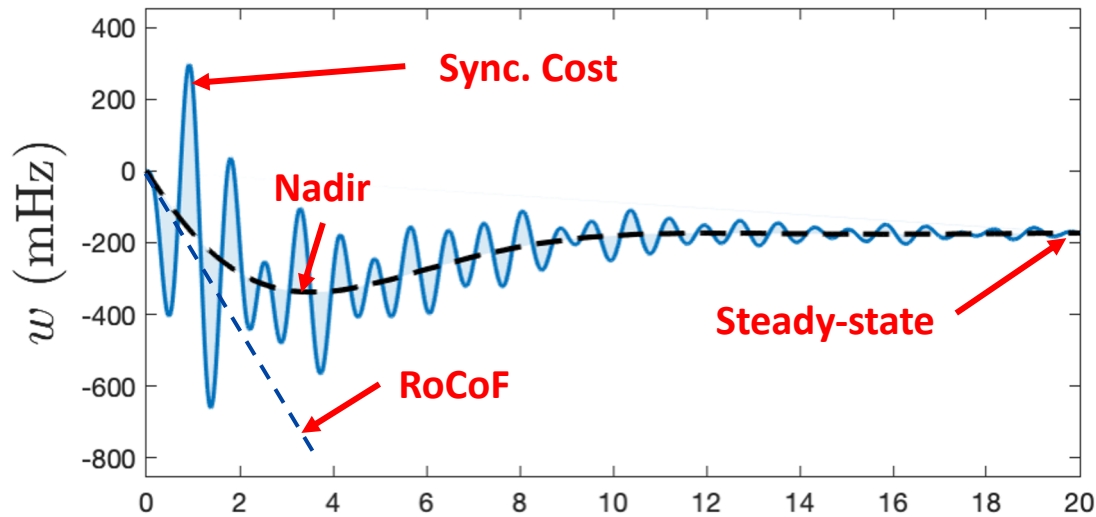
Pros: Provides disturbance rejection, quickly restore steady-state, with reasonable control effort.

Cons?

None, at least for pendulum

Performance Specification

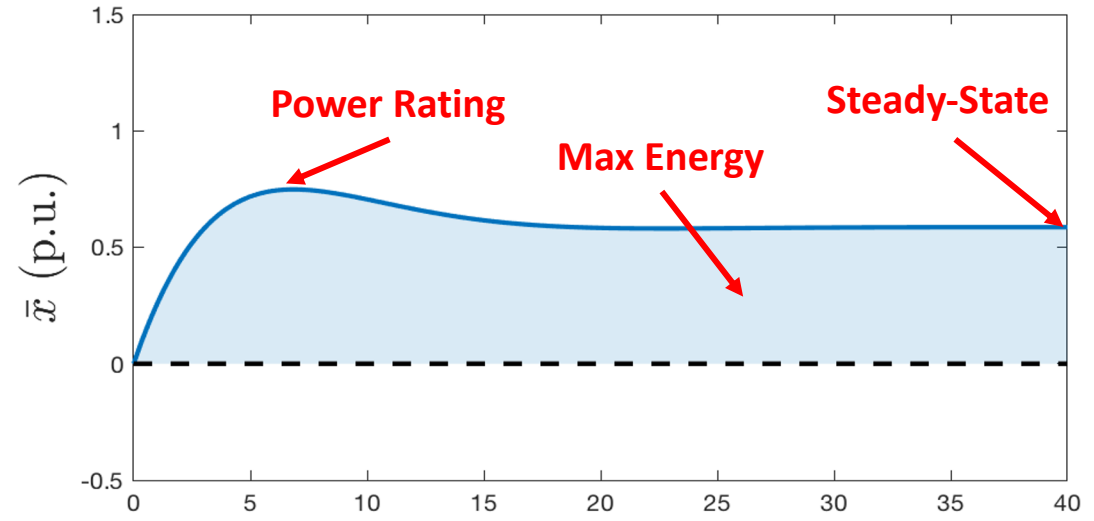
Frequency Response



$$\text{System Freq. : } \bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

$$\text{Sync. Error : } \tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Control Effort



$$\text{Injected Power: } \bar{x}(t) = \sum_i x_i(t)$$

$$\text{Injected Energy: } \dot{E}(t) = \bar{x}(t)$$

Benchmark: Quantify control ability to eliminate overshoot in Nadir

System Frequency w/ Virtual Inertia

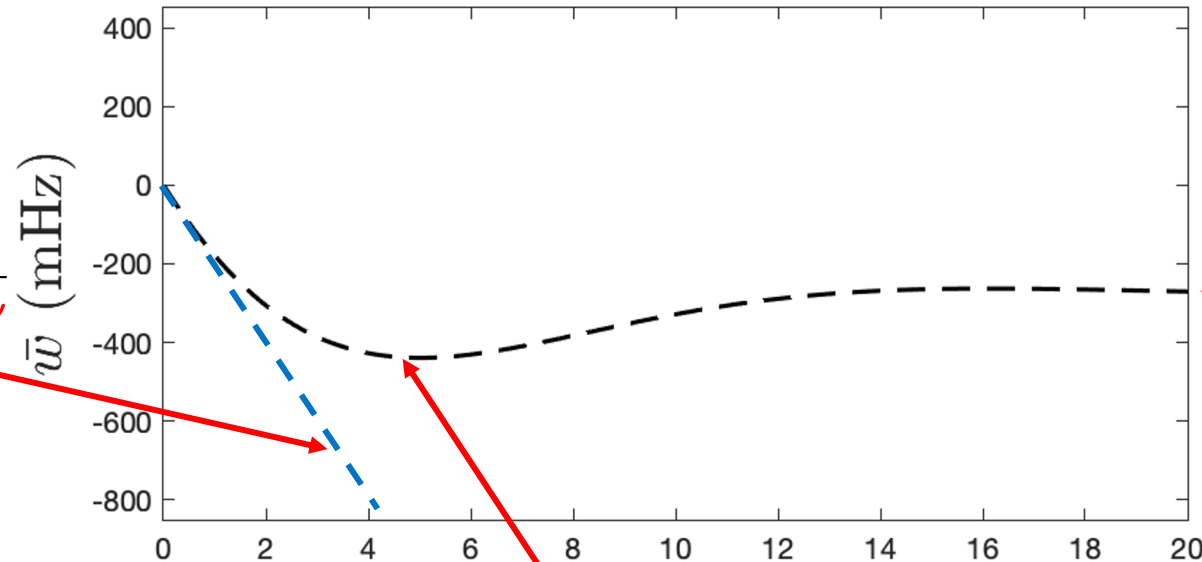
$$C_i : x_i = -f_i(\nu \dot{w}_i + r_r^{-1} w_i)$$

Swing Equations with Turbine: 2nd order response, e.g. underdamped

$$\bar{w}(t) = \frac{\sum_i u_{0i}}{\sum_i f_i} \frac{1}{d + r_g^{-1} + r_r^{-1}} \left[1 - e^{-\eta t} \left(\cos(\omega_d t) - \frac{\gamma - \eta}{\omega_d} \sin(\omega_d t) \right) \right]$$

Maximal RoCoF:
initial response.
Inertia appears directly

$$\|\dot{\bar{w}}\|_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{m + \nu} \bar{w} \text{ (mHz)}$$



Steady-state.
No dependence on inertia

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i} \frac{1}{d + r_g^{-1} + r_r^{-1}}$$

Nadir at overshoot.
Decreases (mildly)
with inertia

$$\|\bar{w}\|_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{d + r_g^{-1} + r_r^{-1}} \left(1 + \sqrt{\frac{\tau r_g^{-1}}{m + \nu}} e^{-\frac{\eta}{\omega_d} (\phi + \frac{\pi}{2})} \right)$$

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, IEEE Transactions on Automatic Control, 2021

System Frequency w/ Virtual Inertia

$$C_i : x_i = -f_i(\nu \dot{w}_i + r_r^{-1} w_i)$$

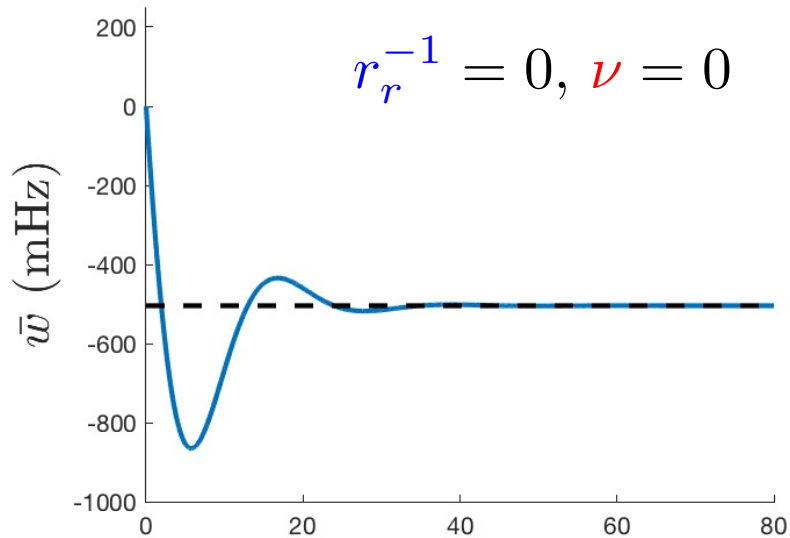
Nadir Overshoot Elimination:

$$\nu \geq \nu_{\min} := \tau_g \left(\sqrt{r_g^{-1}} + \sqrt{d + r_g^{-1} + r_r^{-1}} \right)^2 - m$$

No Control

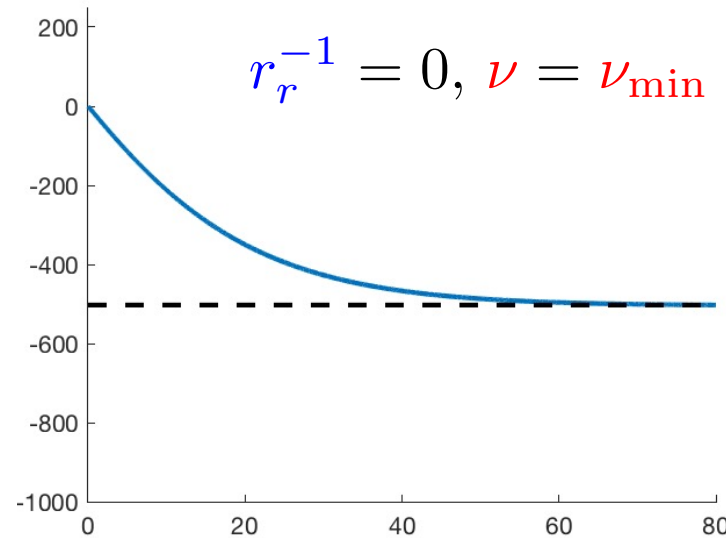
$$r_r^{-1} = 0, \nu = 0$$

System Freq.



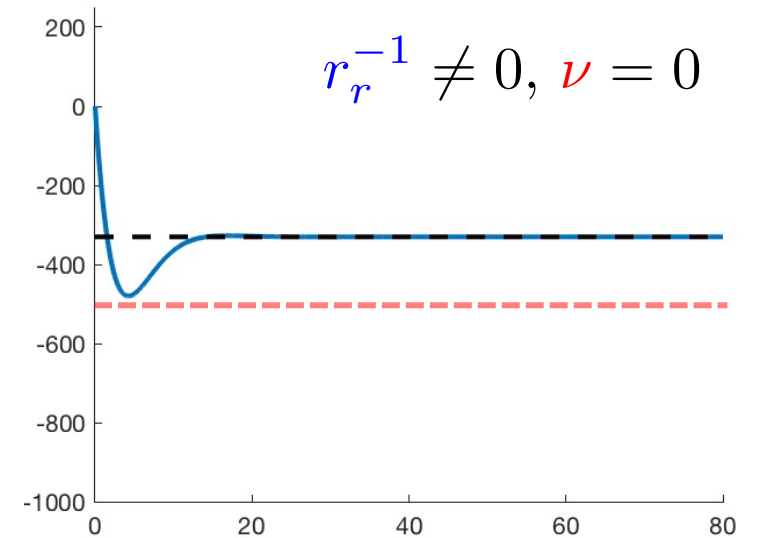
Virtual Inertia

$$r_r^{-1} = 0, \nu = \nu_{\min}$$



Droop Control

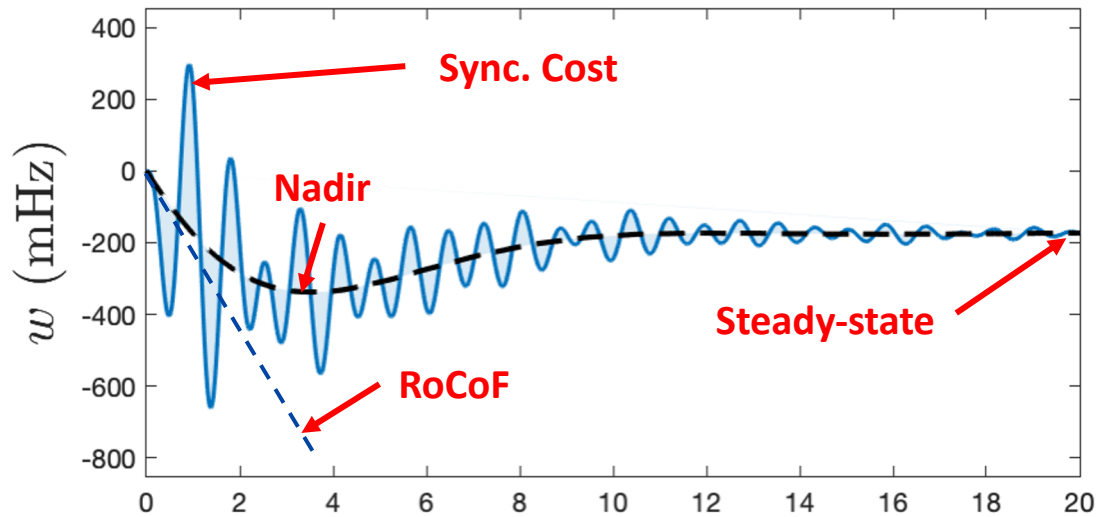
$$r_r^{-1} \neq 0, \nu = 0$$



requires $\nu > 0$ in low inertia systems (low m)

Performance Specification

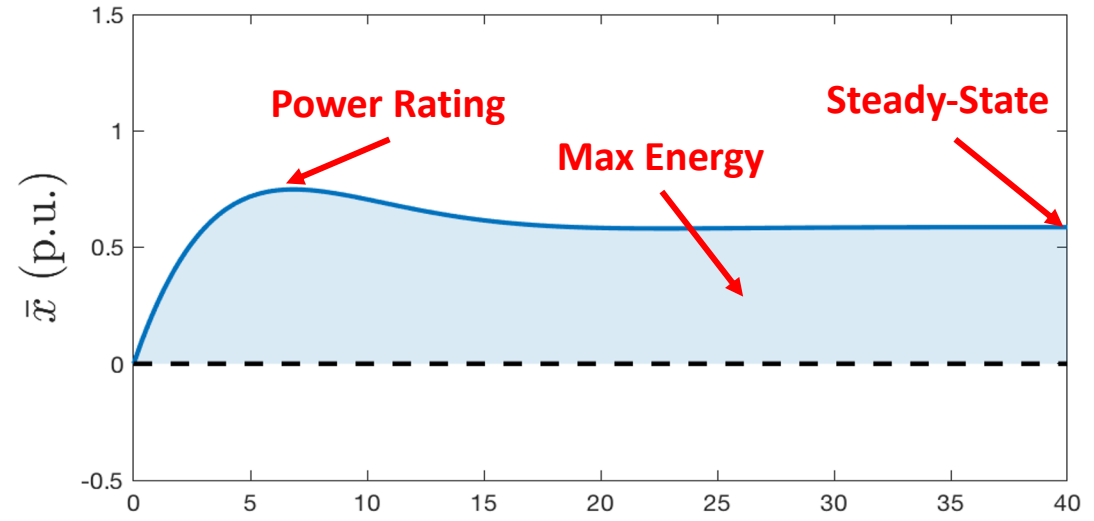
Frequency Response



$$\text{System Freq. : } \bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

$$\text{Sync. Error : } \tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Control Effort



$$\text{Injected Power: } \bar{x}(t) = \sum_i x_i(t)$$

$$\text{Injected Energy: } \dot{E}(t) = \bar{x}(t)$$

Benchmark: Quantify control ability to eliminate overshoot in Nadir

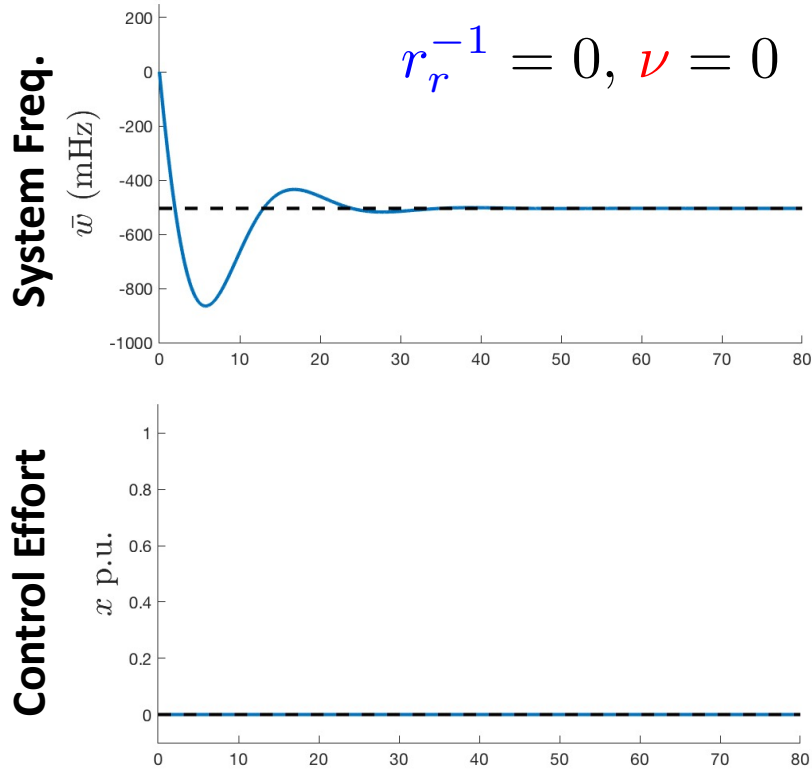
Control Effort

$$C_i : x_i = -f_i(\nu \dot{w}_i + r_r^{-1} w_i)$$

$$\bar{x}(t) = \sum_i x_i(t)$$

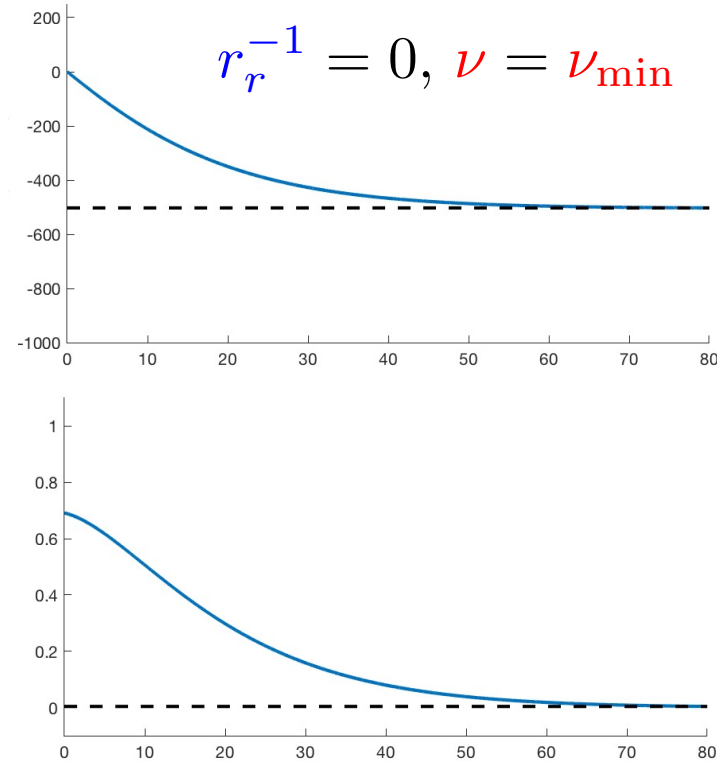
No Control

$$r_r^{-1} = 0, \nu = 0$$



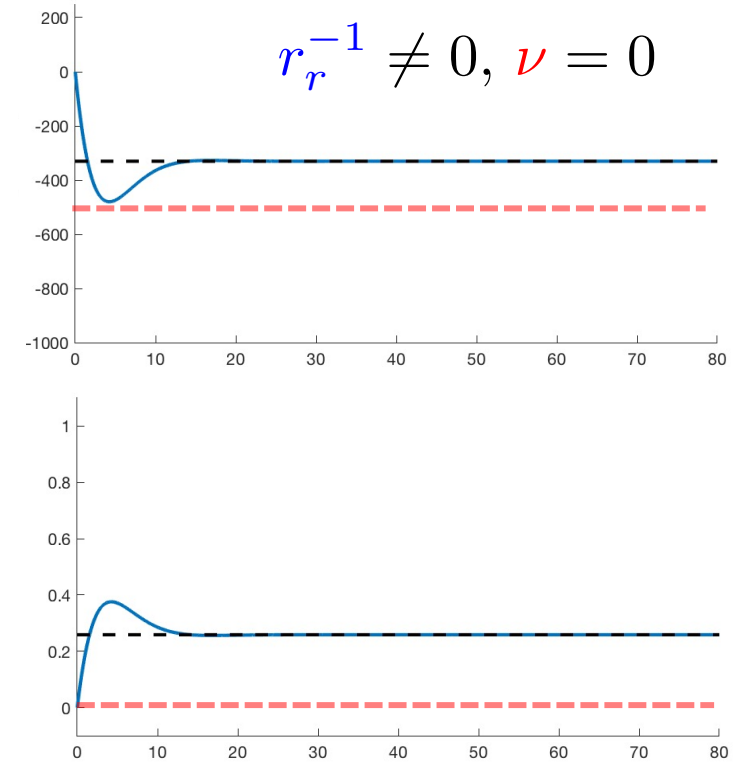
Virtual Inertia

$$r_r^{-1} = 0, \nu = \nu_{\min}$$



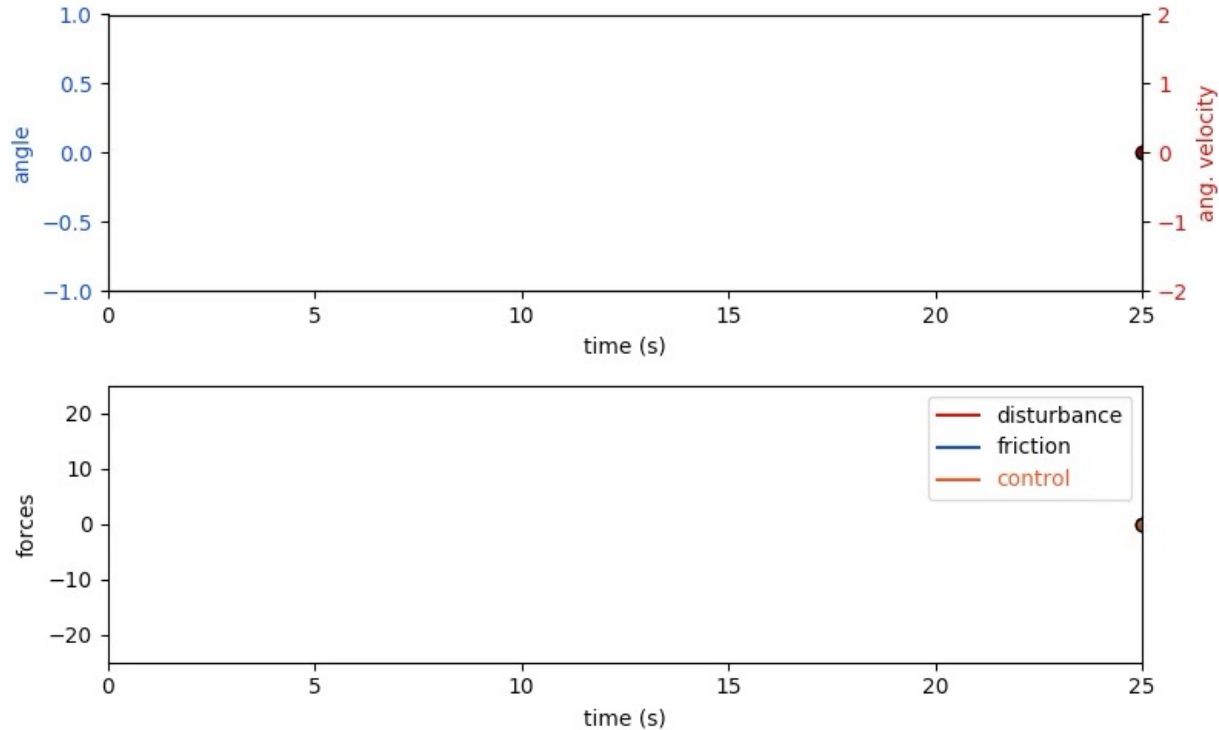
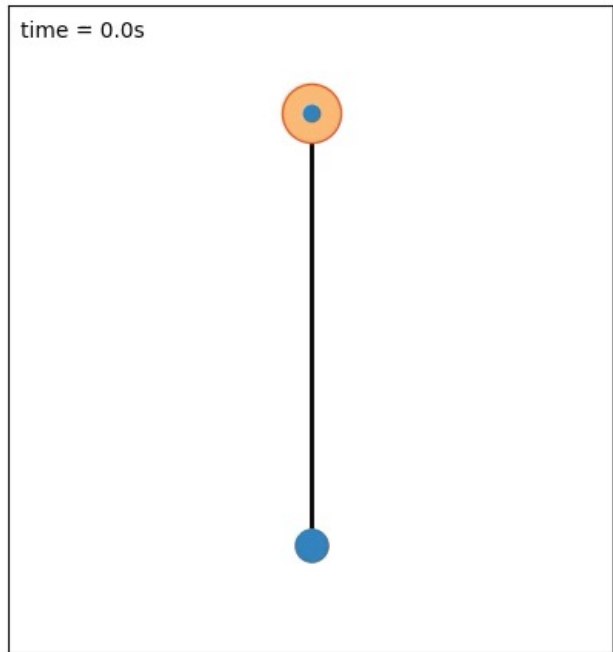
Droop Control

$$r_r^{-1} \neq 0, \nu = 0$$



Control of **Low** Inertia Pendulum

Virtual **Friction** Control: $m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f - r^{-1}\dot{\theta}$



Pros: Provides disturbance rejection, quickly restore steady-state, with reasonable control effort.

Cons? Large steady-state effort in power systems

Roadmap to Low Inertia Frequency Control

- Network Performance Analysis
- Limits of Virtual Inertia and Droop Control
- Control Design: Frequency Shaping

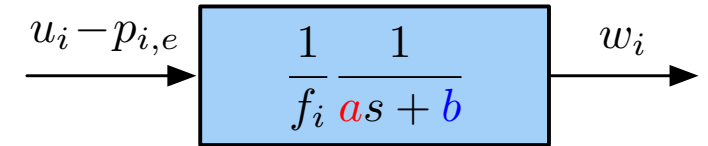
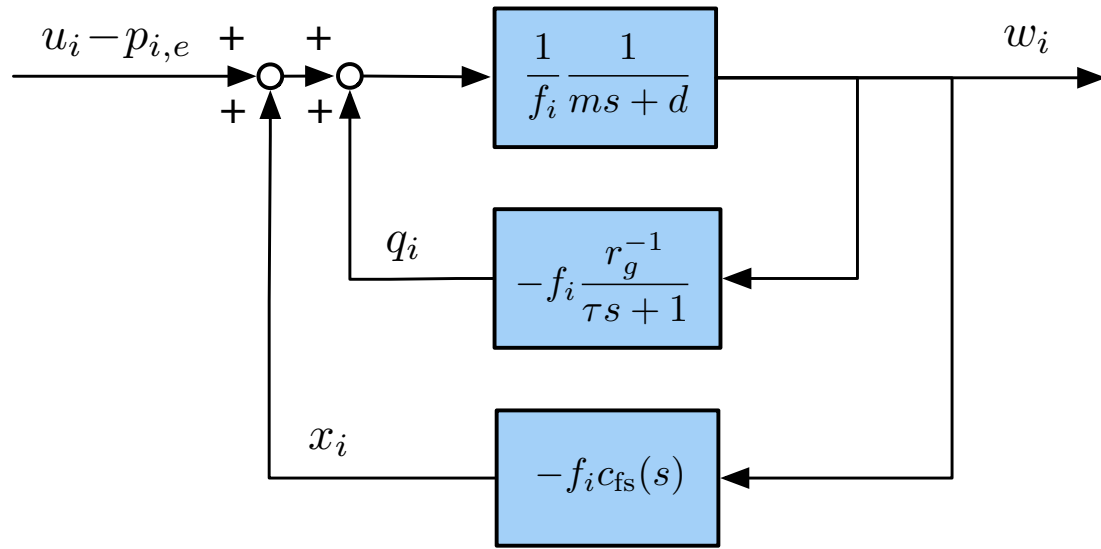
Frequency Shaping

Grid-following Inverters

Grid-forming Inverters

Grid-following Frequency Shaping Control

Key idea: use model matching control (at each bus/area)



Leads to Col Frequency \bar{w} with:

Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i} \frac{1}{b}$$

RoCoF:

$$\|\dot{\bar{w}}\|_{\infty} = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{a}$$

Col Response:

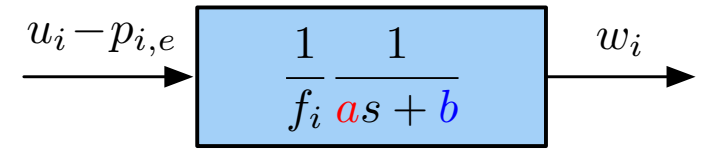
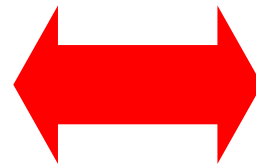
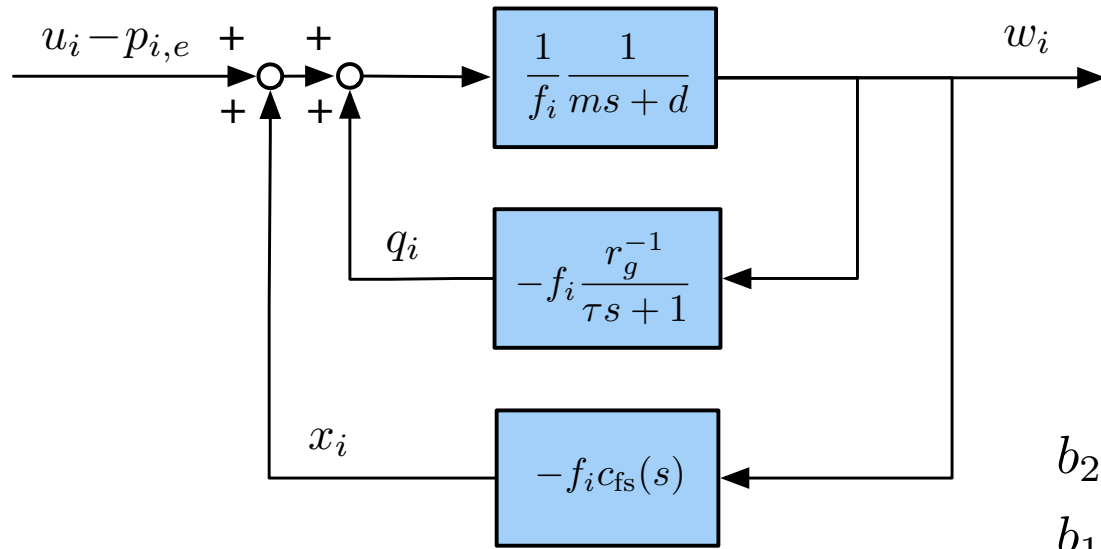
$$\bar{w}(t) = \frac{\sum_i u_{0,i}}{\sum_i f_i} \frac{1}{b} \left(1 - e^{-\frac{b}{a} t} \right)$$

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, IEEE Transactions on Automatic Control, 2021

[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, IEEE Transactions on Power Systems, 2021

Grid-following Frequency Shaping Control

Key idea: use model matching control (at each bus/area)



Leads to Col Frequency \bar{w} with:

Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i} \frac{1}{b}$$

RoCoF:

$$\|\dot{\bar{w}}\|_{\infty} = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{a}$$

$$b_2 = \tau (a - m)$$

$$b_1 = (b - d)\tau + a - m$$

$$b_0 = b - r_g^{-1} - d$$

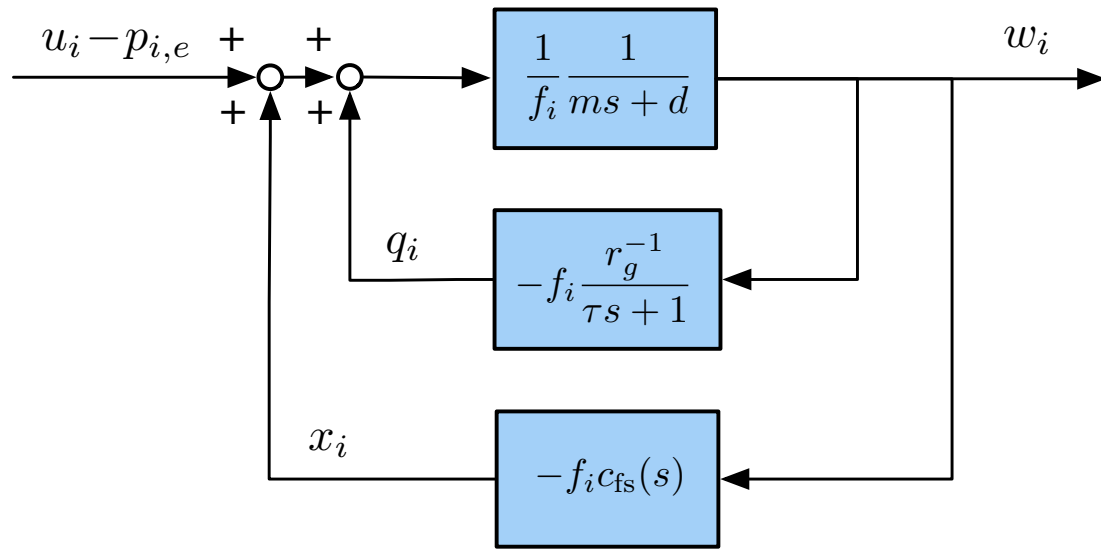
$$\tau' = \tau$$

$$c_{fs}(s) := \frac{b_2 s^2 + b_1 s + b_0}{\tau' s + 1}$$

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, IEEE Transactions on Automatic Control, 2021
 [TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, IEEE Transactions on Power Systems, 2021

Special case: Dynamic Droop (iDroop)

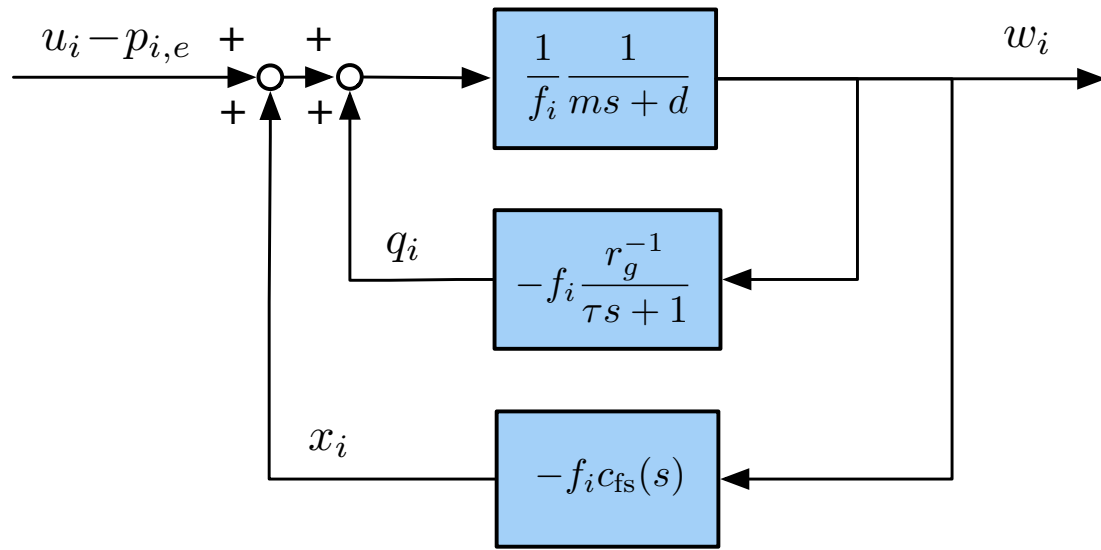
Setting $b_2 = 0$, $b_1 = \nu' \tau'$, and $b_2 = r_r^{-1}$ leads to:



$$c_{fs}(s) := \frac{b_2 s^2 + b_1 s + b_0}{\tau' s + 1}$$

Special case: Dynamic Droop (iDroop)

Setting $b_2 = 0$, $b_1 = \nu' \tau'$, and $b_2 = r_r^{-1}$ leads to:



$$c_{fs}(s) := \frac{\tau' \nu' s + r_r^{-1}}{\tau' s + 1}$$

Inverter Control

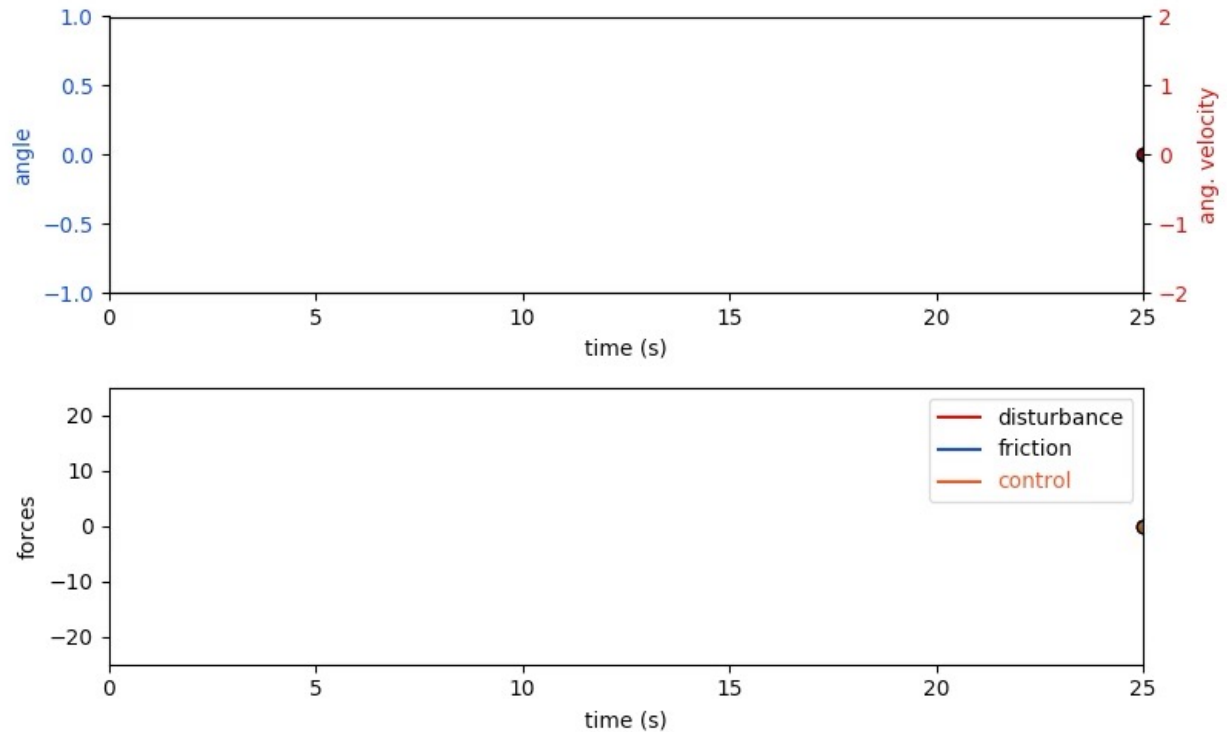
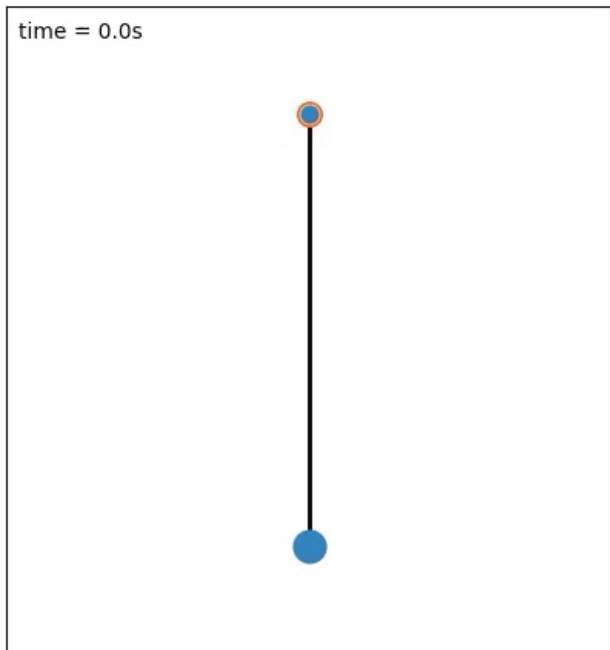
$$c_i(s) = -f_i c_{fs}(s) = -f_i \left(\frac{\tau' \nu' s + r_r^{-1}}{\tau' s + 1} \right)$$

$$c_i : \left\{ \begin{aligned} \tau' \dot{x}_i &= -x_i - f_i (r_r^{-1} w_i + \tau' \nu' \dot{w}_i) \end{aligned} \right.$$

Control of **Low** Inertia Pendulum

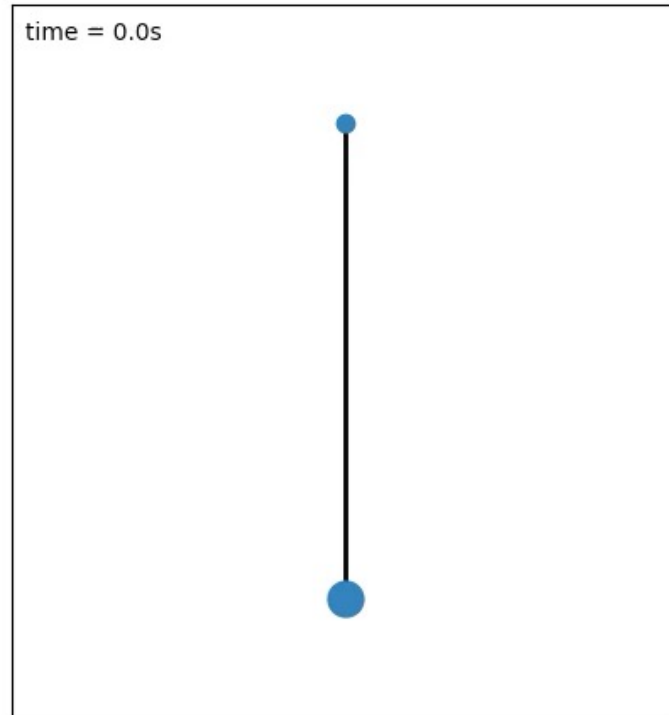
Dynamic Friction Control: $m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f + x$

$$\tau' \dot{x} = -x - (r_r^{-1} \dot{\theta} + \tau' \nu' \ddot{\theta})$$



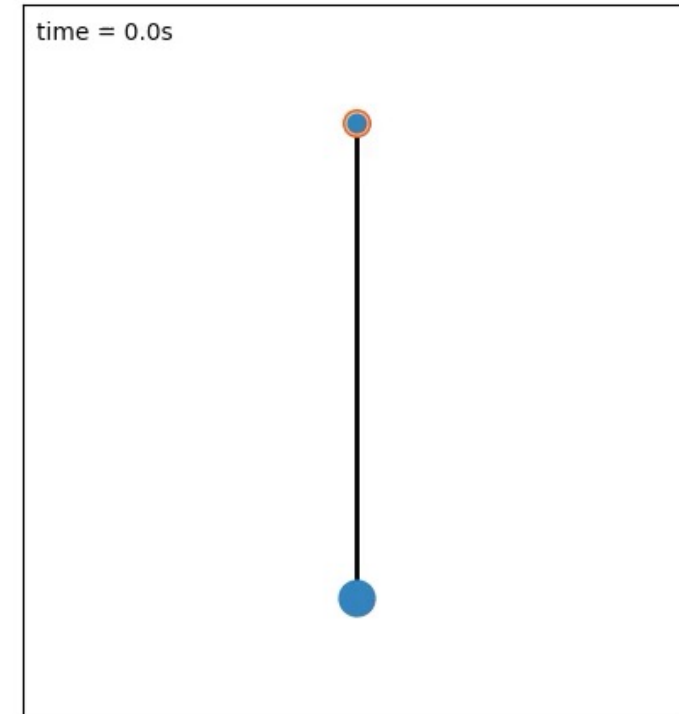
Control of **Low** Inertia Pendulum

No Control



$$m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f$$

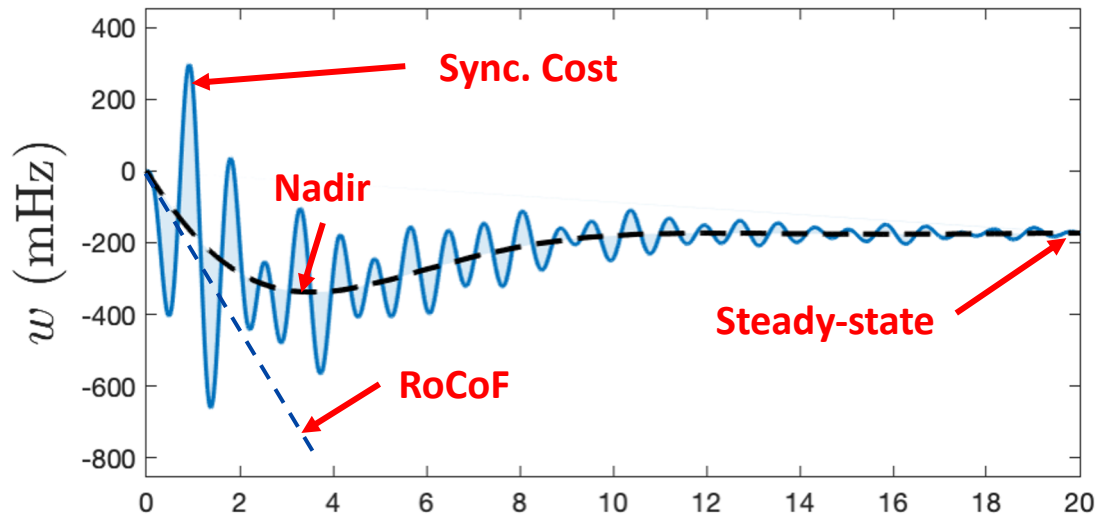
Dynamic Friction Control



$$m\ddot{\theta} = -d\dot{\theta} - mg \sin \theta + f + x$$
$$\tau' \dot{x} = -x - (r_r^{-1} \dot{\theta} + \tau' \nu' \ddot{\theta})$$

Performance Specification

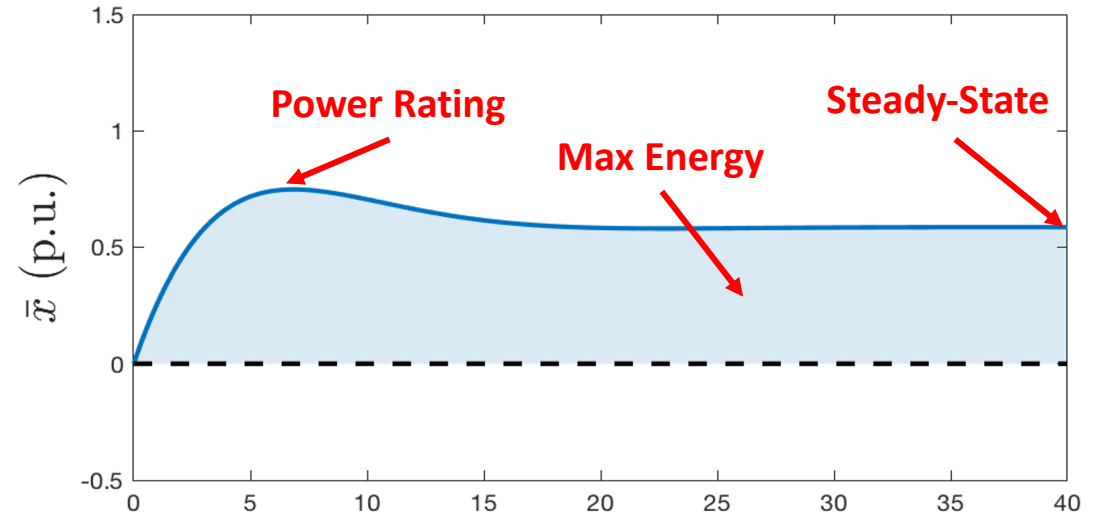
Frequency Response



$$\text{System Freq. : } \bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

$$\text{Sync. Error : } \tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Control Effort



$$\text{Injected Power: } \bar{x}(t) = \sum_i x_i(t)$$

$$\text{Injected Energy: } \dot{E}(t) = \bar{x}(t)$$

Benchmark: Quantify control ability to eliminate overshoot in Nadir

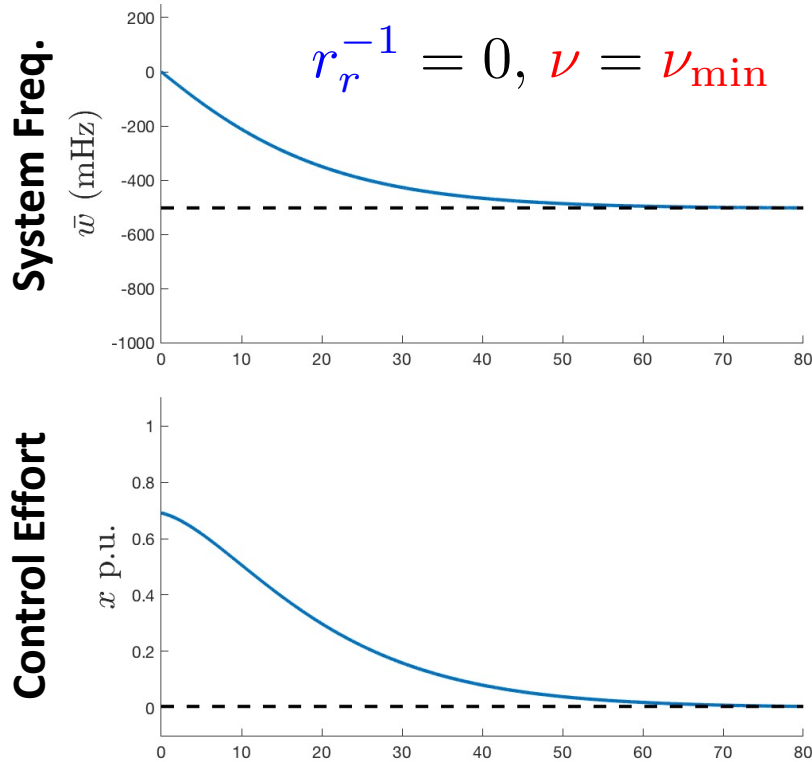
Frequency Shaping w/ iDroop

$$C_i: \tau' \dot{x}_i = -x_i - f_i \left(r_r^{-1} w_i + \tau \nu' \dot{w}_i \right)$$

Whenever $\nu' = r_r^{-1} + r_g^{-1}$ and $\tau' = \tau$

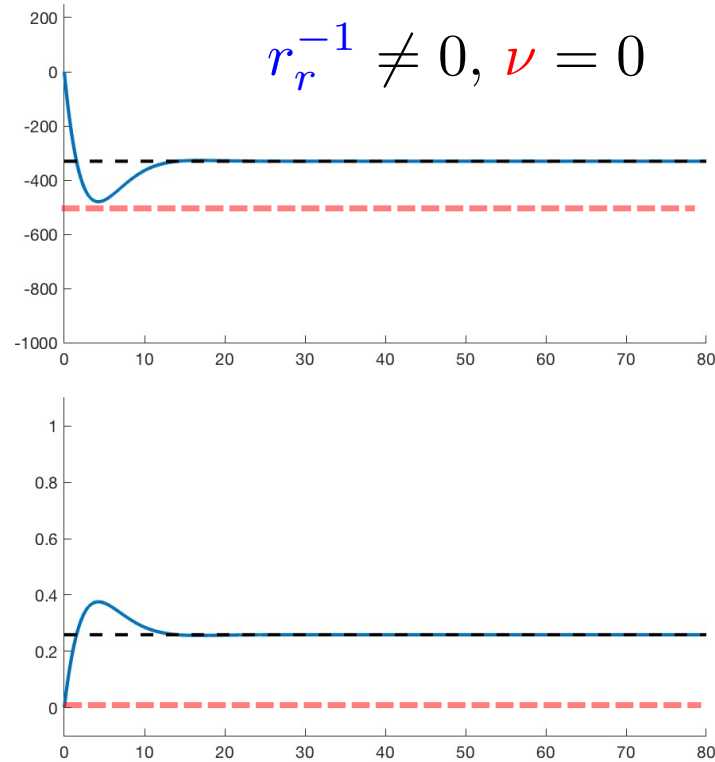
Virtual Inertia

$$r_r^{-1} = 0, \nu = \nu_{\min}$$



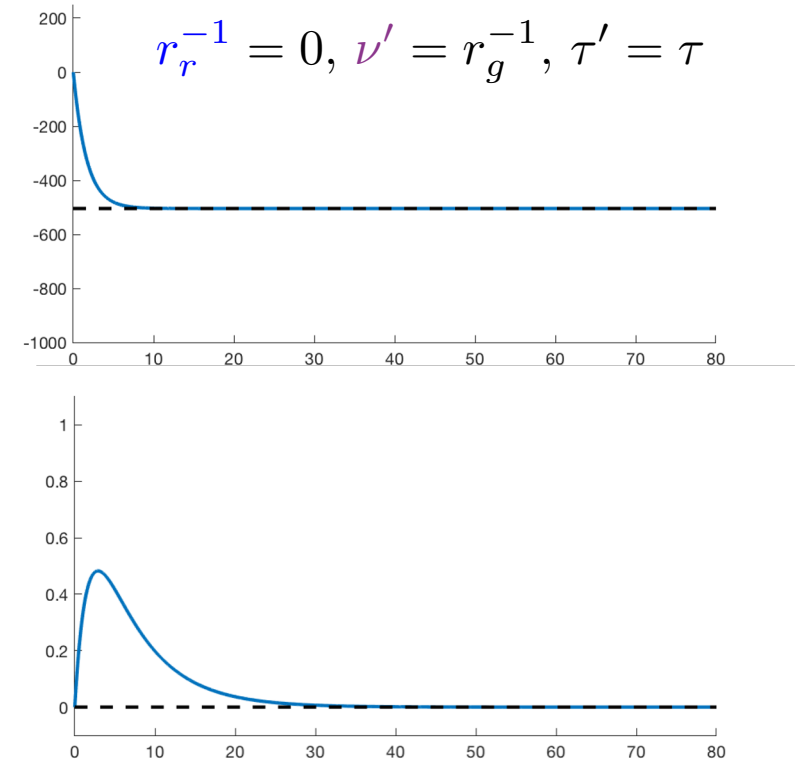
Droop Control

$$r_r^{-1} \neq 0, \nu = 0$$



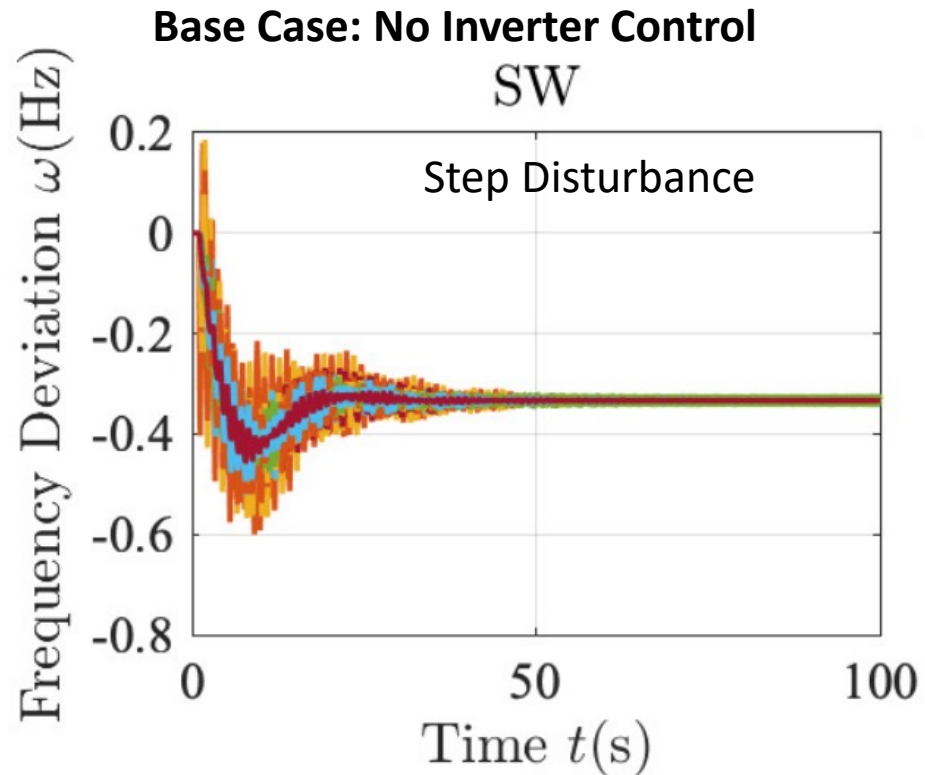
iDroop

$$r_r^{-1} = 0, \nu' = r_g^{-1}, \tau' = \tau$$

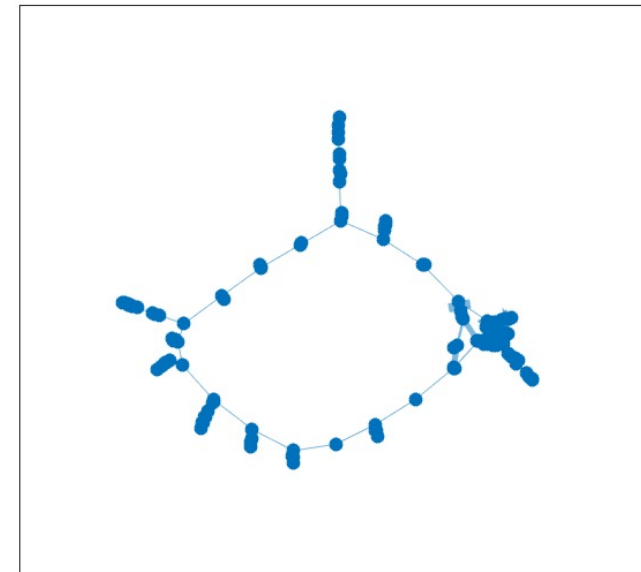


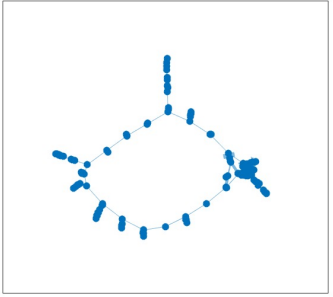
Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)



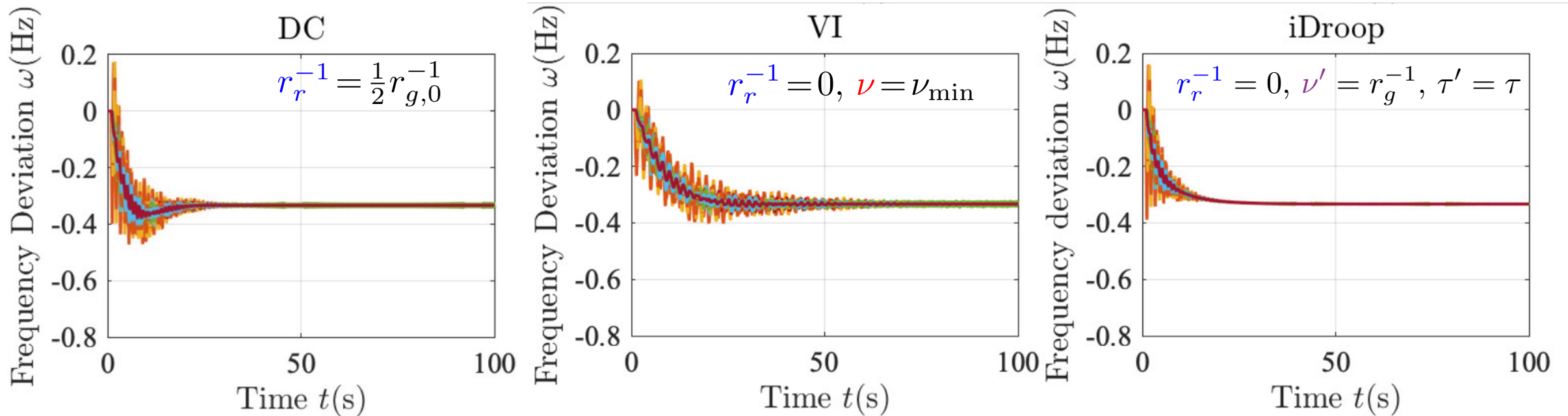
Icelandic Grid

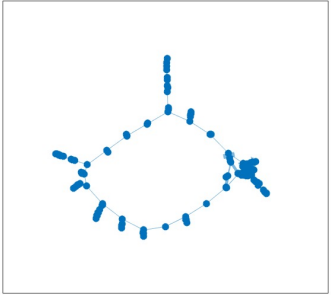




Example: Icelandic Power Grid

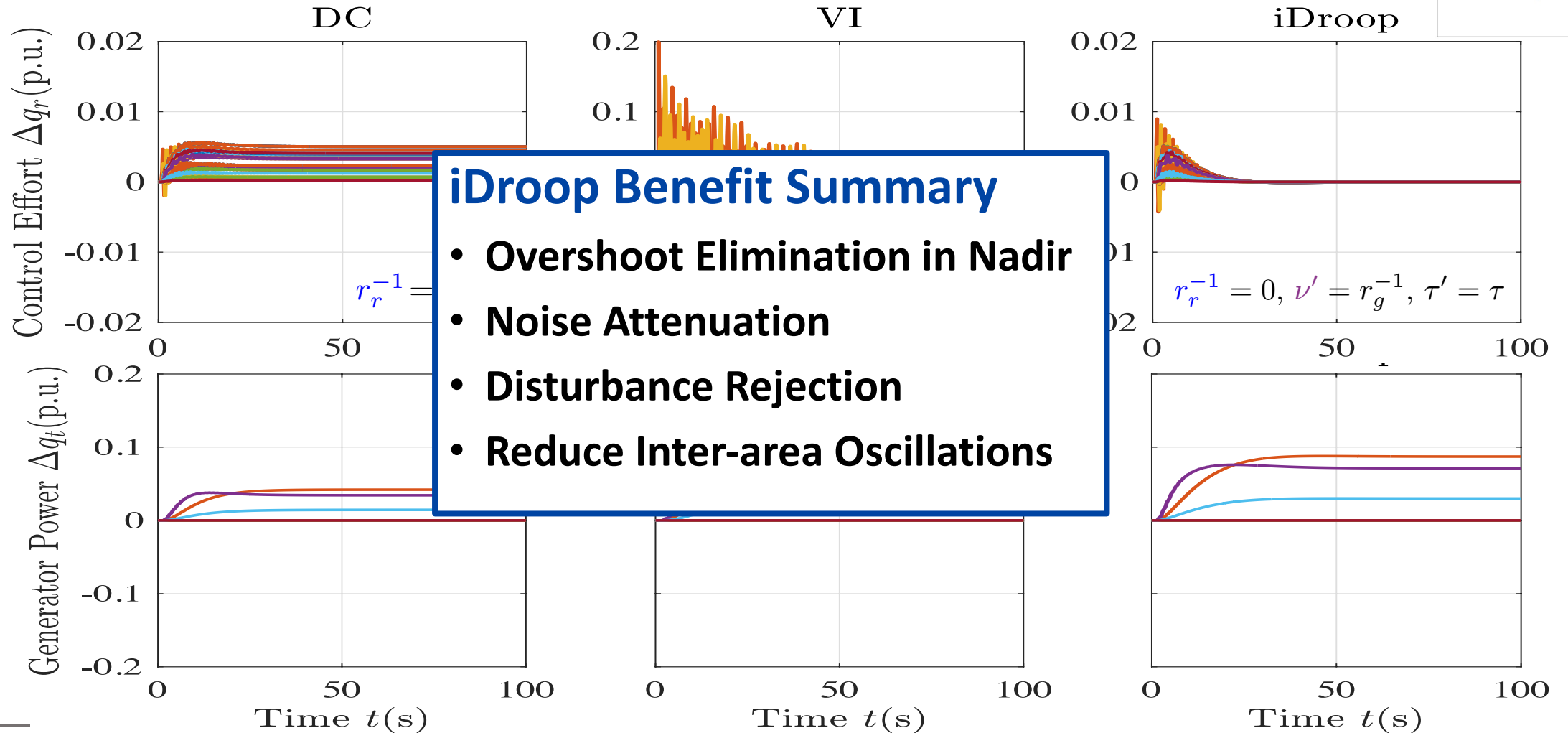
- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)
- Droop equally set for inverters in all cases
- Virtual inertia tuned for **critically damped response** $\nu = \nu_{min}$
- **iDroop** tuned for Frequency Shaping $\nu' = r_g^{-1} + r_r^{-1}$ and $\tau' = \tau$





Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)



[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, IEEE Transactions on Automatic Control, 2021

Trading off Control Effort and RoCoF

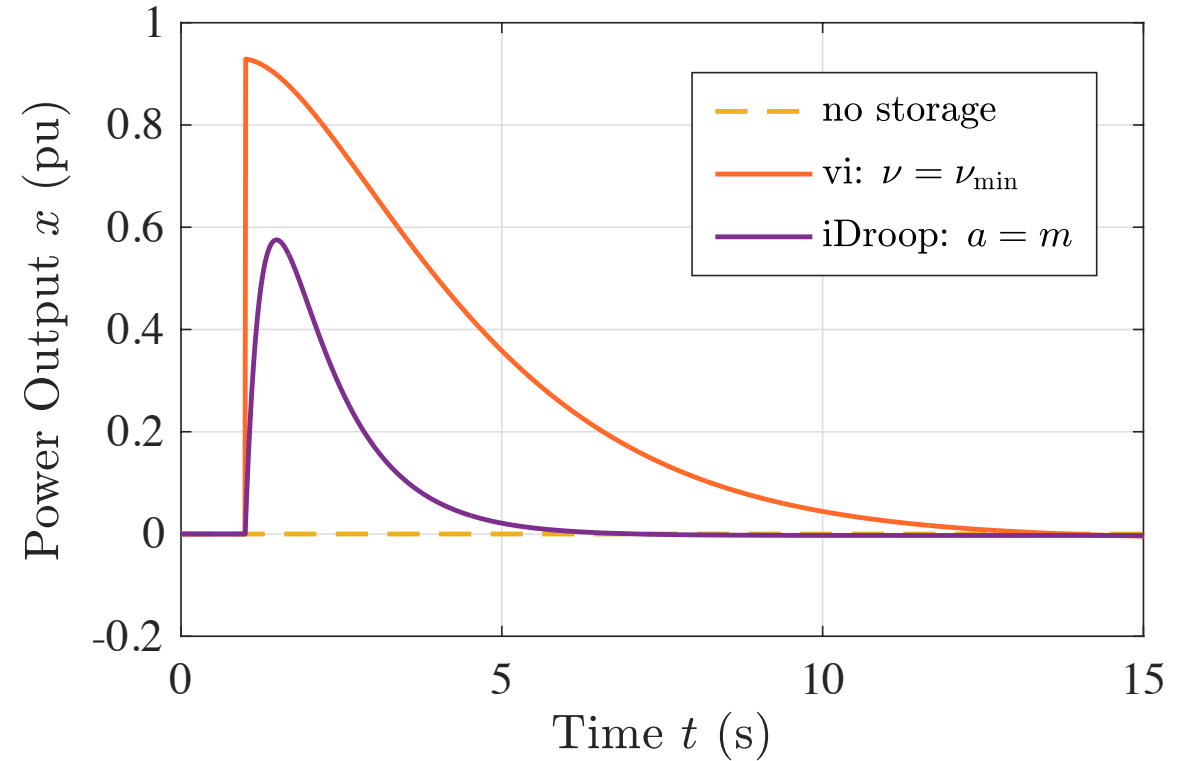
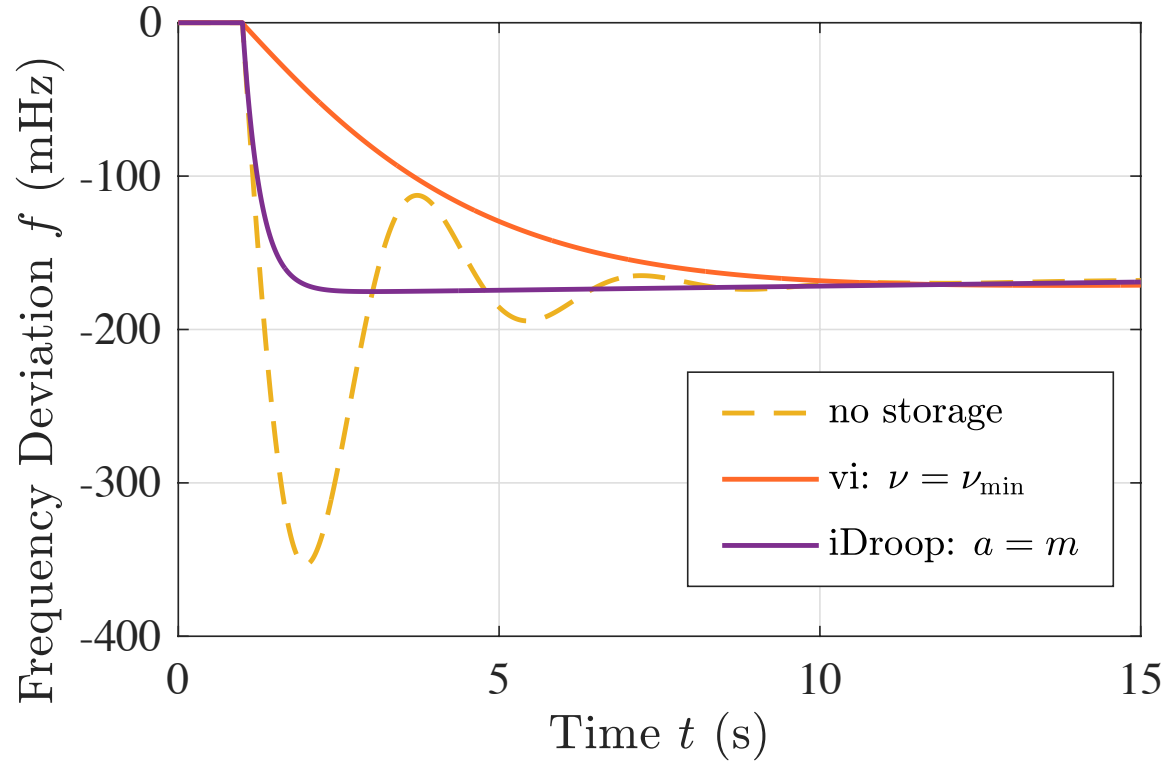
$$c_{fs}(s) := \frac{b_2 s^2 + b_1 s + b_0}{\tau' s + 1}$$

$$b_2 = \tau (a - m)$$

$$b_1 = (b - d)\tau + a - m$$

$$b_0 = b - r_g^{-1} - d$$

$$\tau' = \tau$$



Trading off Control Effort and RoCoF

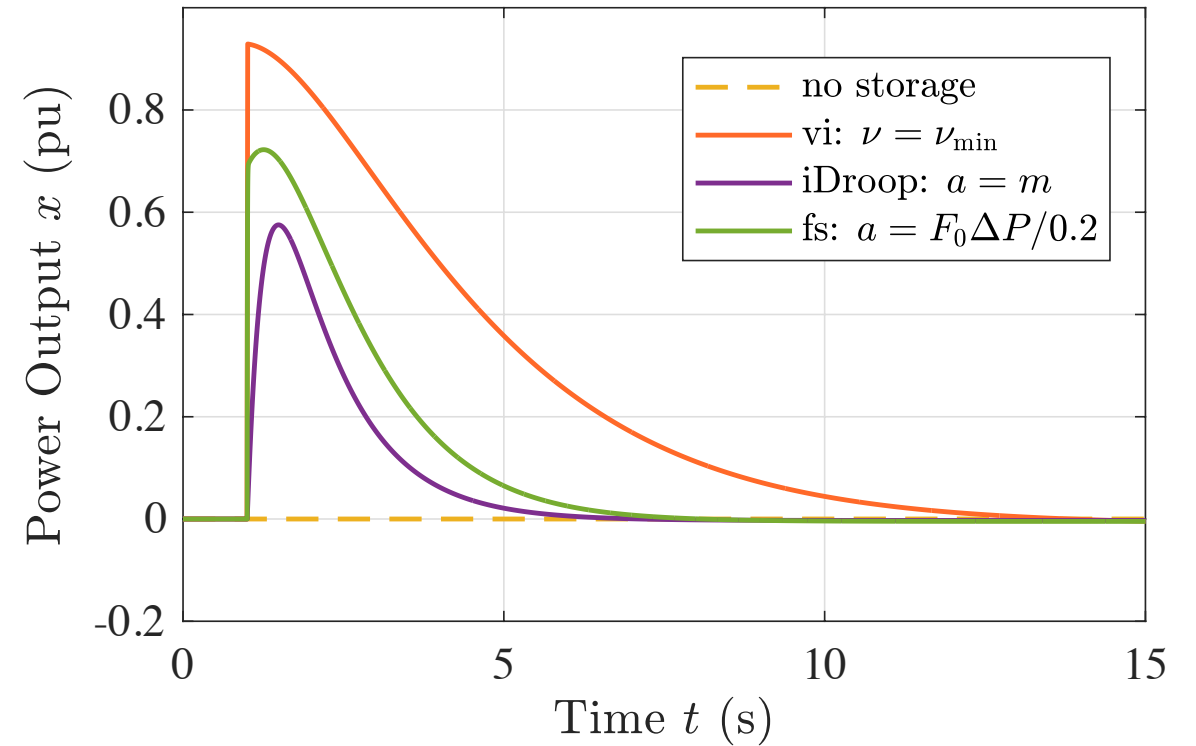
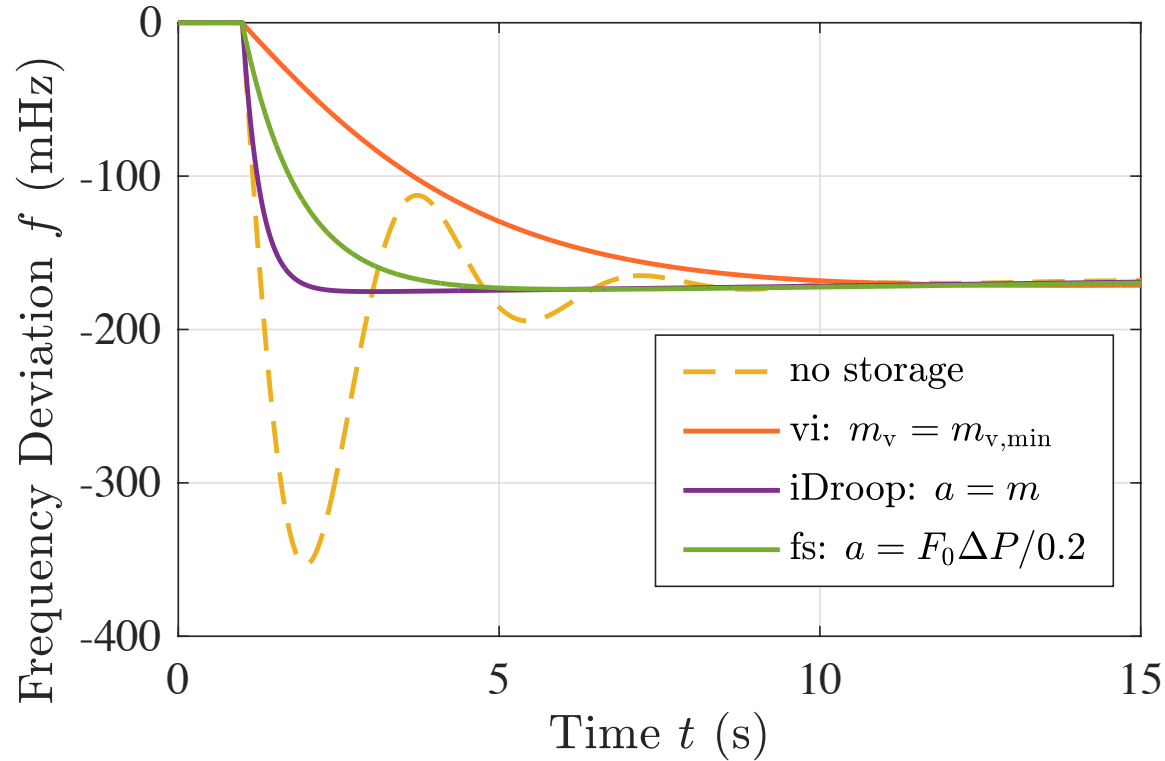
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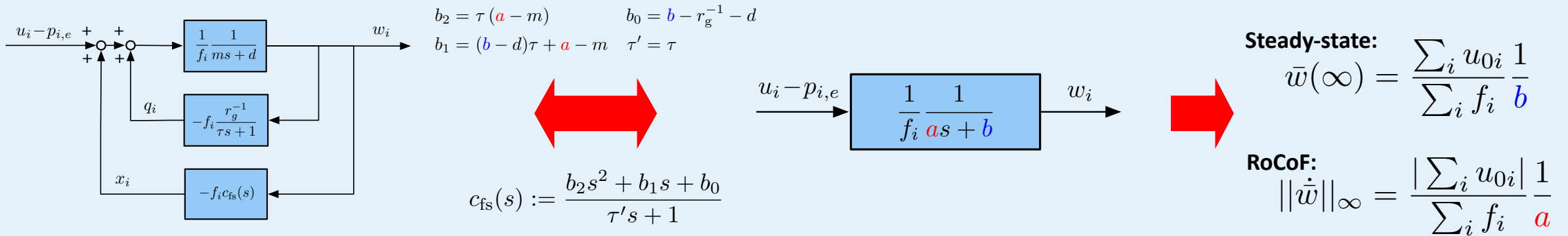
$$b_0 = b - r_g^{-1} - d$$

$$\tau' = \tau$$



Frequency Shaping

Grid-following Inverters: At each bus/area...



Grid-forming Inverters

Center of Inertia /w Grid Forming Inverters

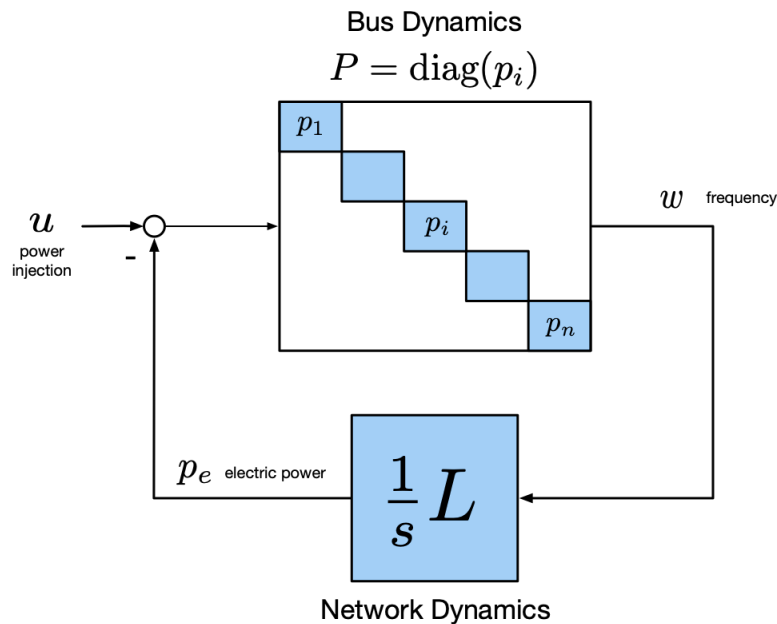
Center of Inertia Freq.

$$\bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

$$p_i(s) = \frac{1}{f_i} p_0(s)$$



$$\bar{w}(s) = p_0(s) \frac{1}{\sum_{i=1}^n f_i} \left(\sum_{i=1}^n u_i(s) \right)$$



Problem: No longer valid grid-forming inverters
Yet...

$$\bar{w}(s) = p_0(s) \frac{1}{\sum_{i=1}^n f_i} \left(\sum_{i=1}^n u_i(s) \right)$$

Center of Inertia /w Grid Forming Inverters

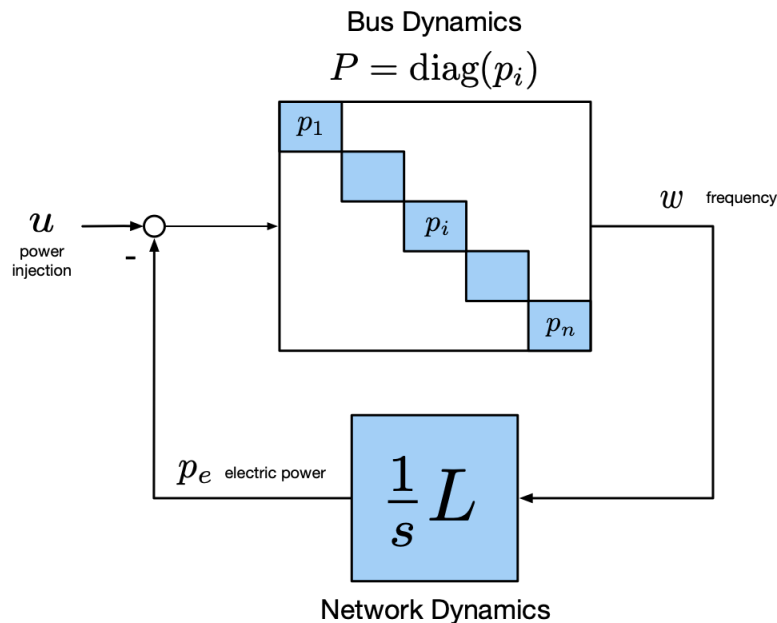
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Problem: No longer valid grid-forming inverters
Yet...

$$\bar{w}(s) = \frac{1}{p_0^{-1}(s)} \frac{1}{\sum_{i=1}^n f_i} \left(\sum_{i=1}^n u_i(s) \right)$$

Center of Inertia /w Grid Forming Inverters

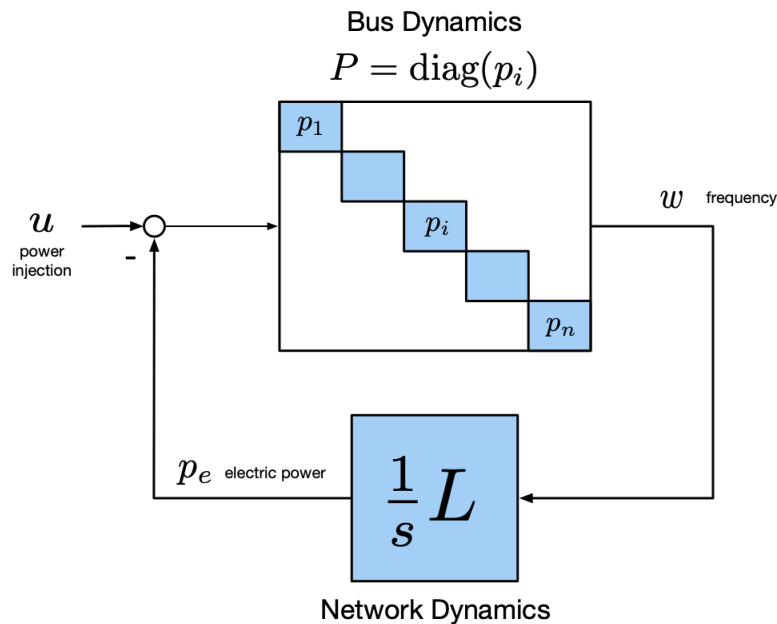
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Center of Inertia /w Grid Forming Inverters

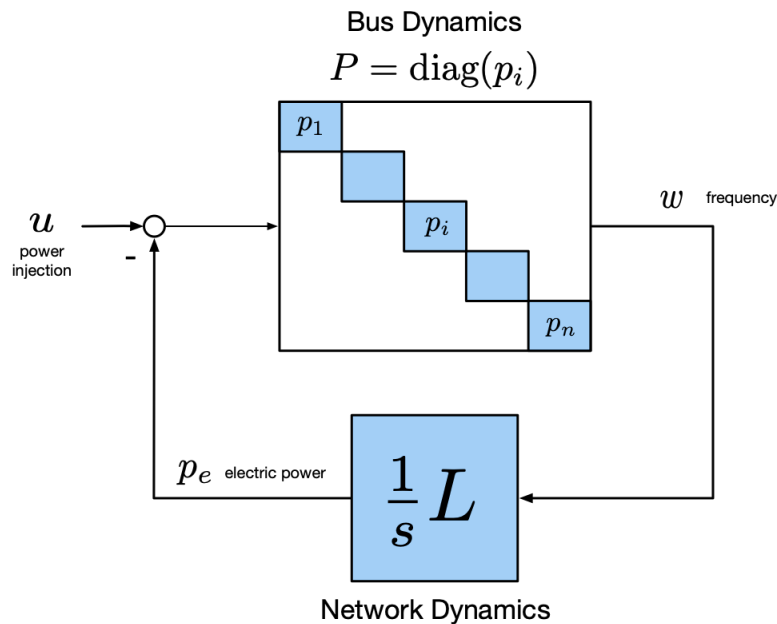
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$$\bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

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$$\bar{w}(s) = p_0(s) \frac{1}{\sum_{i=1}^n f_i} \left(\sum_{i=1}^n u_i(s) \right)$$



Problem: No longer valid grid-forming inverters
Yet...

$$\bar{w}(s) = \left(\sum_{i=1}^n p_i^{-1}(s) \right)^{-1} \left(\sum_{i=1}^n u_i(s) \right)$$

...provides a good approx. of aggregate response!

Generalized Center of Inertia

Define the **Generalized CoI Frequency** by:

$$\bar{w}(s) = \hat{p}(s) \left(\sum_{i=1}^n u_i(s) \right)$$

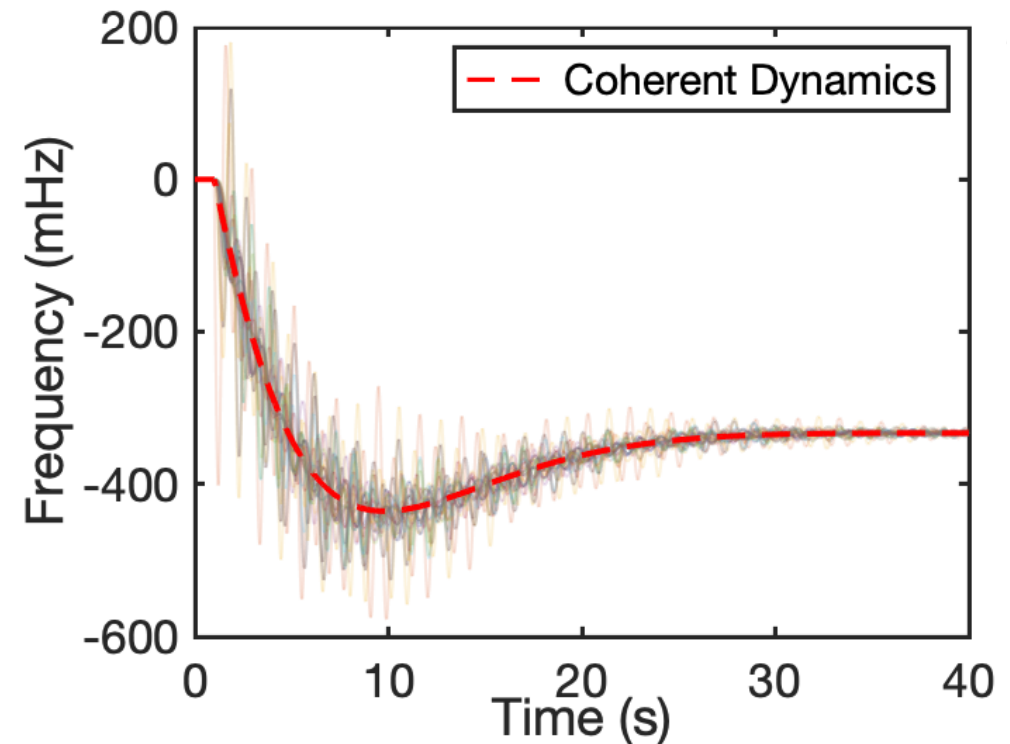
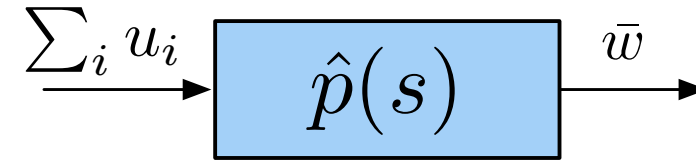
where...

$$\hat{p}(s) = \left(\sum_{i=1}^n p_i^{-1}(s) \right)^{-1}$$

represents the **coherent dynamics**.

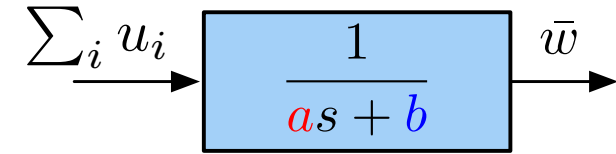
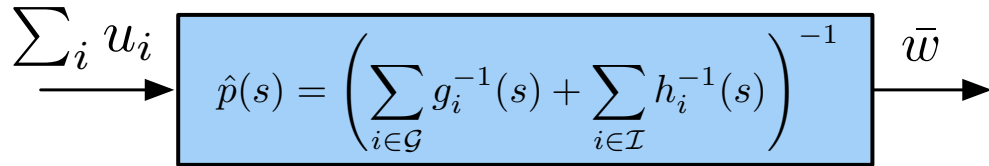
Coherent Dynamics:

- Representation of aggregate response
- Accuracy of approximation:
 - is frequency dependent
 - increases with network connectivity
- Provides excellent template for reduced order models (via balance-truncations)
- More details in [TAC 22]



Grid-forming Frequency Shaping Control

Key idea: use model matching control on coherent dynamics



Generation:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}, \quad i \in \mathcal{G}$$

Inverters:

$$h_i(s) = \frac{1}{m_i s + d_i + c_i(s)}, \quad i \in \mathcal{I}$$

$$a := \sum_{i \in \mathcal{G}} m_i + \sum_{i \in \mathcal{I}} m_i$$

$$b := \sum_{i \in \mathcal{G}} (d_i + r_i^{-1}) + \sum_{i \in \mathcal{I}} d_i$$

$$\sum_{i \in \mathcal{I}} c_i(s) = \sum_{i \in \mathcal{G}} \frac{r_i^{-1} \tau_i s}{\tau_i s + 1}$$

RoCoF:

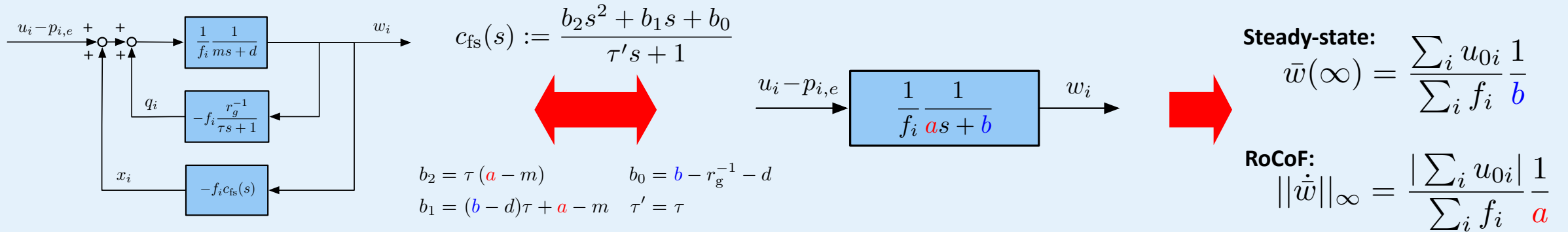
$$\|\dot{\bar{\omega}}\|_{\infty} = \frac{|\sum_i u_{0i}|}{a}$$

Steady-state:

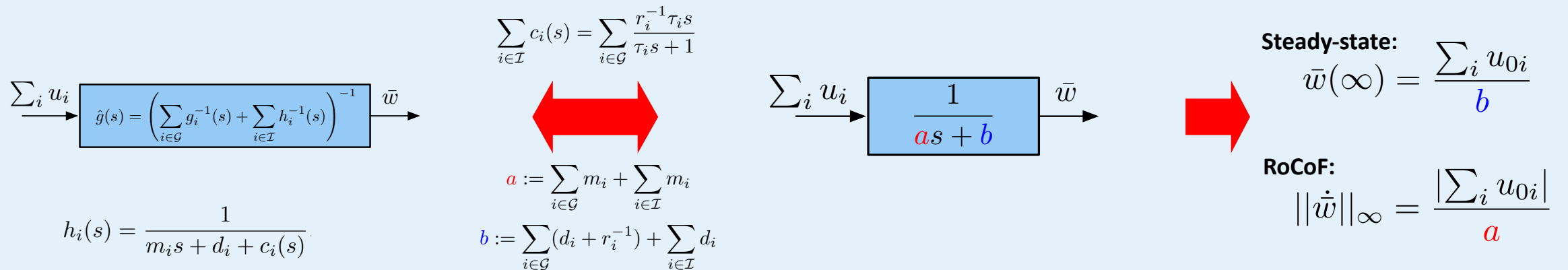
$$\bar{\omega}(\infty) = \frac{\sum_i u_{0i}}{b}$$

Frequency Shaping

Grid-following Inverters: At each bus/area...



Grid-forming Inverters



Summary

- Developed dynamic performance metrics that are analytically tractable and give insight on the effect of inertia, damping, network, etc.
 - To bridge the theory-practice gap, we cover: heterogeneous machines, step response and stochastic metrics
 - Generalized notion of Col: based on a novel coherence analysis
- Take away messages
 - Role of inertia less dramatic than in conventional wisdom. **Lighter systems are also faster to control.** Short term damping d is a more crucial parameter.
 - Flexibility in the control design provides more opportunities:
 - **Dynamic Droop Control** (iDroop) can cancel Nadir, Sync. Cost, and attenuate frequency variance.
 - **Frequency Shaping Control** can further trade off between RoCoF and Control Effort
- What's missing
 - Stability in the faster timescales (compatibility btwn. converters and gens. voltage regulation)
 - Performance improvement depends on knowledge on system -> Robust Performance

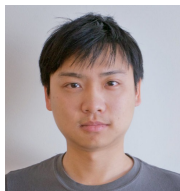
Thanks!

Related Publications:

- Paganini and M, “Global analysis of synchronization performance for power systems: bridging the theory-practice gap,” **IEEE TAC 2020**
- Jiang, Pates, M, “Dynamic Droop Control for Low Inertia Power Systems,” **IEEE TAC 2021**
- Jiang, Cohn, Vorobev, M “Storage-Based Frequency Shaping Control,” **IEEE TPS 2021**
- Min, Paganini, M, “Accurate Reduced Order Models for Coherent Synchronous Generators,” **L-CSS 2020**



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Petr Vorobev



Richard Pates



Fernando Paganini



Backup Slides

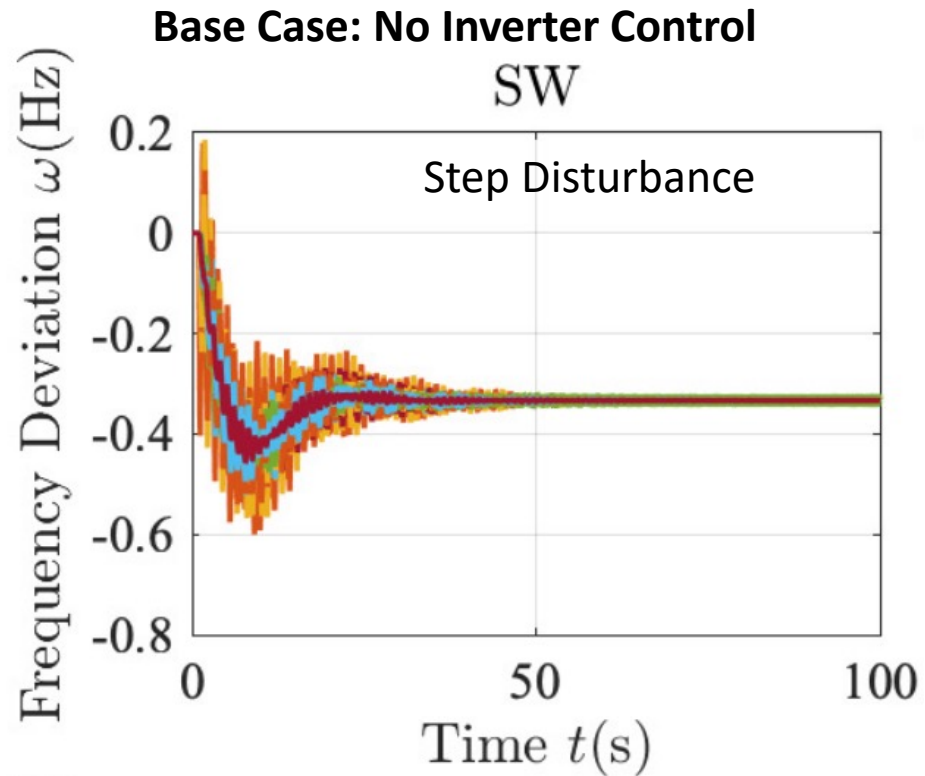
Numerical Examples

Modal Decomposition

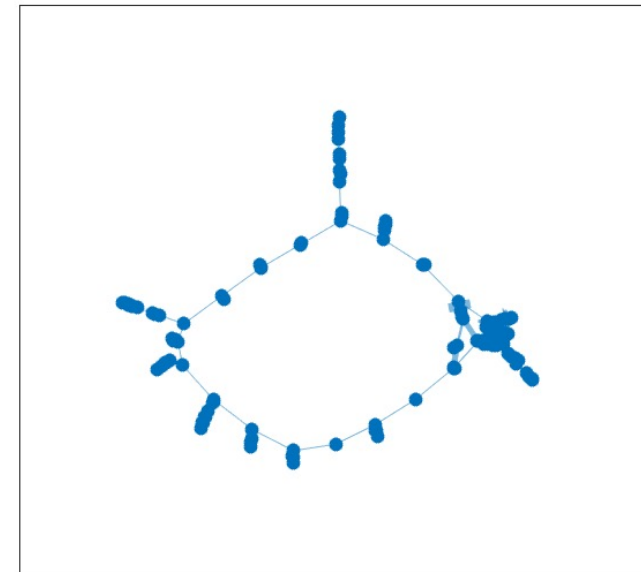
Coherence

Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

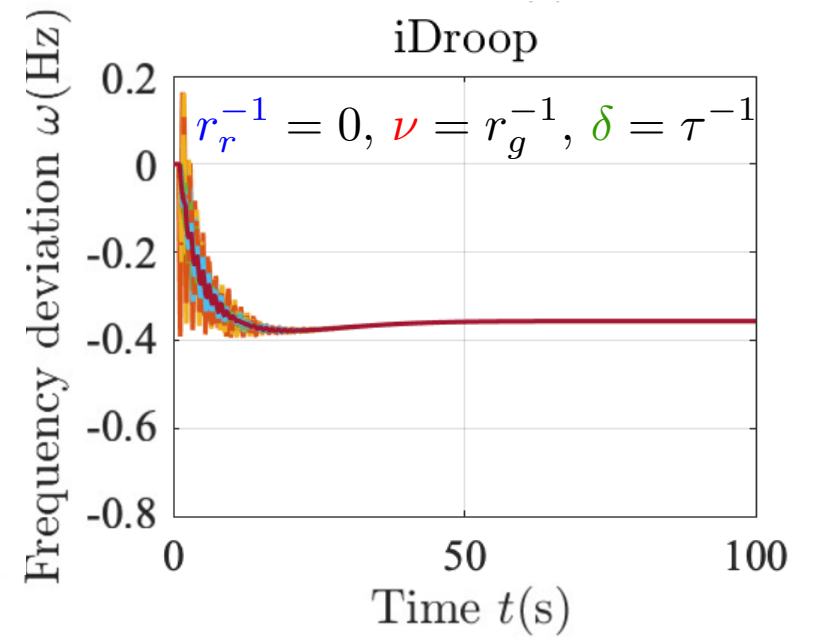
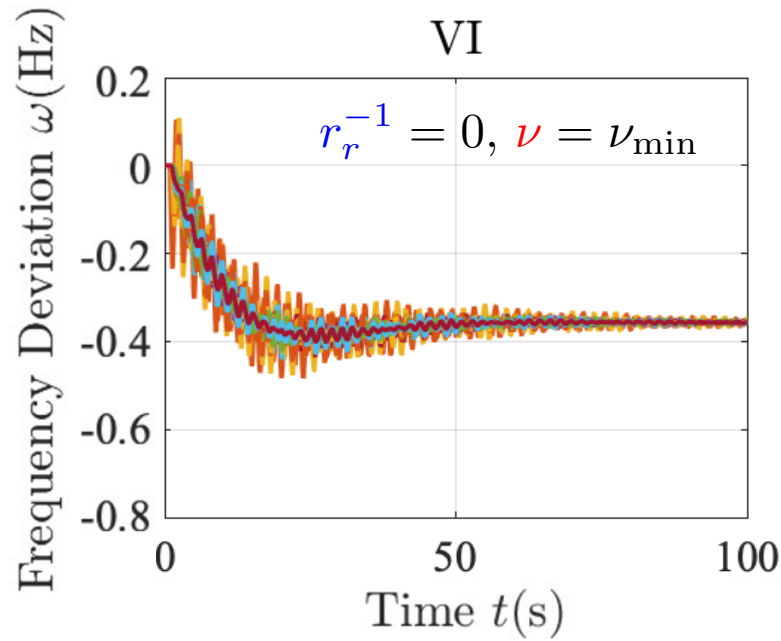
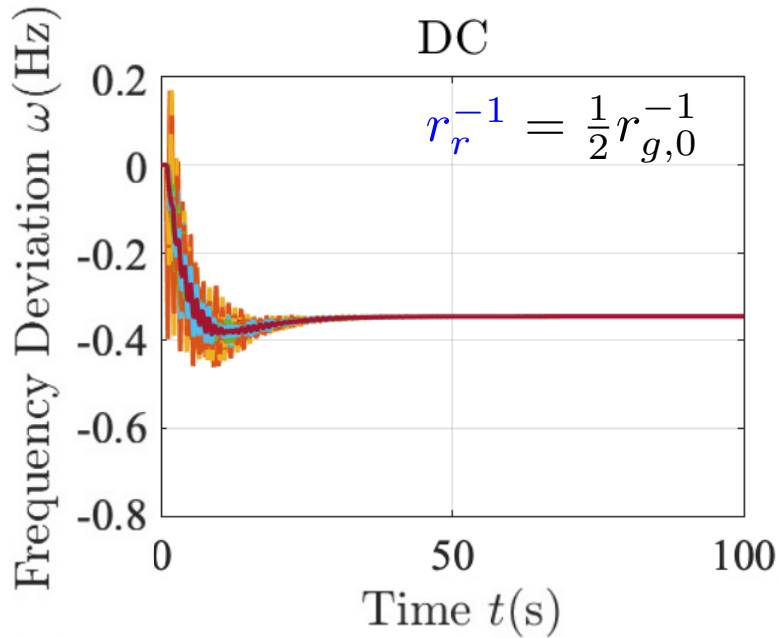
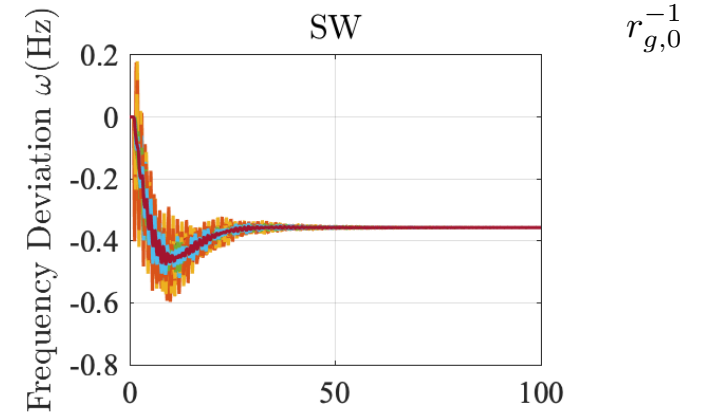


Icelandic Grid



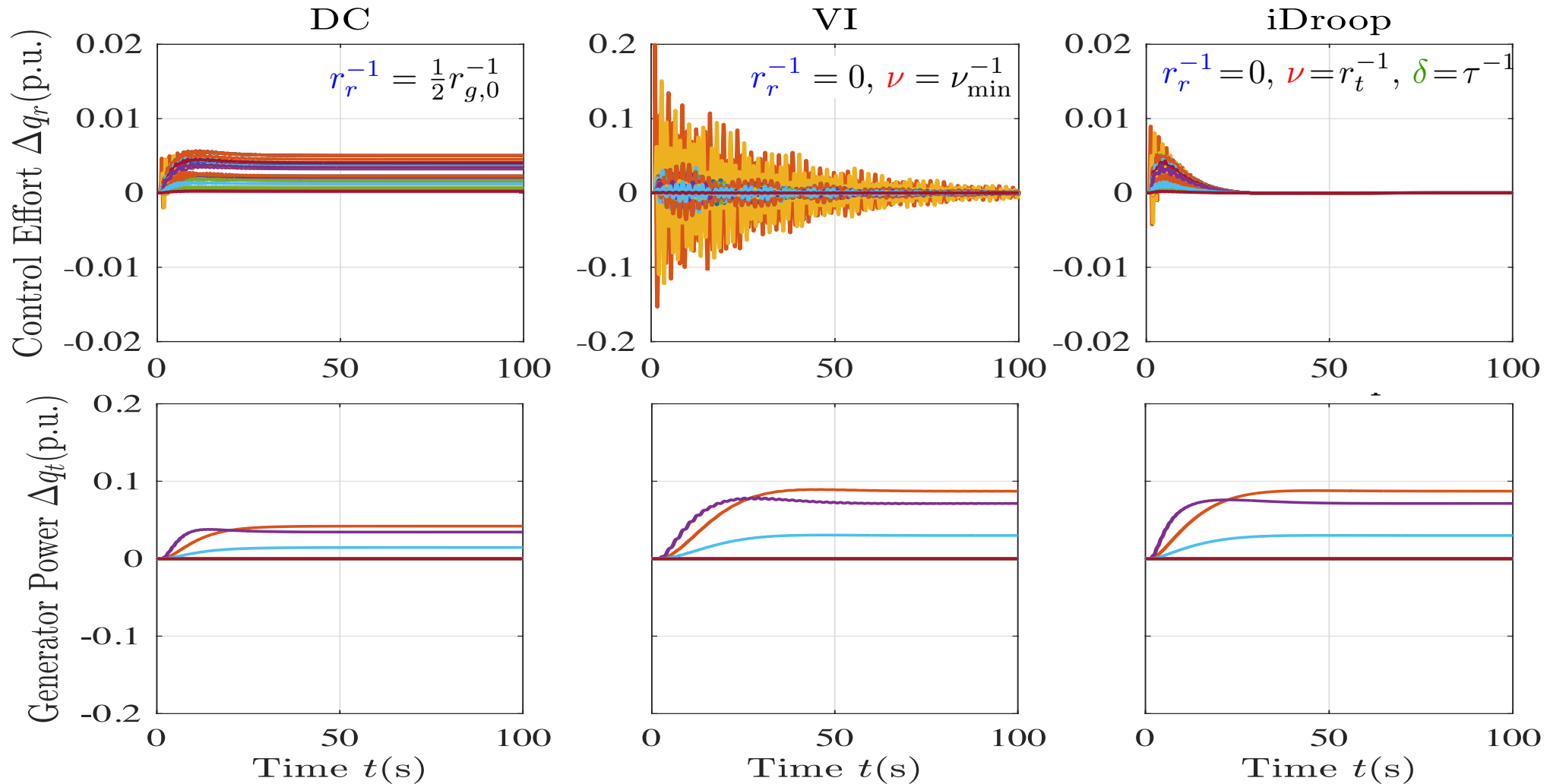
Step Disturbance - Frequency

- Droop equally shared between gens. and inverters.
- Virtual inertia tuned for **critically damped response** $\nu = \nu_{min}$
- **iDroop** tuned for Nadir elimination

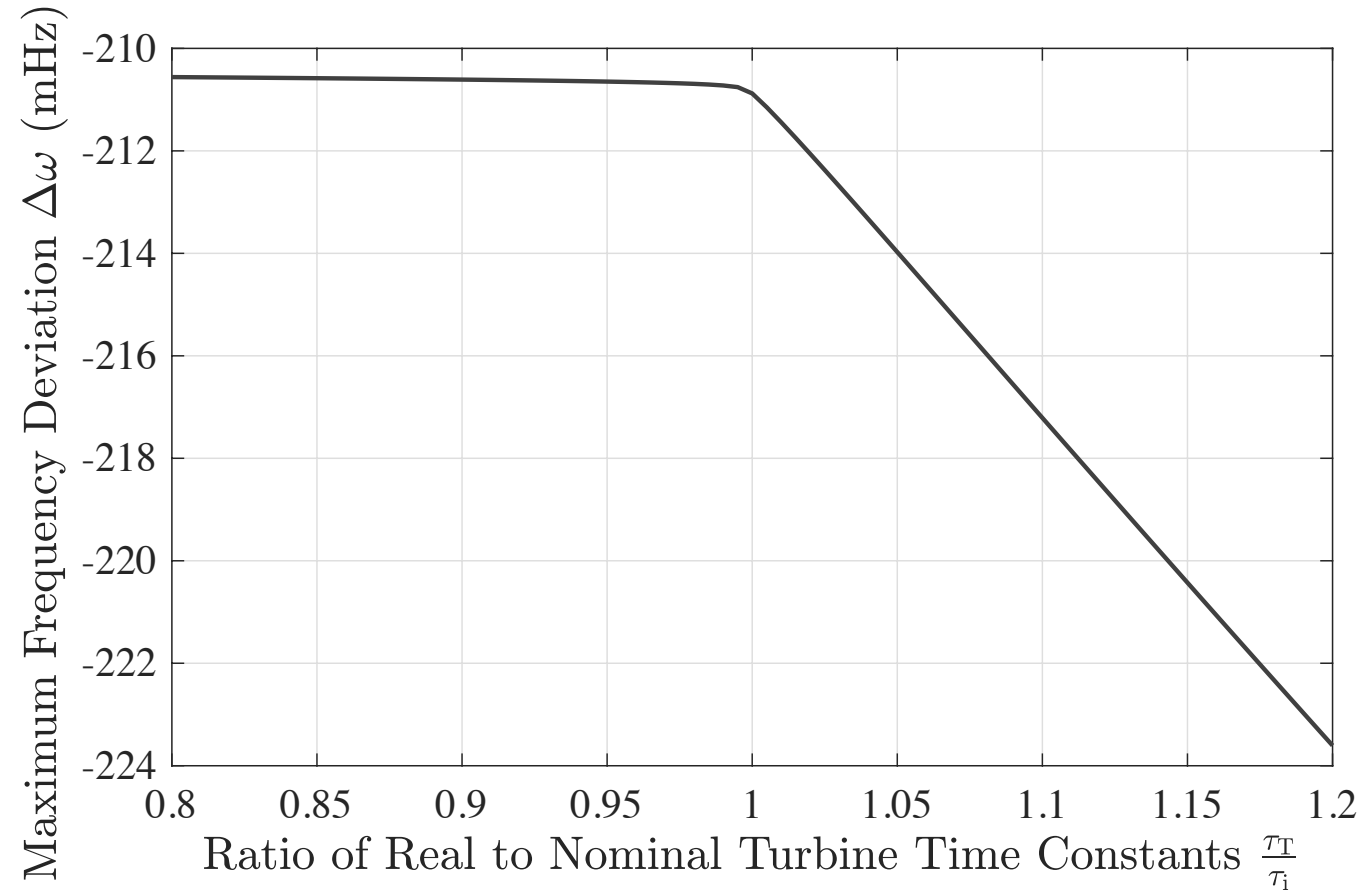


Step Disturbance – Control Effort

Inertia parameters of iDroop and VI are set to achieve zero overshoot.



Parameter Uncertainty



Step Disturbance

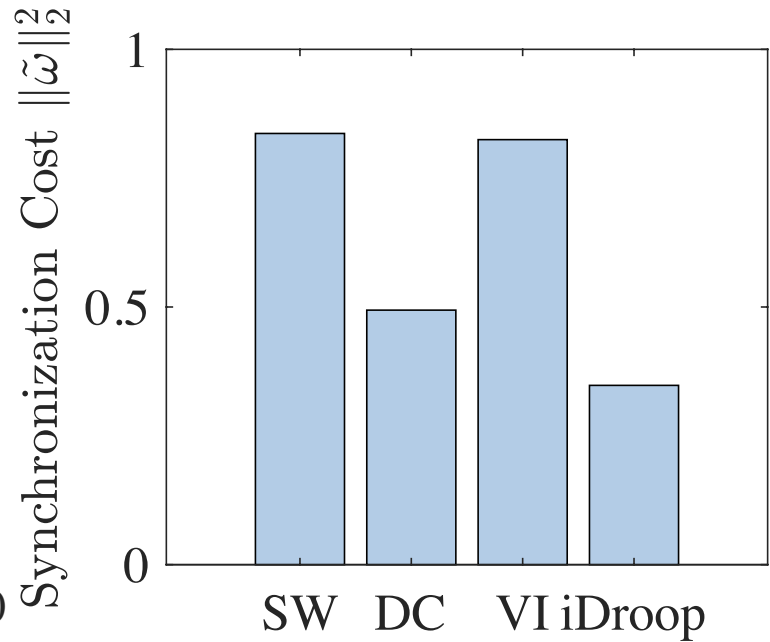
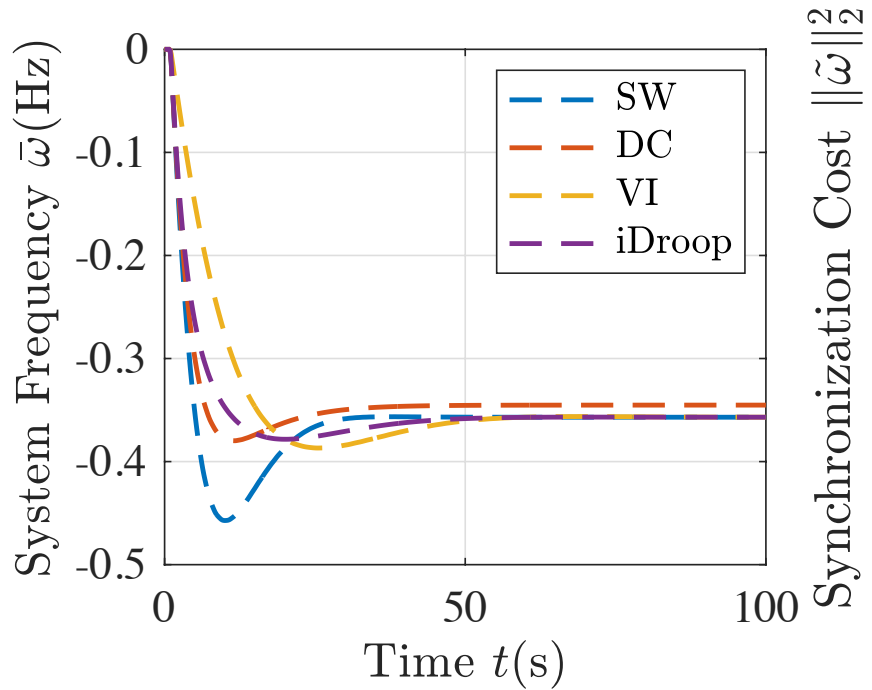
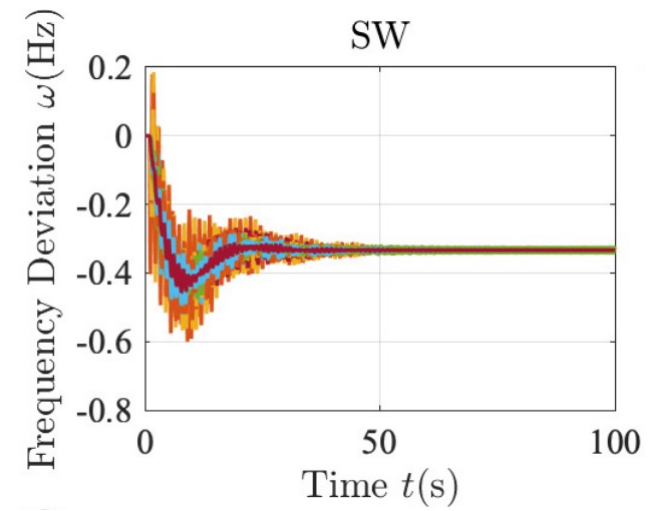
Inertia parameters of iDroop and VI are set to achieve zero overshoot.

δ^*	0.331
ν^*	0.0054
ν_{VI}	0.035

Zero Nadir Tuning

$$\delta = \tau^{-1}$$

$$\nu = r_r^{-1} + r_g^{-1}$$



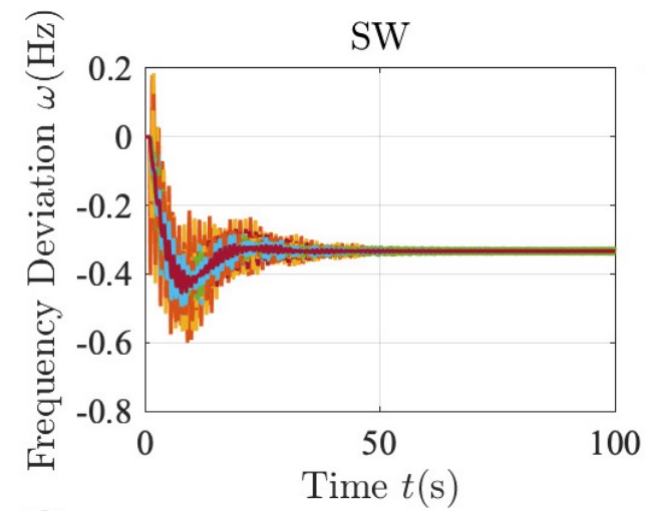
Step Disturbance

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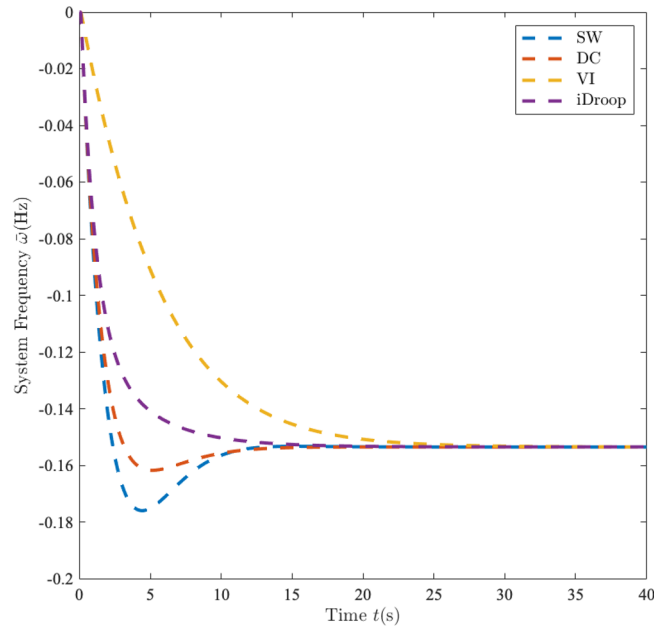
δ^*	0.331
ν^*	0.0054
ν_{VI}	0.035



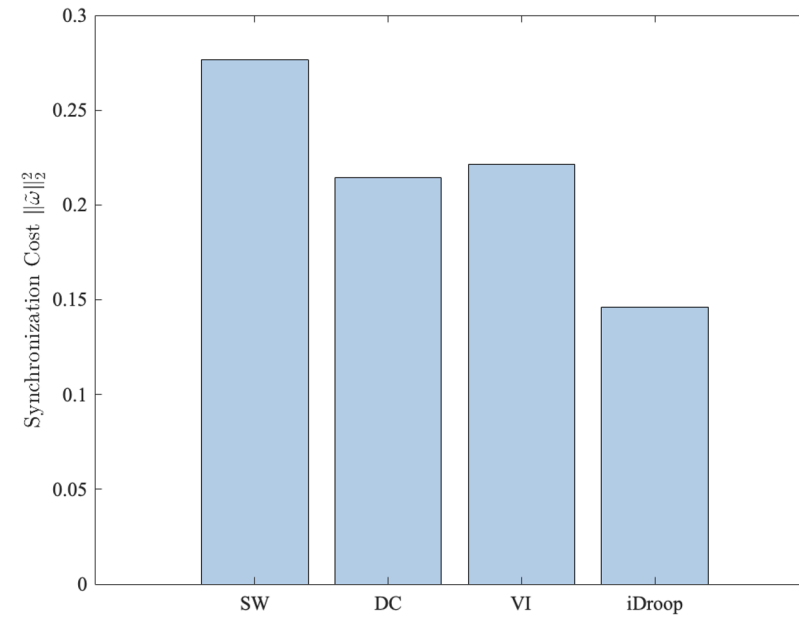
δ^*	0.331
ν^*	0.0108
ν_{VI}	0.070



System Frequency



Synchronization Cost



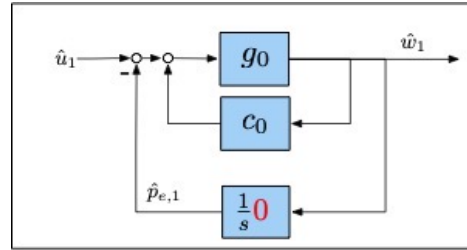
Back

Modal Decomposition for Multi-Rated Machines

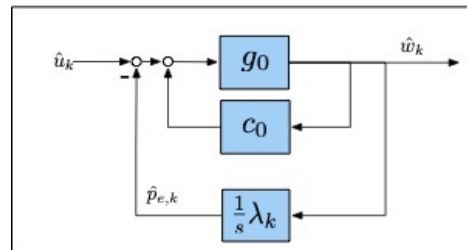
Assumption: Let f_i be the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

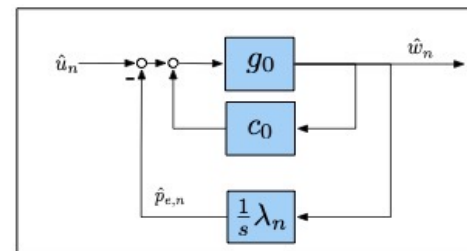
$$c_i(s) = f_i c_0(s)$$



⋮



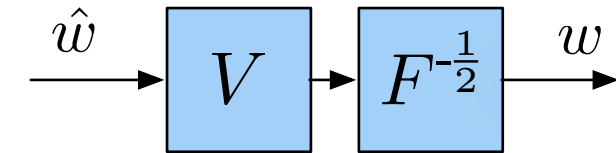
⋮



System Frequency

$$\bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

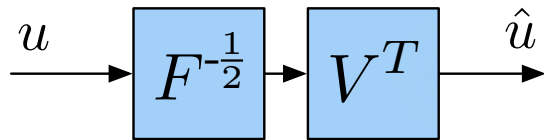
Change of Vars.



Sync Error

$$\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Change of Vars.



$$F = \text{diag}(f_i)$$

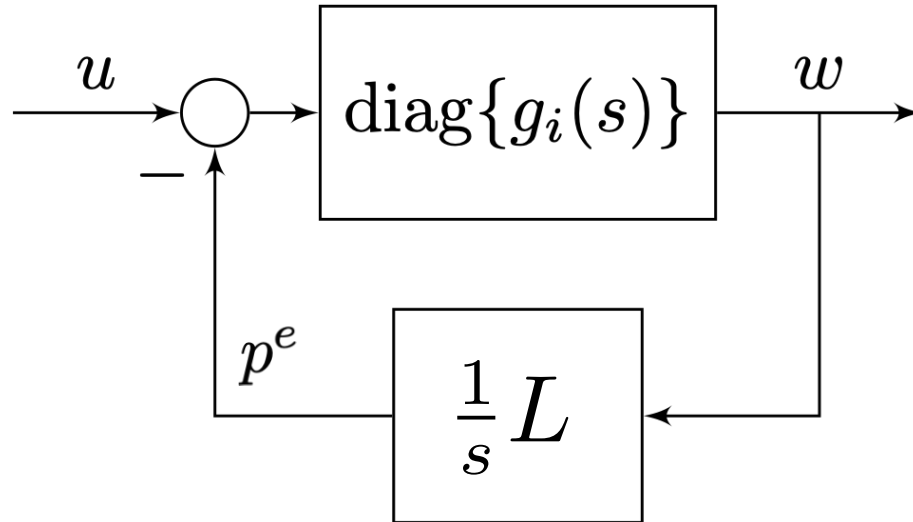
Eigenvalues of: $L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}$

$$0 = \lambda_0 \leq \lambda_1 \leq \dots \leq \lambda_{n-1}$$

[Paganini M '17, Guo Low 18']

Coherent Dynamics

Block Diagram:



$g_i(s), i = 1, \dots, n$: Generator dynamics,

$$L = [L_{ij}],$$

$$L_{ij} = \frac{\partial}{\partial \theta_j} \sum_{k=1}^n |V_i| |V_k| b_{ik} \sin(\theta_i - \theta_k) \Big|_{\theta=\theta_0}$$

L symmetric, $0 = \lambda_1(L) \leq \lambda_2(L) \leq \dots \leq \lambda_n(L)$

Generator dynamics:

$$w_i(s) = g_i(s)(u_i(s) - p_i^e(s))$$

$$i = 1, \dots, n$$

Power flow equation:

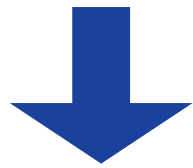
$$p^e(s) = \frac{1}{s}Lw(s)$$

Characterization of Coherent Dynamics

Assume all generators from a coherent group

$$w_i(s) = g_i(s)(u_i(s) - p_i^e(s)) \quad i = 1, \dots, n$$

Assume generators
"output" identical
frequencies



$$w_i(s) = \hat{w}(s)$$

$$g_i^{-1}(s)\hat{w}(s) = u_i(s) - p_i^e(s) \quad i = 1, \dots, n$$

Sum from 1 to n:

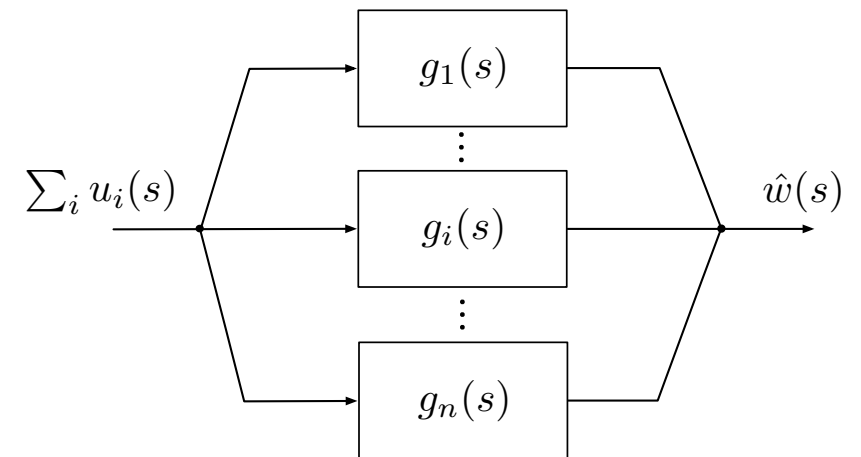
$$\left(\sum_{i=1}^n g_i^{-1}(s) \right) \hat{w}(s) = \sum_{i=1}^n u_i(s) - \underbrace{\sum_{i=1}^n p_i^e(s)}_{=0}$$

Coherent Dynamics:

$$\hat{w}(s) = \left(\sum_{i=1}^n g_i^{-1}(s) \right)^{-1} \sum_{i=1}^n u_i(s)$$

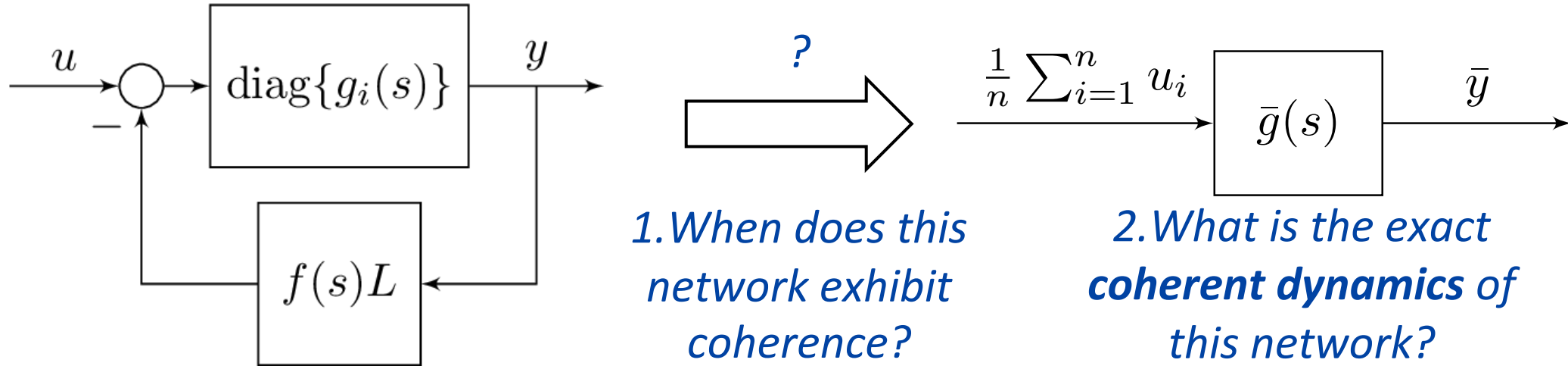
$$\hat{g}(s) = \left(\sum_{i=1}^n g_i^{-1}(s) \right)^{-1}$$

parallel impedance formula



Coherence in networked dynamical systems

Block Diagram:



1. When does this network exhibit coherence?

2. What is the exact coherent dynamics of this network?

1. Coherence can be understood as a **low rank** property the **closed-loop transfer matrix**
2. It emerges as the **effective algebraic connectivity** increases
3. The coherent dynamics is given by the **harmonic mean** of nodal dynamics

$$\bar{g}(s) = \left(\frac{1}{n} \sum_{i=1}^n g_i^{-1}(s) \right)^{-1}$$

Justification for Previous Derivation

Theorem. *Assume all $g_i(s)$ are positive real. Let the transfer matrix from u to w be $T(s)$, then for any $\eta_0 > 0$:*

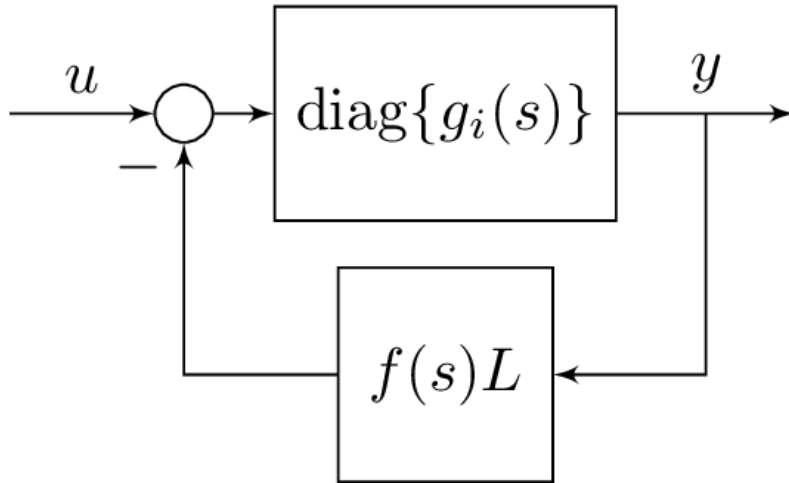
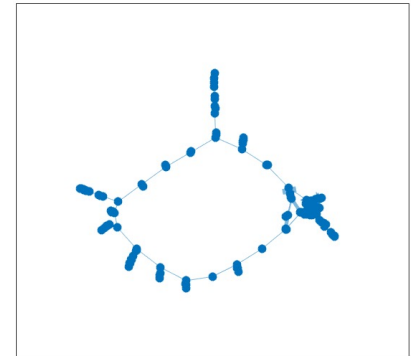
$$\lim_{\lambda_2(L) \rightarrow +\infty} \sup_{\eta \in [-\eta_0, \eta_0]} \|T(j\eta) - \hat{g}(j\eta)\mathbb{1}\mathbb{1}^T\| = 0, \quad j = \sqrt{-1}.$$

- Extension of recent result (CDC '19) on coherence in **networked dynamical systems**
- Convergence on imaginary axis is **related to time-domain response** by Inverse Laplace Transform
- Algebraic connectivity of L is an indicator of **level of coherence**
- $\hat{g}(s)$ accurately represents the aggregate dynamics in the asymptotic sense

Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

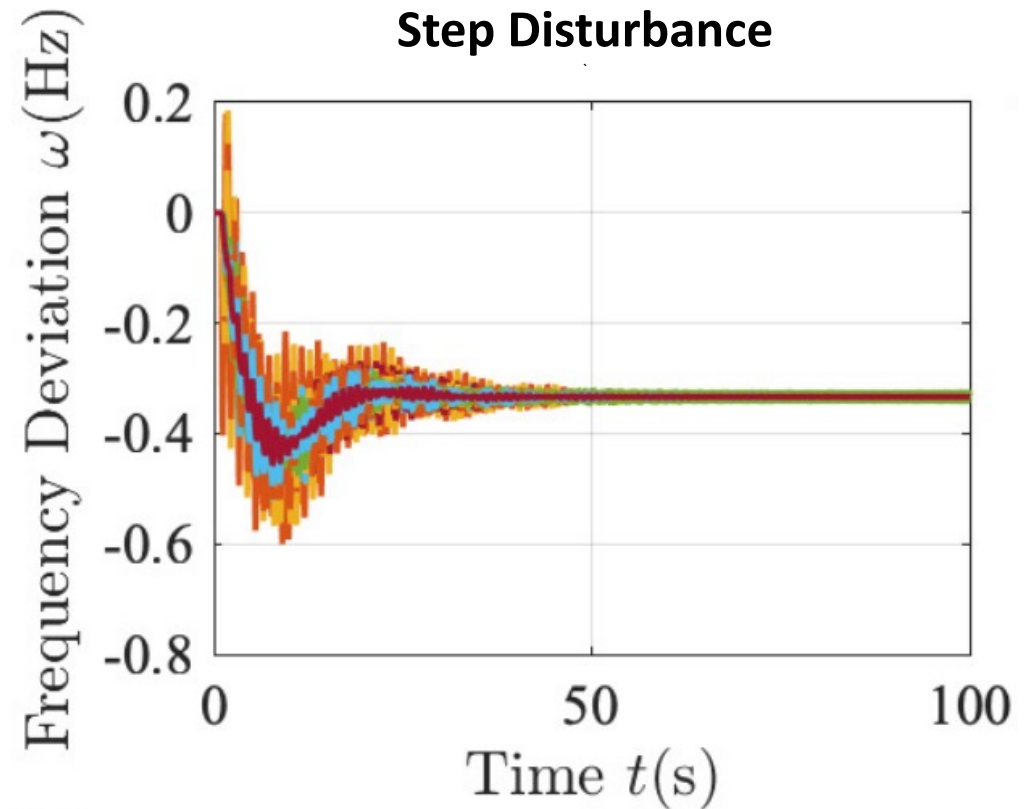
Icelandic Grid



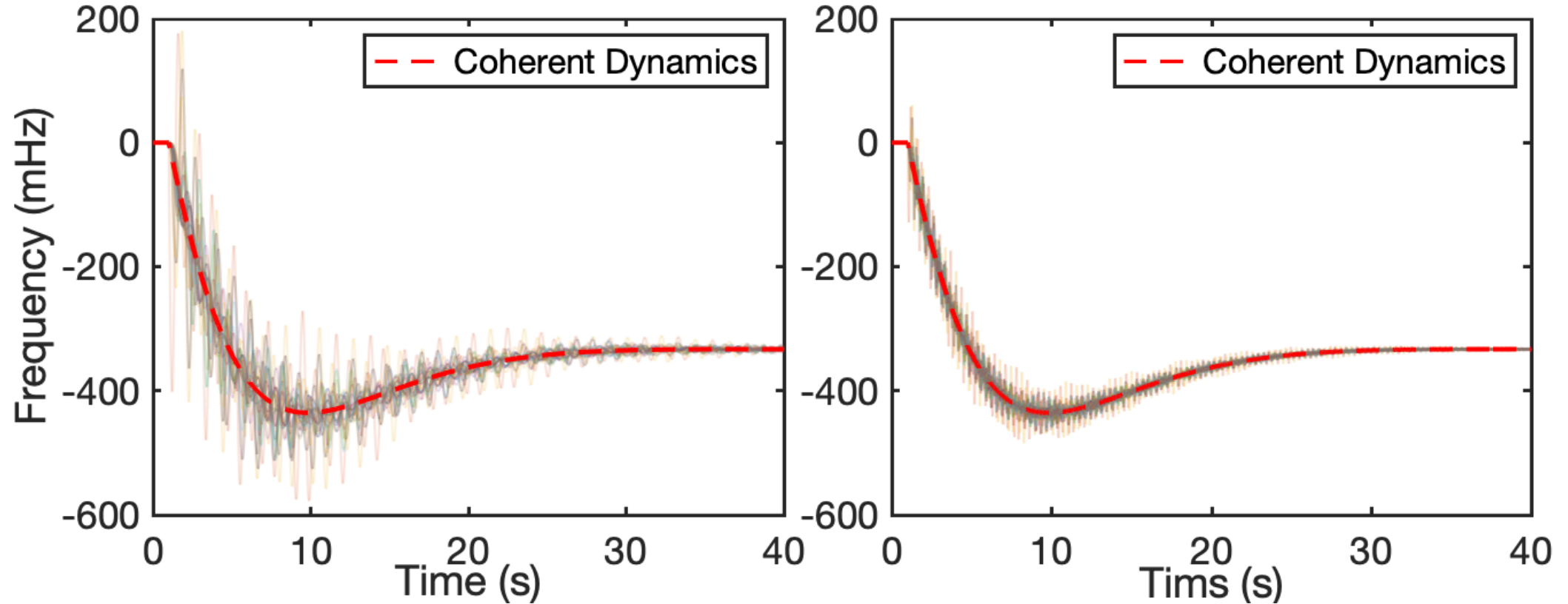
$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

$$f(s) = \frac{1}{s}$$

Step Disturbance



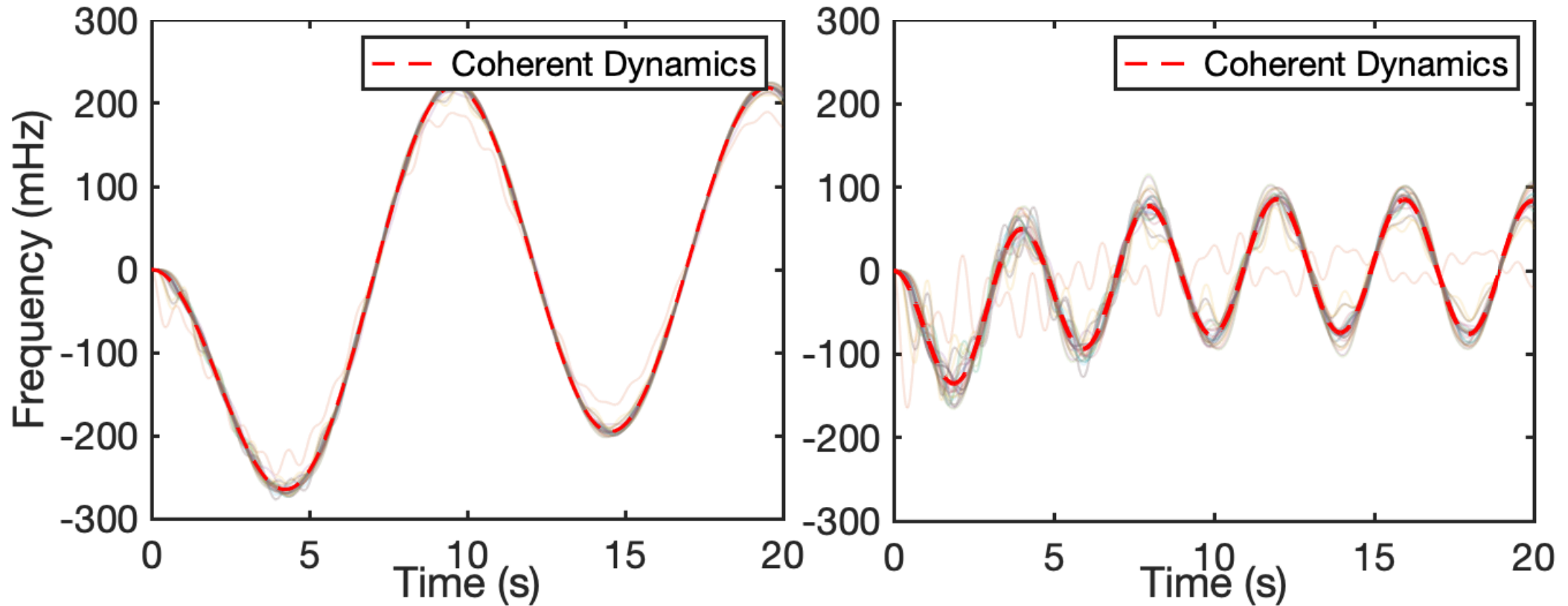
Example: Effect of Network Algebraic Connectivity $\lambda_2(L) \uparrow$



Coherent dynamics acts as a more accurate version of the Center of Inertia (CoI)

Example: Sinusoidal Disturbances: $\sin(\omega_d t)$

$\omega_d \uparrow$



Challenges on Aggregating Coherent Generators

For generator dynamics given by a swing model with turbine control:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

The aggregate dynamics:

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \sum_{i=1}^n \frac{r_i^{-1}}{\tau_i s + 1}}$$

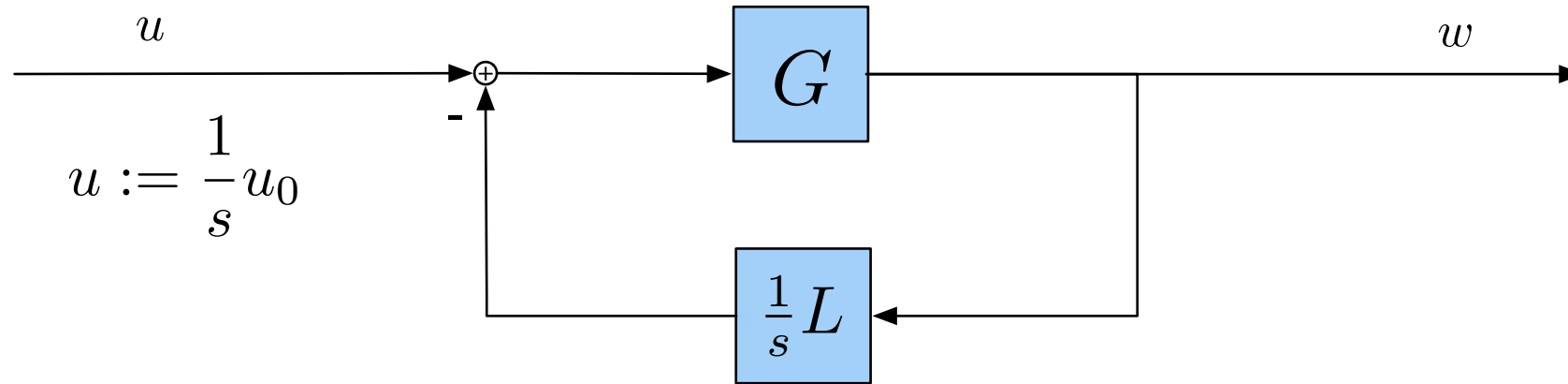
high-order if τ_i are heterogeneous

Need to find a low-order approximation of $\hat{g}(s)$

Diagonalization for Step Disturbances

$$u = \frac{1}{s} u_0$$

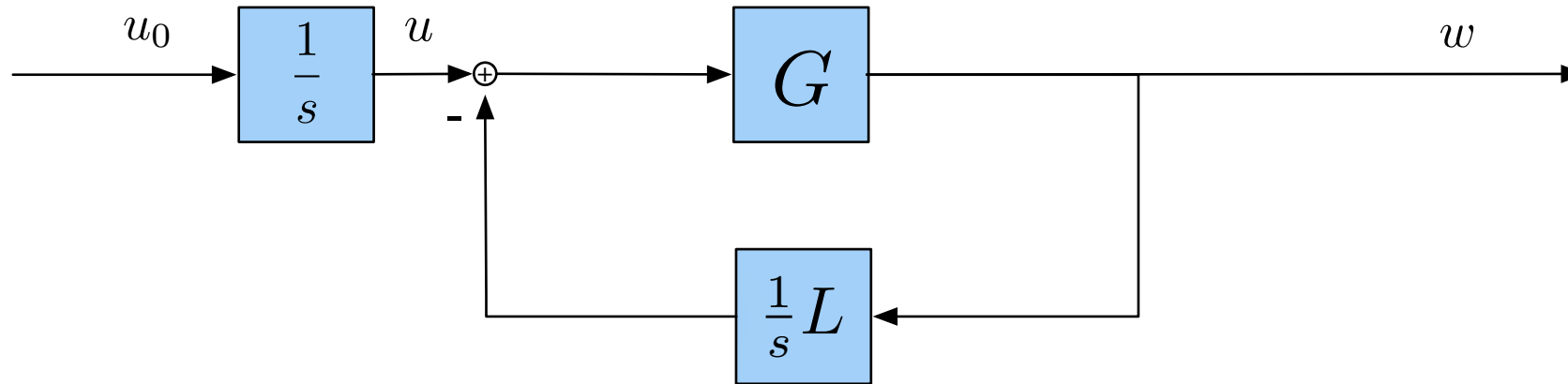
step disturbance



Diagonalization for Step Disturbances

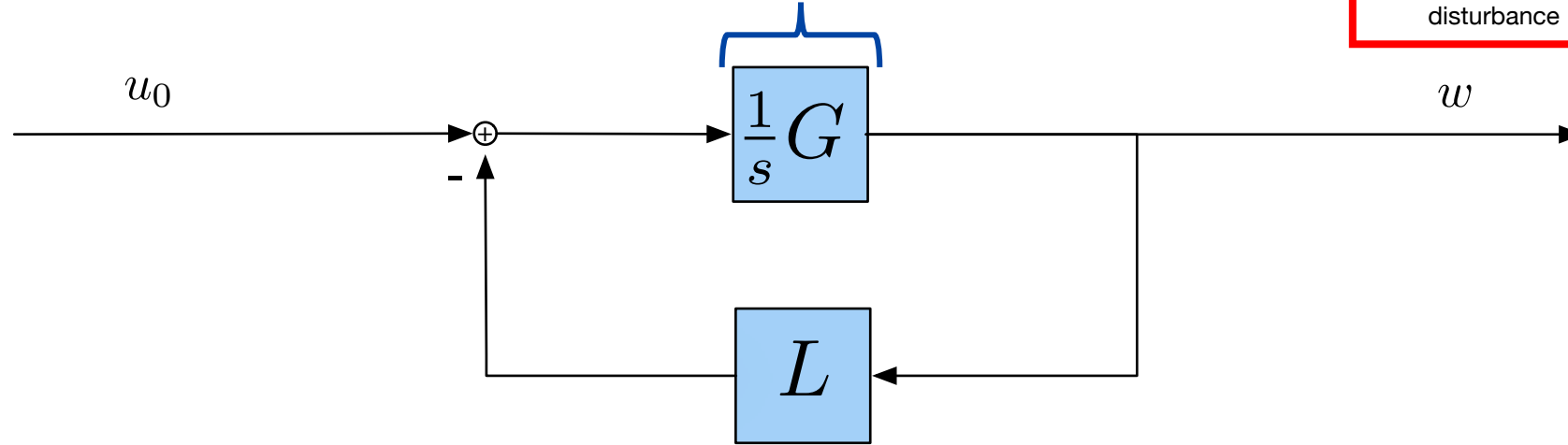
$$u = \frac{1}{s} u_0$$

step disturbance



Diagonalization for Step Disturbances

$$G := g_0(s)F^{-1}$$



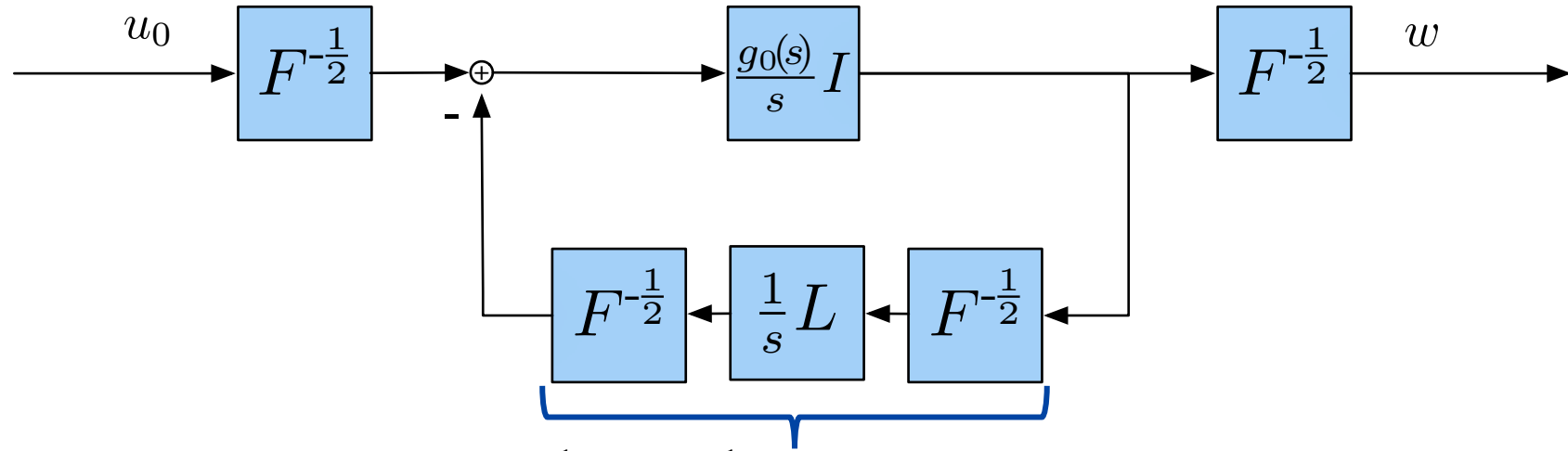
$$u = \frac{1}{s}u_0$$

step
disturbance

Diagonalization for Step Disturbances

$$u = \frac{1}{s} u_0$$

step
disturbance



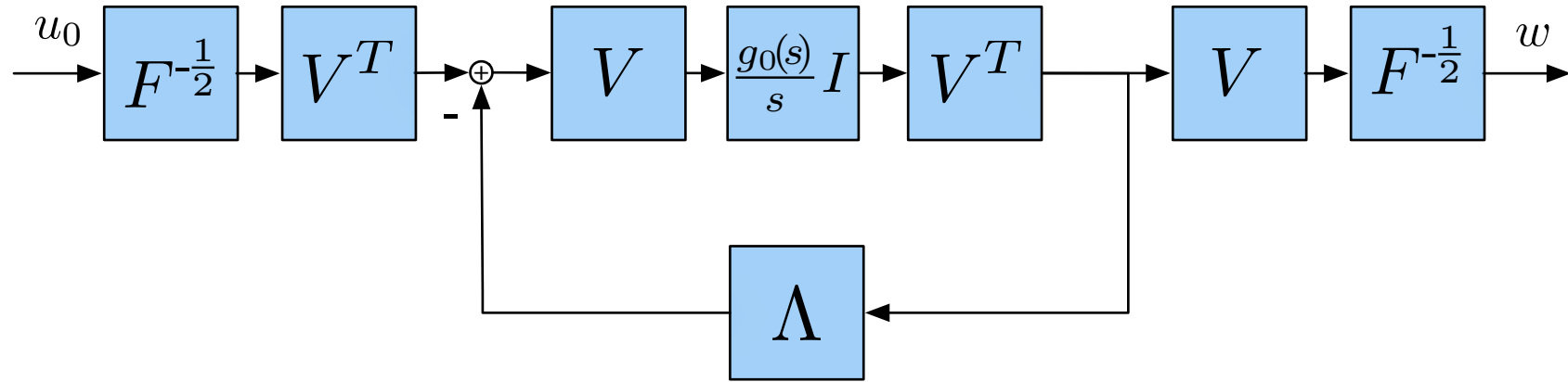
$$F^{-\frac{1}{2}} L F^{-\frac{1}{2}} := L_F = V \Lambda V^T$$

$$\Lambda = \text{diag}(\lambda_i), \lambda_0 = 0 \leq \lambda_1 \leq \dots \leq \lambda_{n-1}$$

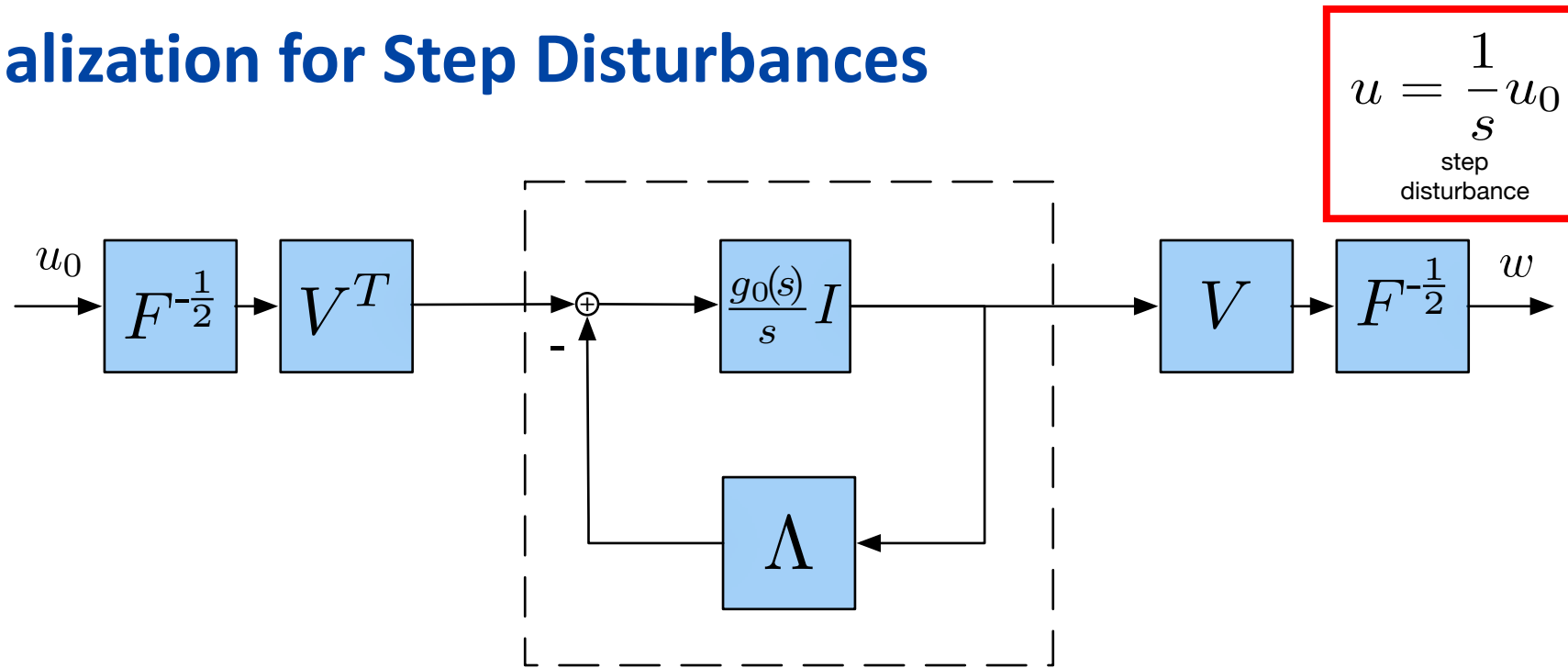
Diagonalization for Step Disturbances

$$u = \frac{1}{s} u_0$$

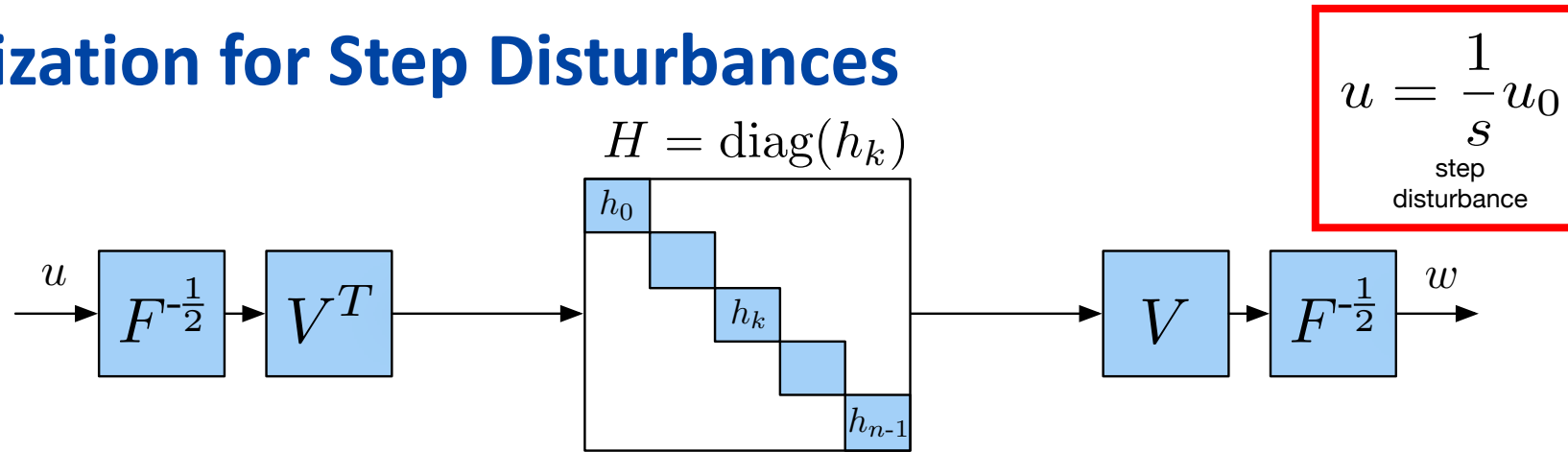
step disturbance



Diagonalization for Step Disturbances



Diagonalization for Step Disturbances



where $h_k(s) = \frac{g_0(s)}{s + \lambda_k g_0(s)}$, with

$$\lambda_0 = 0 \leq \lambda_1 \leq \dots \leq \lambda_{n-1}$$

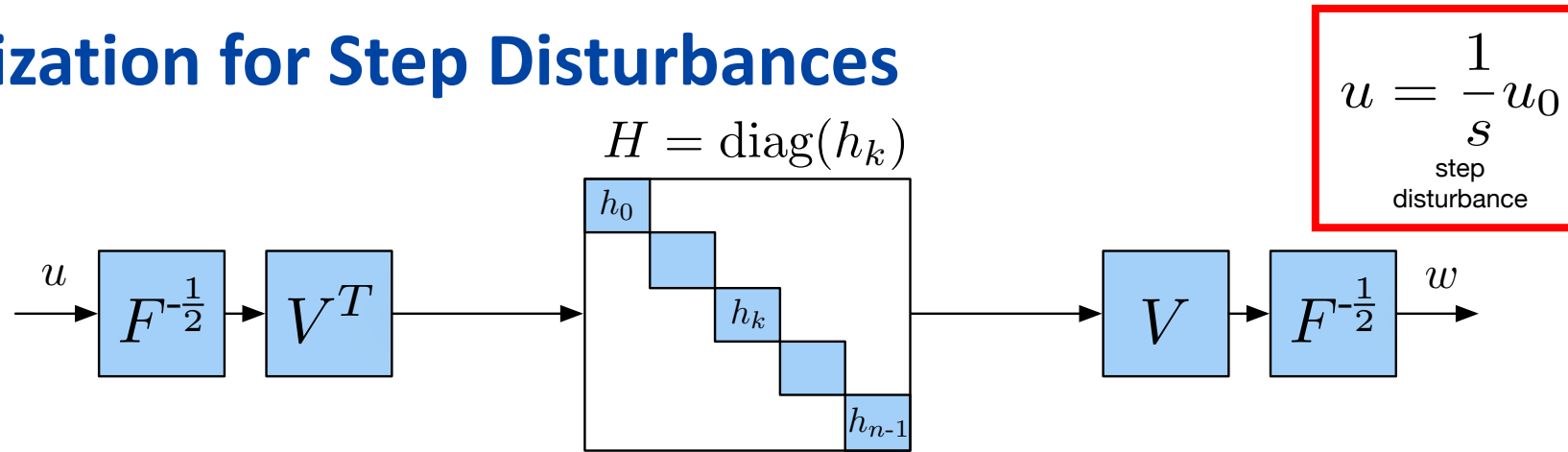
**System
Frequency:**

$$\bar{w}(s) = \frac{1}{\sum_i f_i} h_0(s) \underbrace{1^T u_0}_{\text{Imbalance}}$$

Global Response

Imbalance

Diagonalization for Step Disturbances



where $h_k(s) = \frac{g_0(s)}{s + \lambda_k g_0(s)}$, with

$$\lambda_0 = 0 \leq \lambda_1 \leq \dots \leq \lambda_{n-1}$$

**System
Frequency:**

$$\bar{w}(s) = \frac{1}{\sum_i f_i} h_0(s) \underbrace{1^T u_0}_{\text{Imbalance}}$$

Global Response

**Synchronization
Error:**

$$\tilde{w}(s) = \underbrace{F^{-\frac{1}{2}} V_{\perp} \tilde{H}(s) V_{\perp}^T F^{-\frac{1}{2}}}_{\text{Network Dependent Response}} u_0$$

Network Dependent Response

$$\tilde{H}(s) := \text{diag}(h_1(s), \dots, h_{n-1}(s))$$

$$V := [v_0 \ V_{\perp}]$$

Synchronization Cost

$$\|\tilde{w}\|_2^2 = \int_0^\infty \tilde{w}(t)^T \tilde{w}(t) dt$$

$$\tilde{w}(t) := \sum_{k=1}^{n-1} h_k(t) F^{-\frac{1}{2}} v_k v_k^T F^{-\frac{1}{2}} u_0$$

Proposition:

$$\|\tilde{w}\|_2^2 = z_0^T Y z_0$$

where $Y \in \mathbb{R}^{(n-1) \times (n-1)}$, with elements

$$y_{kl} = \gamma_{kl} \int_0^\infty h_k(t) h_l(t) dt, \quad \Gamma = (\gamma_{kl}) := V_\perp^T F^{-1} V_\perp \quad \text{and} \quad z_0 = V_\perp^T F^{-1} u_0$$

Depends on Generators and Controls.

Depends on Network and Inertia.

Average over step direction: $E_{u_0} [\|\tilde{w}\|_2^2]$ where $E [u_0 u_0^T] = \Sigma^u$

Synchronization Cost (II)

Swing Dynamics

$$\int_0^\infty h_k(t)h_l(t)dt = \frac{2d}{m(\lambda_k - \lambda_l)^2 + 2(\lambda_k + \lambda_l)d^2}$$

Homogeneous Case: $F = I \implies \Gamma = I$

$$E_{u_0} [||\tilde{w}||_2^2] = \frac{1}{2d} \sum_{k=1}^{n-1} \frac{1}{\lambda_k}$$

Independent of Inertia!

Heterogeneous case:

- High Inertia: $m \rightarrow \infty$ $||\tilde{w}||_2^2 = \sum_{k=1}^{n-1} \frac{\gamma_{kk} z_{0k}^2}{2d\lambda_k}$
- Small Inertia: $m \rightarrow 0$ $||\tilde{w}||_2^2 = \sum_{k,l=1}^{n-1} \frac{\gamma_{kl} z_{0k} z_{0l}}{d(\lambda_k + \lambda_l)}$

Inertia has limited effect

$$||\tilde{w}||_2^2 = z_0^T Y z_0 \quad y_{kl} = \gamma_{kl} \int_0^\infty h_k(t)h_l(t)dt$$

Swing Dynamics with Turbines

Homogenous Case:

$$E_{u_0} [||\tilde{w}||_2^2] = \sum_{k=1}^{n-1} ||h_k||^2$$

Moderate Improvement w/ Inertia

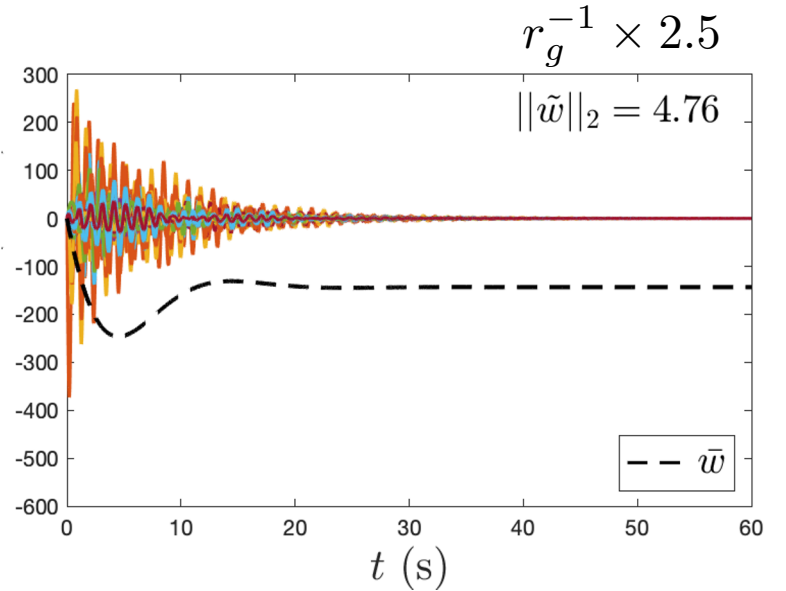
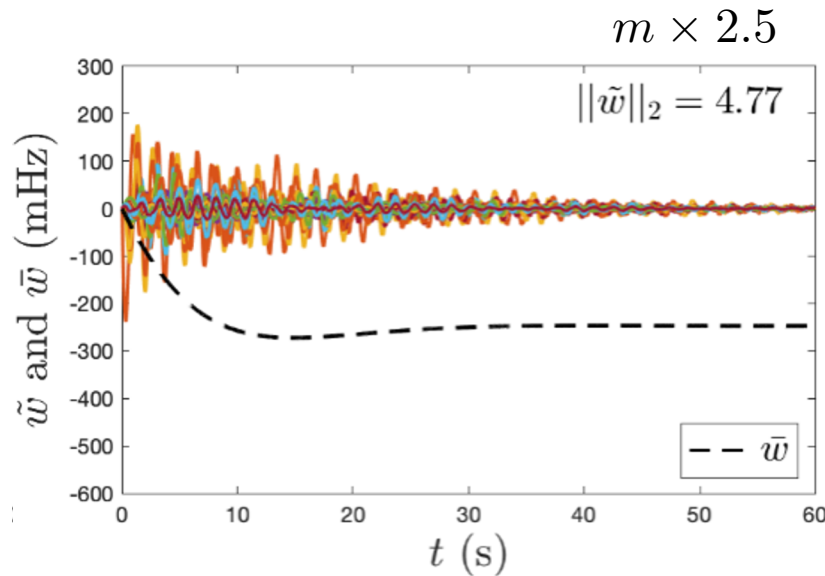
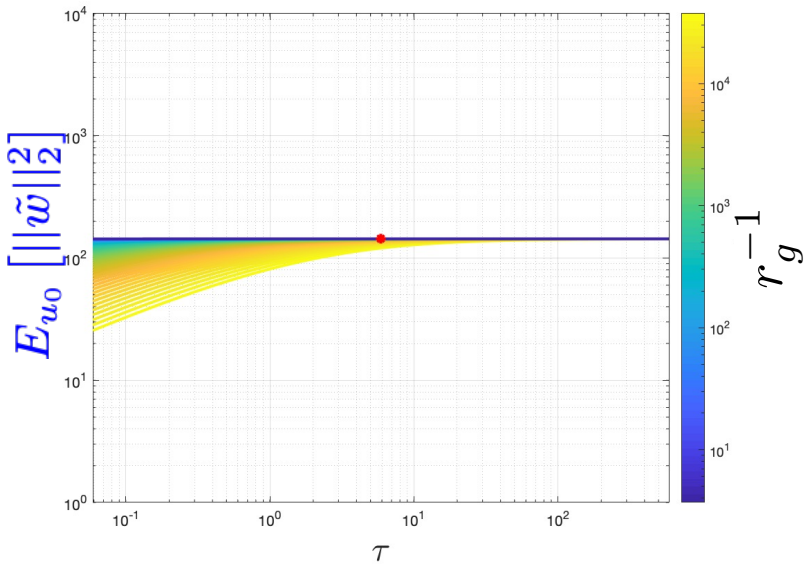
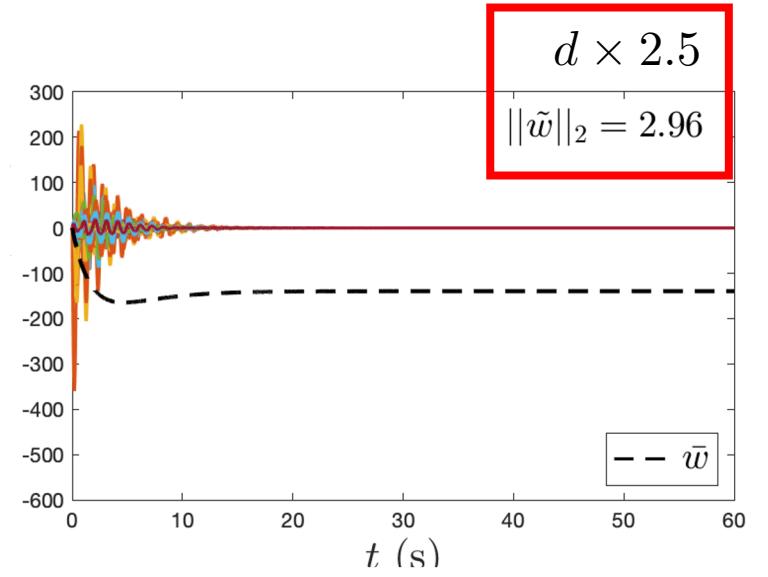
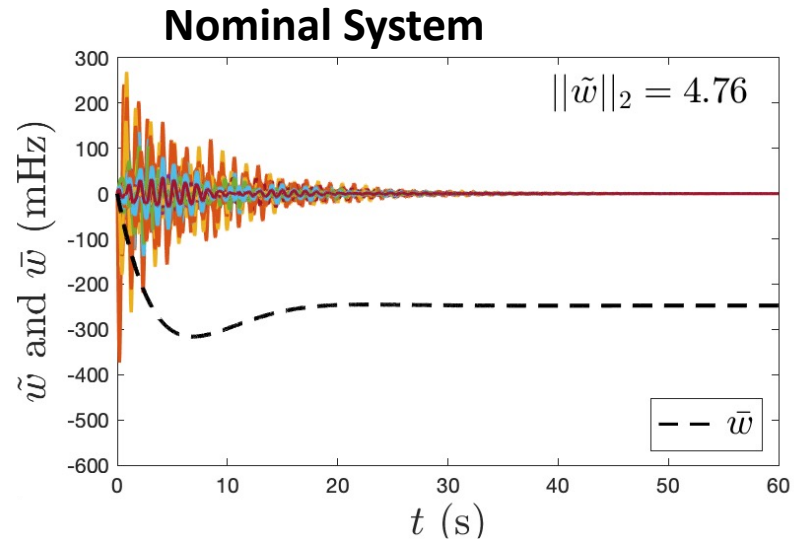
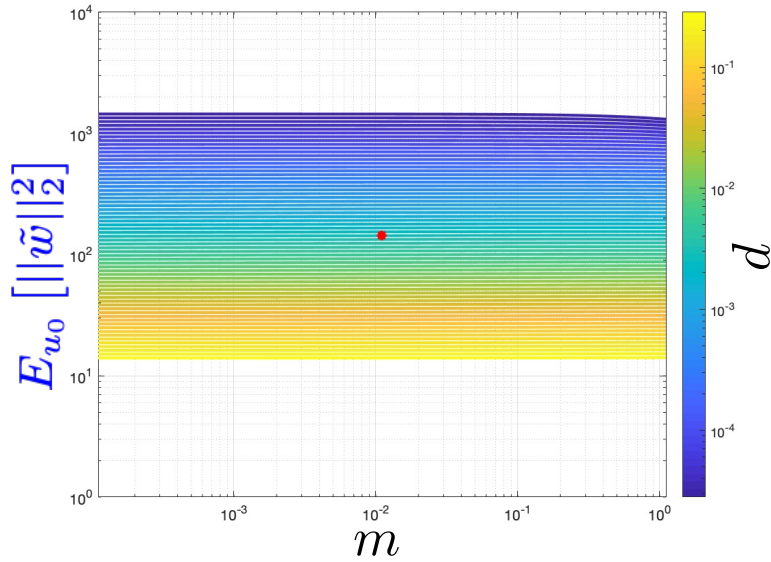
$$\lim_{m \rightarrow 0} ||h_k||^2 > \lim_{m \rightarrow \infty} ||h_k||^2$$

Heterogeneous Case:

- *More involved expressions...*
- *Limits are always **finite***

- High Inertia: $||\tilde{w}||_2^2 = \sum_{k=1}^{n-1} \frac{\gamma_{kk} z_{0k}^2}{2d\lambda_k} \frac{d}{d + r_g^{-1}}$

Synchronization Cost (II)



Red dots: nominal values