Embracing Low Inertia for Power System Frequency Control

A Frequency Shaping Approach

Enrique Mallada

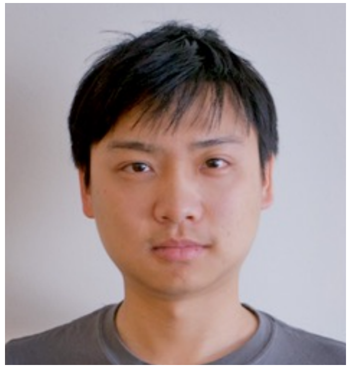
JOHNS HOPKINS
UNIVERSITY

Berkeley
DREAM/CPAR Seminar

April 25, 2022

Acknowledgements

Yan Jiang



Hancheng Min

Eliza Cohn

JOHNS HOPKINS
UNIVERSITY

Petr Vorobev

Skoltech
Skolkovo Institute of Science and Technology

Richard Pates

LUND UNIVERSITY

Fernando Paganini

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JOHNS HOPKINS
ENVIRONMENT, ENERGY,
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INSTITUTE

Decarbonization of electricity is key to mitigate climate change

California lifts renewable energy target to 73% by 2032

The California Public Utilities Commission raised renewable energy procurement targets, plans for a more aggressive decarbonization plan, and includes increased reliability provisions.

FEBRUARY 14, 2022 **RYAN KENNEDY**

New York mandates 70% renewable energy by 2030

By Kelsey Misbrener | October 15, 2020

Vermont House passes 75% by 2032 renewable energy mandate

Published March 11, 2015

Oregon bill targets 100% clean power by 2040, with labor and environmental justice on board

After Democratic cap-and-trade bills faltered in the face of GOP revolts, an electricity-focused, consensus-driven bill gains ground in Oregon.

23 June 2021

ENVIRONMENT

Maryland bill mandating 50% renewable energy by 2030 to become law, but without Gov. Larry Hogan's signature

By Scott Dance
Baltimore Sun • May 22, 2019 at 6:40 pm

Virginia becomes the first state in the South to target 100% clean power

The state's Democratic majority is doing what Democratic majorities do.

By David Roberts | @drvolts | Updated Apr 13, 2020, 2:56pm EDT

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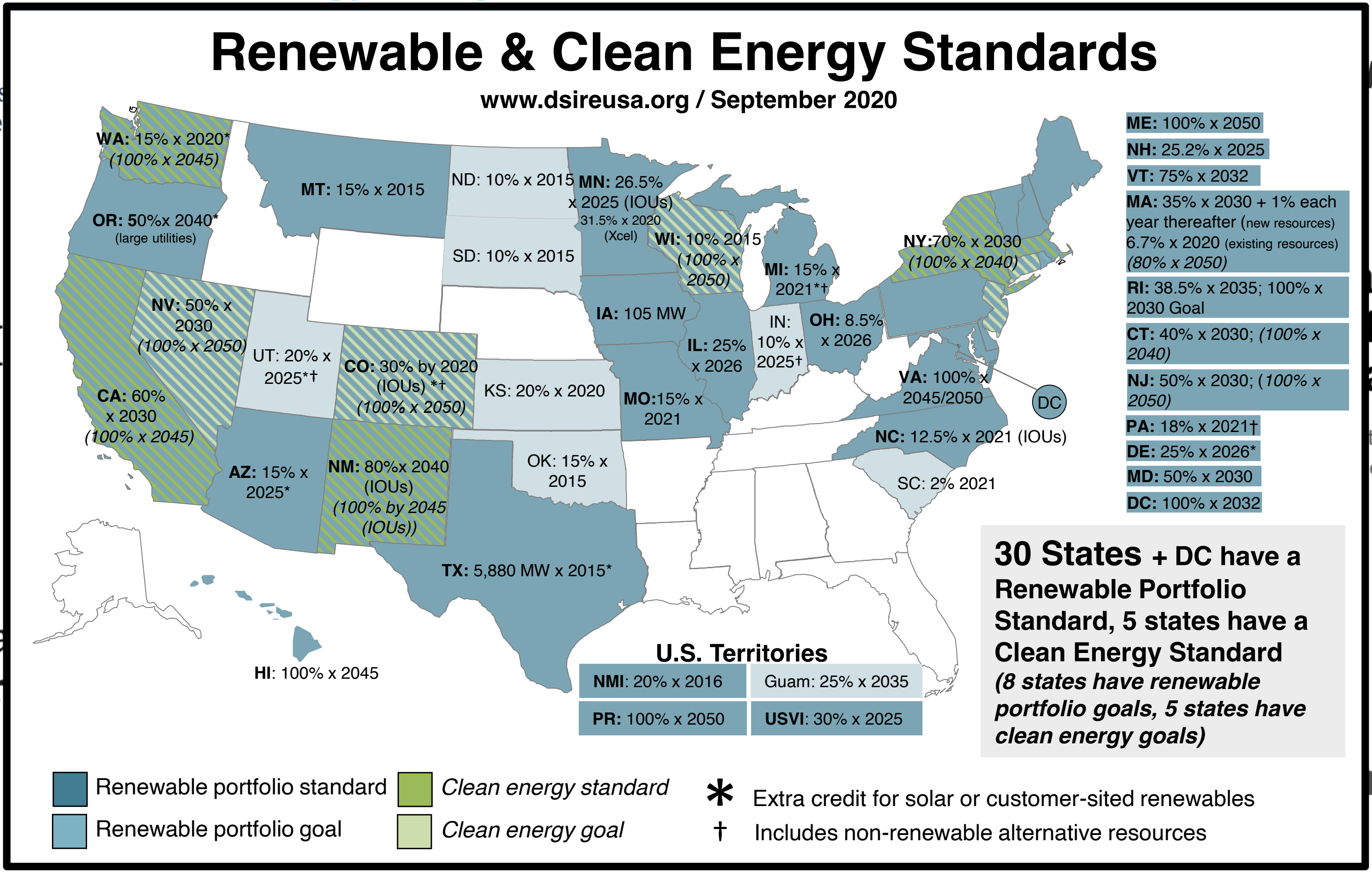

Renewable & Clean Energy Standards

www.dsireusa.org / September 2020

ME: 100% x 2050
NH: 25.2% x 2025
VT: 75% x 2032
MA: 35% x 2030 + 1% each year thereafter (new resources) 6.7% x 2020 (existing resources) *(80% x 2050)*
RI: 38.5% x 2035; 100% x 2030 Goal
CT: 40% x 2030; *(100% x 2040)*
NJ: 50% x 2030; *(100% x 2050)*
PA: 18% x 2021†
DE: 25% x 2026*
MD: 50% x 2030
DC: 100% x 2032

U.S. Territories

NMI: 20% x 2016	Guam: 25% x 2035
PR: 100% x 2050	USVI: 30% x 2025

30 States + DC have a Renewable Portfolio Standard, 5 states have a Clean Energy Standard
(8 states have renewable portfolio goals, 5 states have clean energy goals)

■ Renewable portfolio standard
□ Renewable portfolio goal
■ *Clean energy standard*
□ *Clean energy goal*
* Extra credit for solar or customer-sited renewables
† Includes non-renewable alternative resources

able energy

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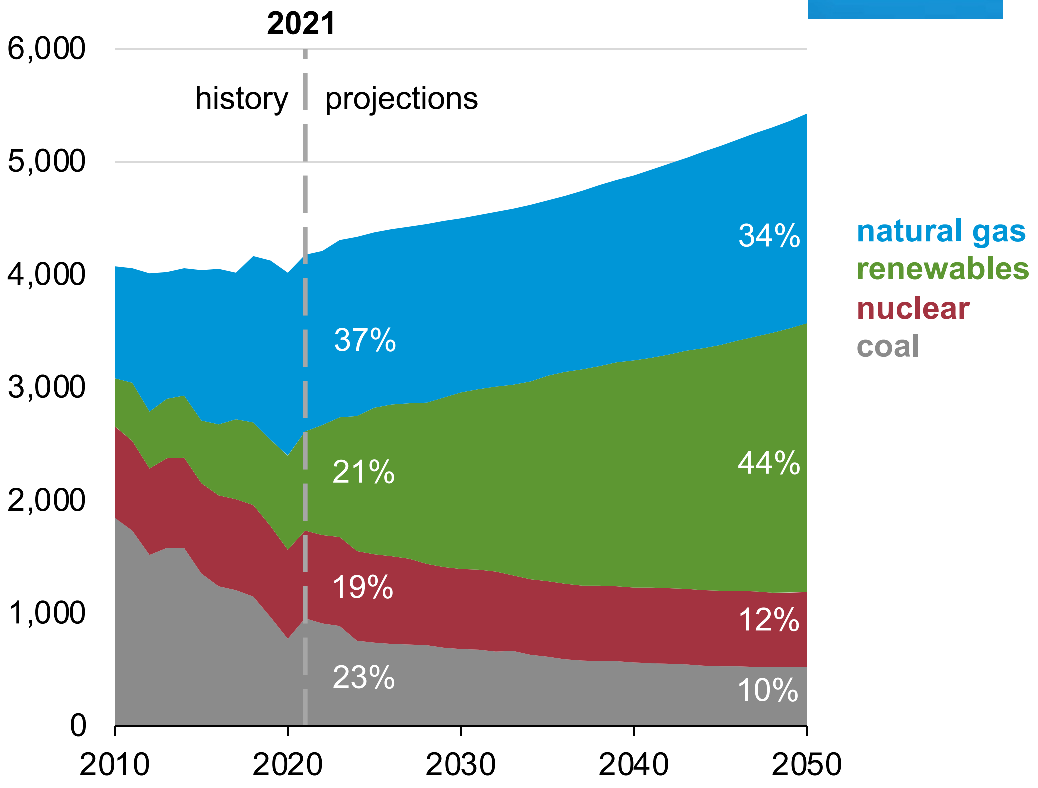
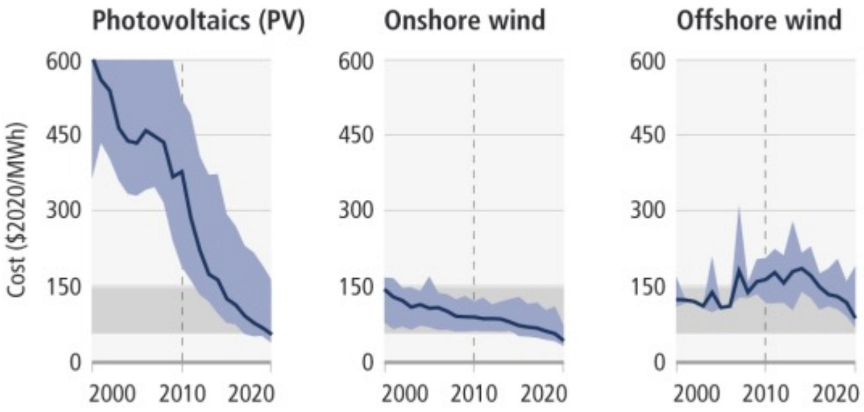
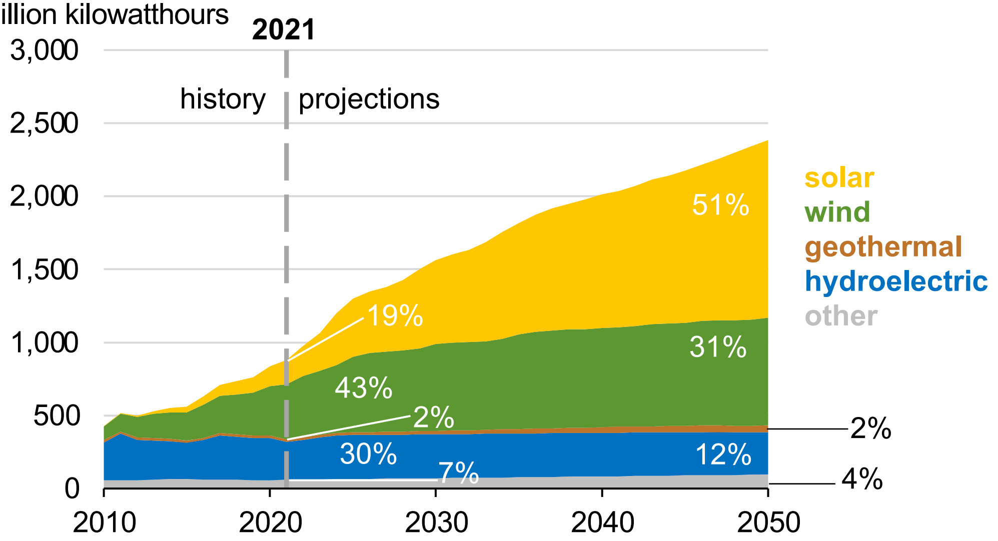
Decarbonization of electricity is key to mitigate climate change

U.S. electricity generation from selected fuels
AEO2022 Reference case
billion kilowatthours



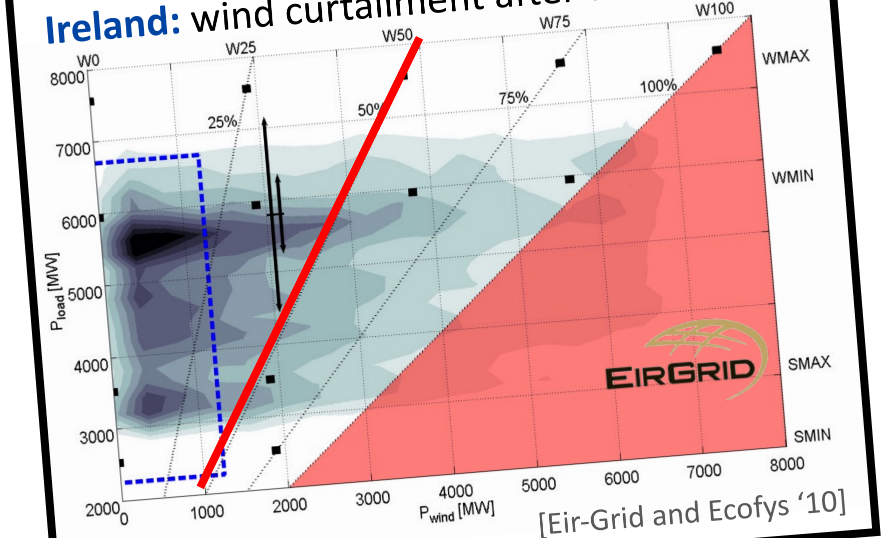


U.S. renewable electricity generation, including end use
AEO2022 Reference case
billion kilowatthours



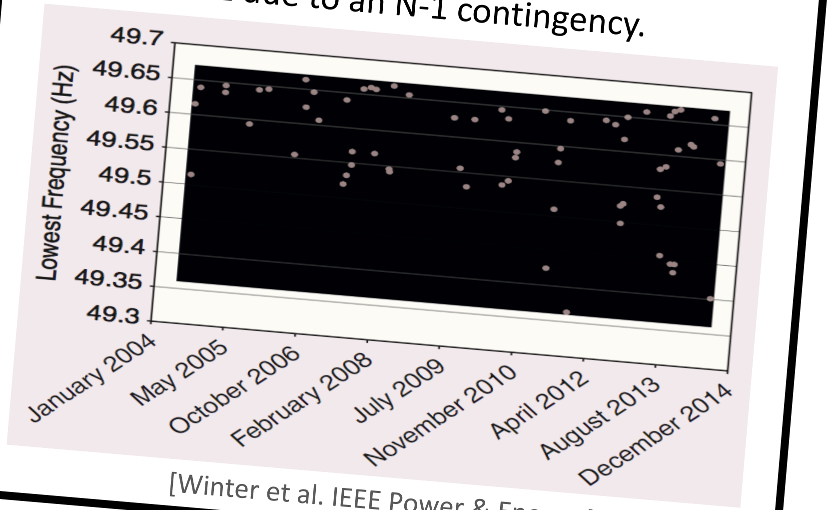
Dynamic Degradation

Ireland: wind curtailment after 50% of load



[Eir-Grid and Ecofys '10]

Nordic system: disturbances with frequencies under 49.7 Hz due to an N-1 contingency.



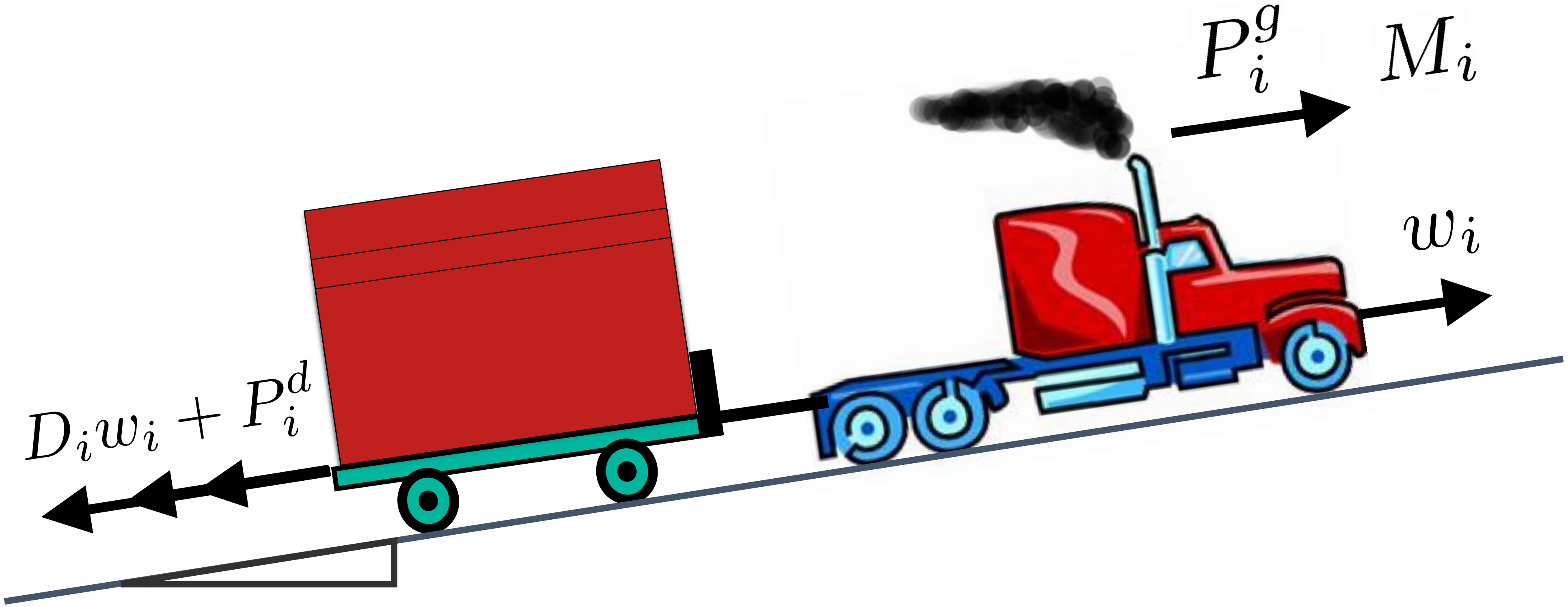
[Winter et al. IEEE Power & Energy Magazine '15]

In the United States: FERC Federal Energy Regulatory Commission

"While the three [contiguous] U.S. interconnections currently exhibit adequate frequency response performance above their interconnection frequency response obligations, there has been a significant decline in the frequency response performance of the Western and Eastern Interconnections," FERC said.

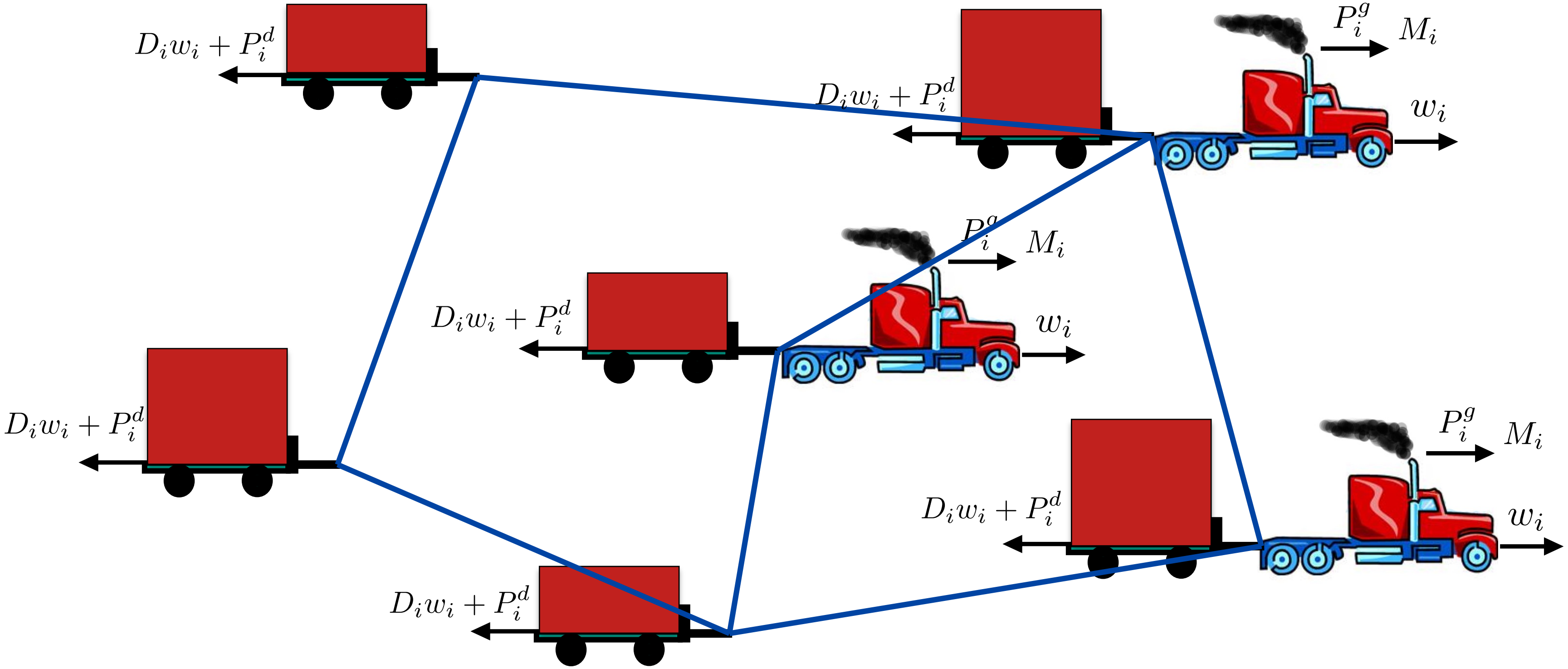
[FERC, Nov. 16]

Understanding Frequency Control



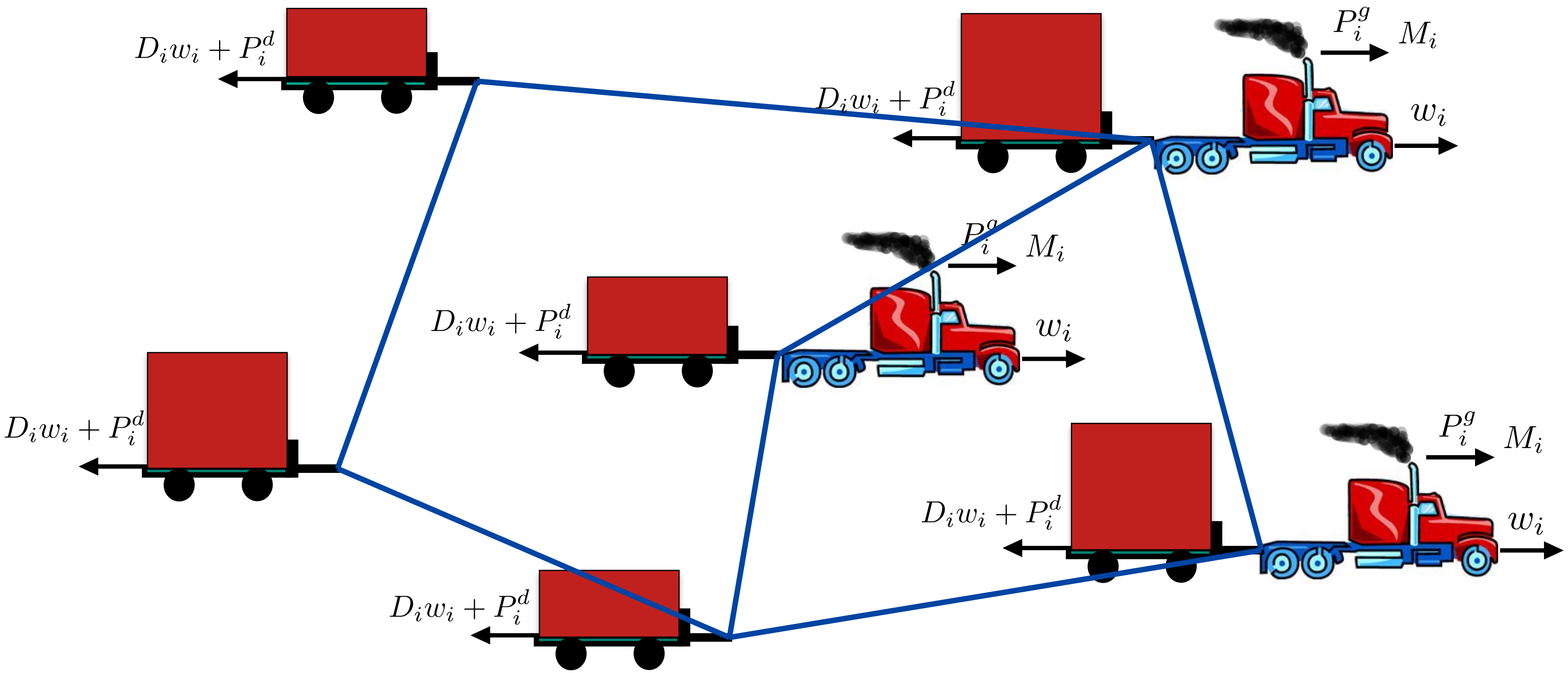
Goal: Maintain speed w_i close to the nominal (60/50 Hz)

Understanding Frequency Control



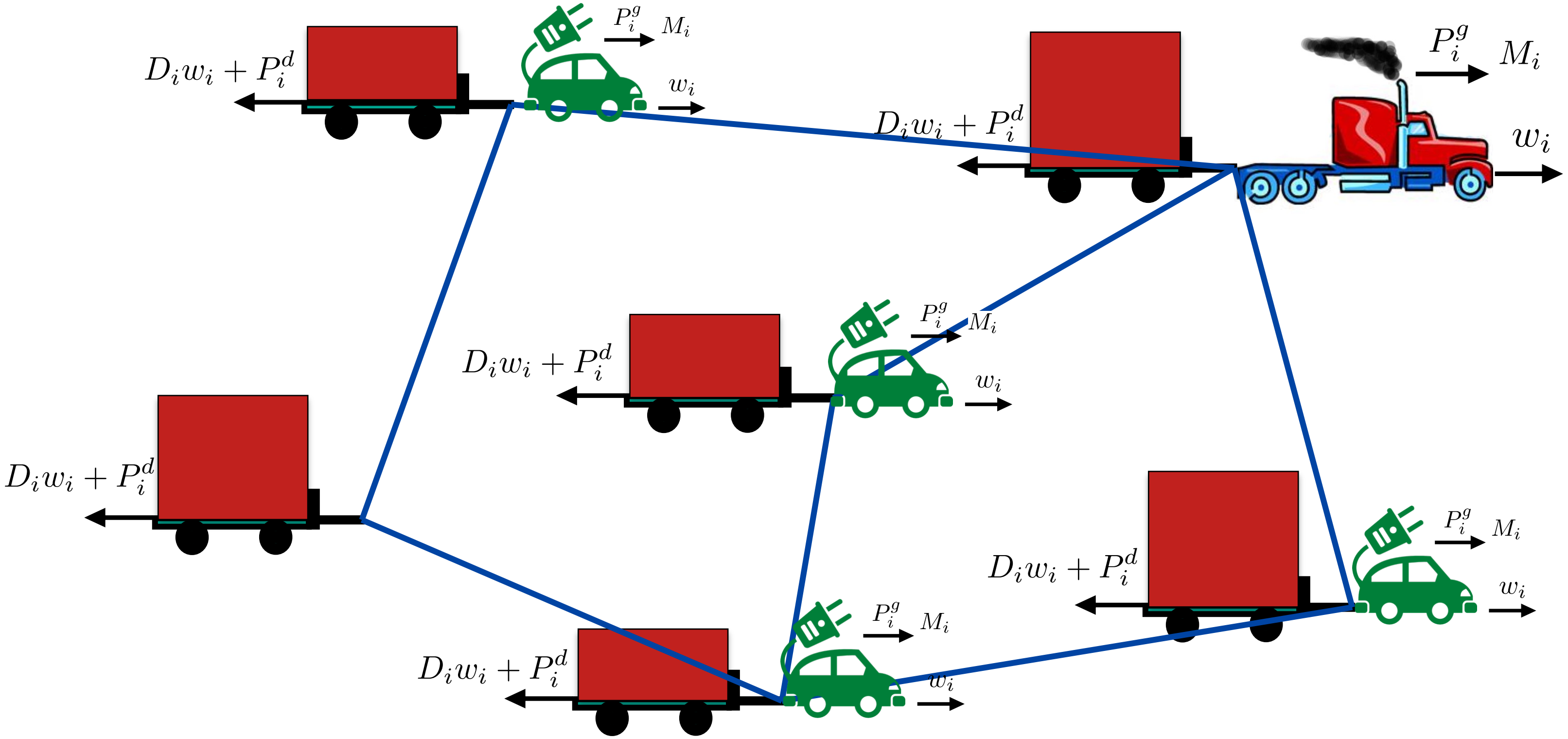
Understanding Low Inertia Frequency Control

How should we control low inertia power systems?



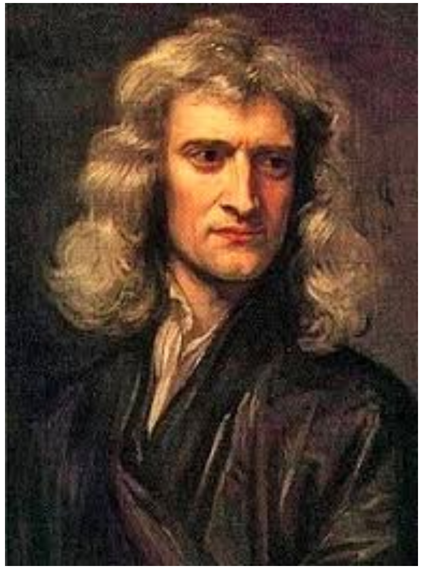
Understanding Low Inertia Frequency Control

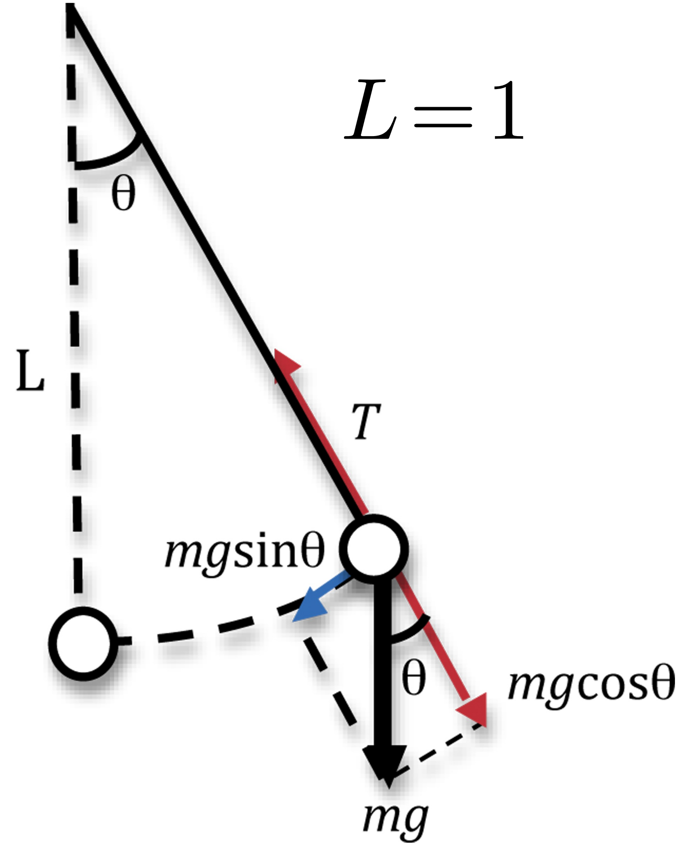
How should we control low inertia power systems?



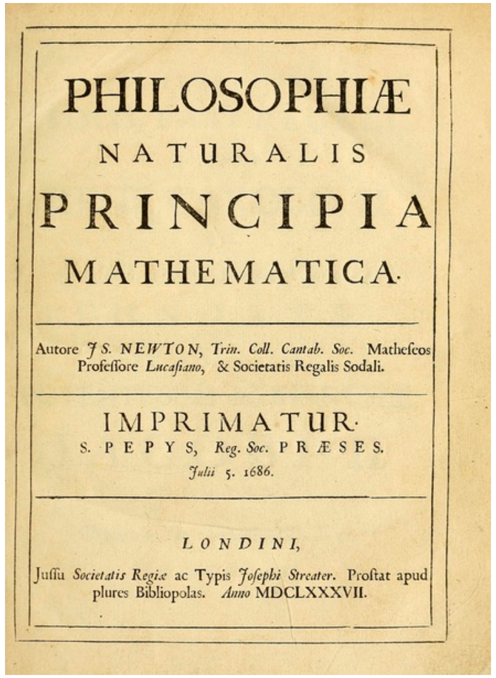
it is important to understand the merits and trade-offs of low inertia!

Merits and Trade-offs of Inertia





$$L=1$$



PHILOSOPHIÆ
NATURALIS
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MATHEMATICA.

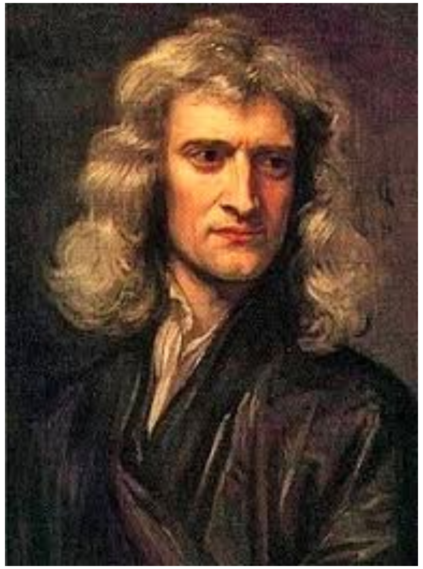
Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheſeos Profeſſore Lucaſiano, & Societatis Regalis Sodali.

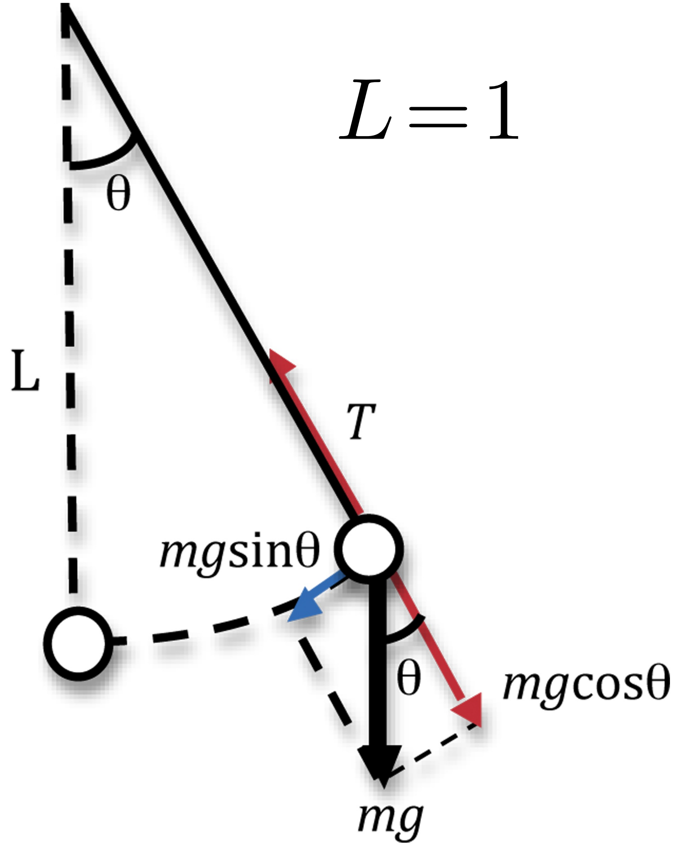
IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu Societatis Regiæ ac Typis Joſephi Streater. Proſtat apud plures Bibliopolas. Anno MDCLXXXVII.

$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f$$

Merits and Trade-offs of Inertia





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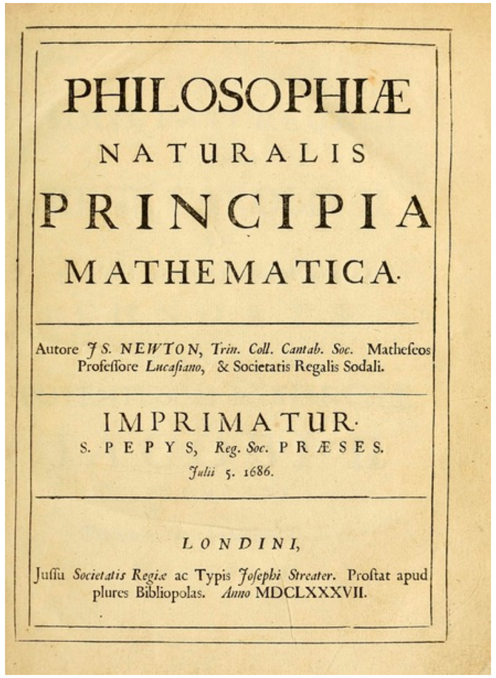

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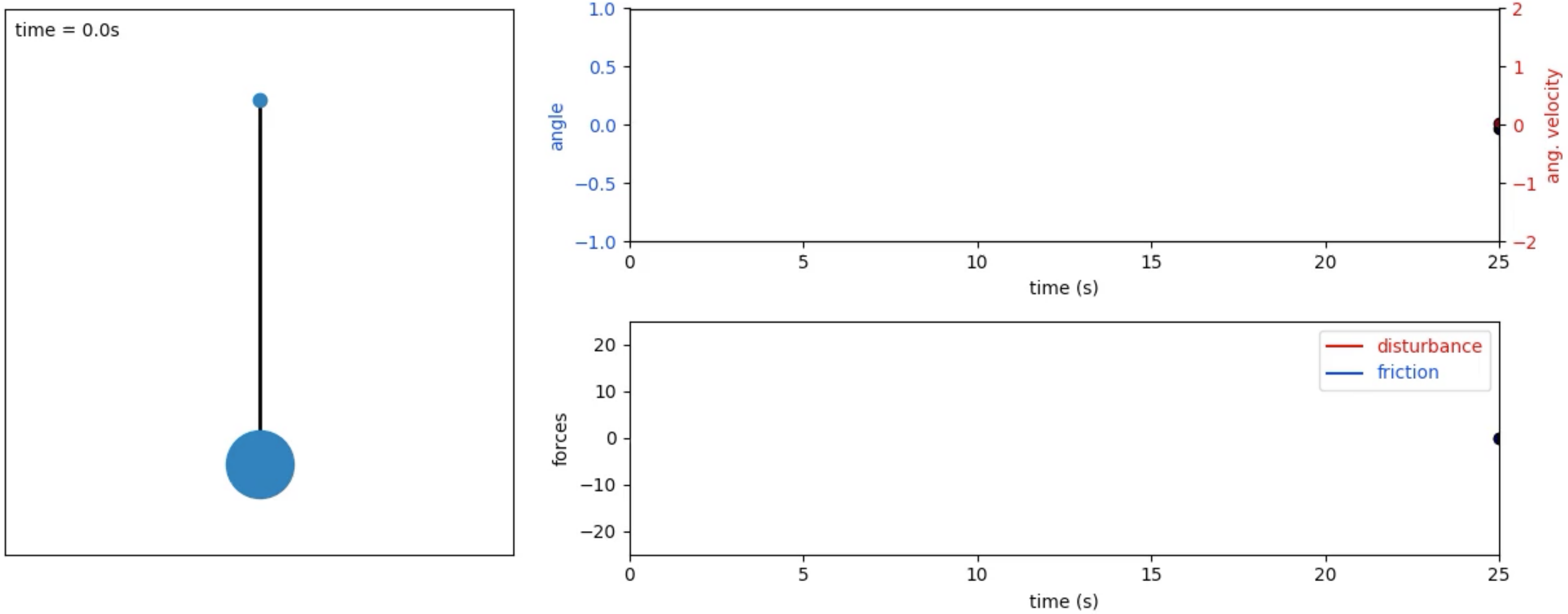
$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$

Merits and Trade-offs of Inertia

$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$

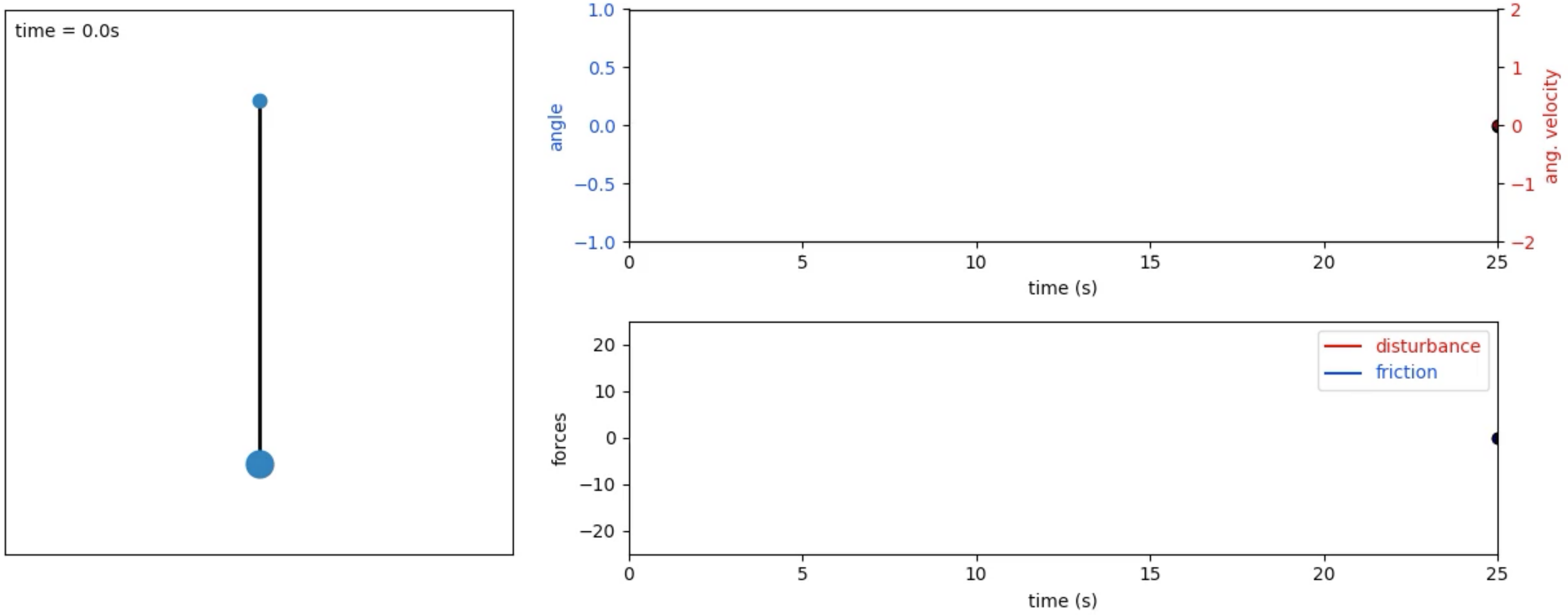


Pros: Provides natural disturbance rejection

Cons: Hard to regain steady-state

Merits and Trade-offs of Low Inertia

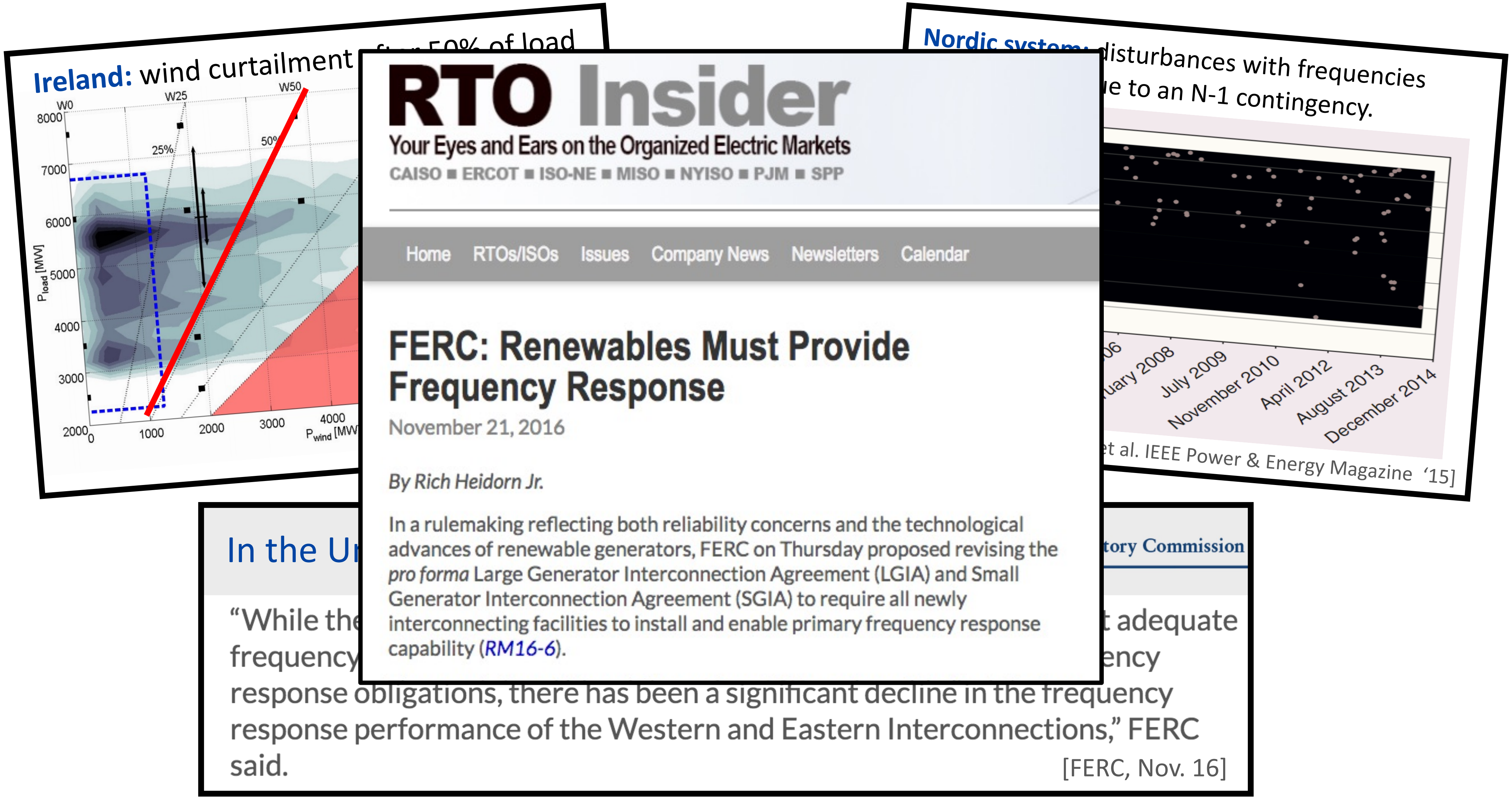
$$\ddot{\theta} = -\frac{d}{m}\dot{\theta} - g\sin\theta + \frac{f}{m}$$



Cons: Susceptible to disturbances

Pros: Regains steady-sate faster

Dynamic Degradation



RTO Insider

Your Eyes and Ears on the Organized Electric Markets

CAISO ■ ERCOT ■ ISO-NE ■ MISO ■ NYISO ■ PJM ■ SPP

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FERC: Renewables Must Provide Frequency Response

November 21, 2016

By Rich Heidorn Jr.

In a rulemaking reflecting both reliability concerns and the technological advances of renewable generators, FERC on Thursday proposed revising the *pro forma* Large Generator Interconnection Agreement (LGIA) and Small Generator Interconnection Agreement (SGIA) to require all newly interconnecting facilities to install and enable primary frequency response capability (RM16-6).

In the U

tory Commission

“While the

t adequate

frequency

ency

response obligations, there has been a significant decline in the frequency response performance of the Western and Eastern Interconnections,” FERC said.

[FERC, Nov. 16]

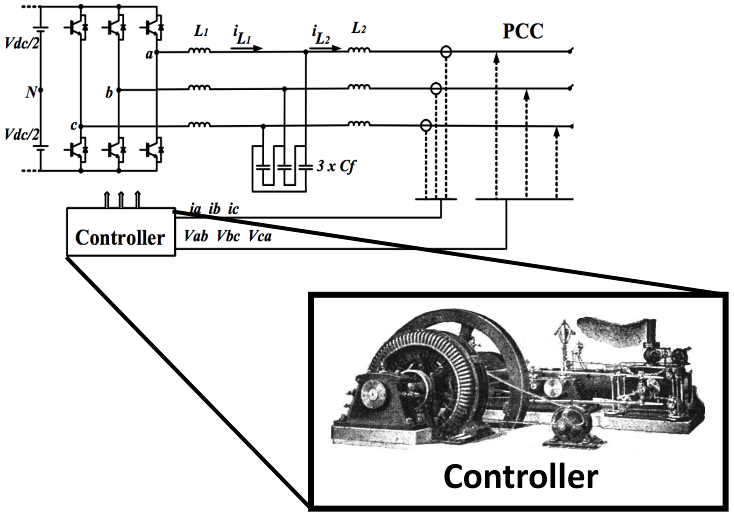
Inverter-based Control

Challenges

- Measurements with noise and delays
- Stability + robustness (plug & play)
- Lack of incentives

Current approach: Use inverter-based control to **mimic generators response**

Virtual Synchronous Generator



Telecom Analogy

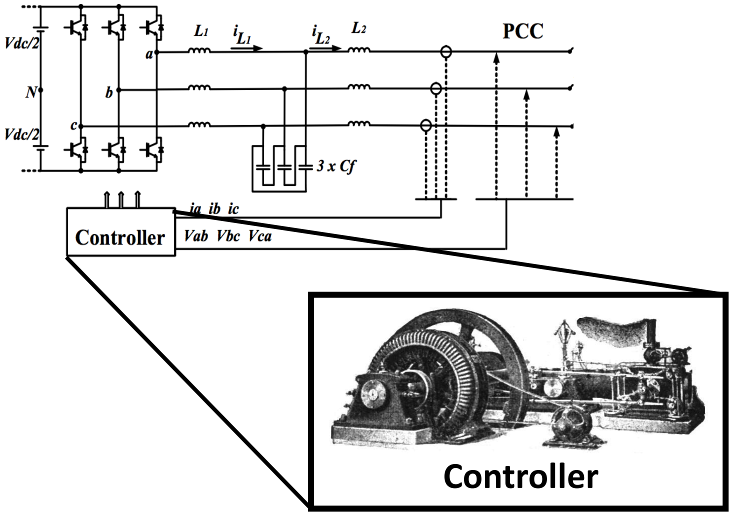
Inverter-based Control

Challenges

- Measurements with noise and delays
- Stability + robustness (plug & play)
- Lack of incentives

Current approach: Use inverter-based control to **mimic generators response**

Virtual Synchronous Generator



Telecom Analogy

It works, but perhaps there is something better…

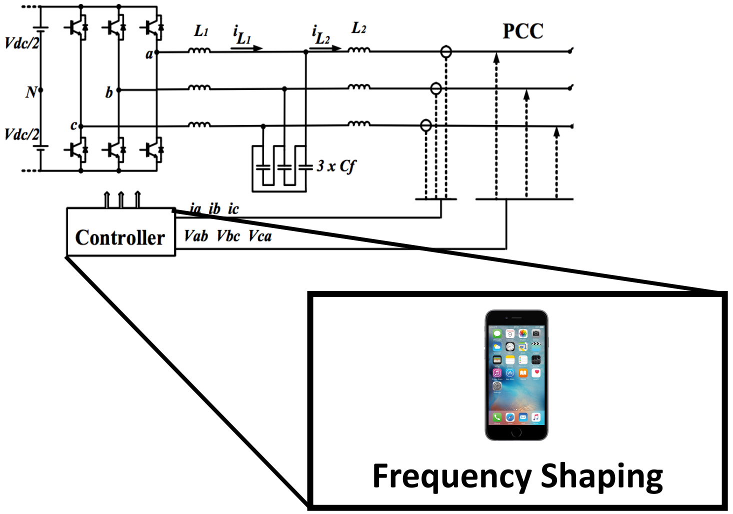
Inverter-based Control

Challenges

- Measurements with noise and delays
- Stability + robustness (plug & play)
- Lack of incentives

Our approach: Design and tune of controllers rooted on **sound control principles**

Frequency Shaping Control



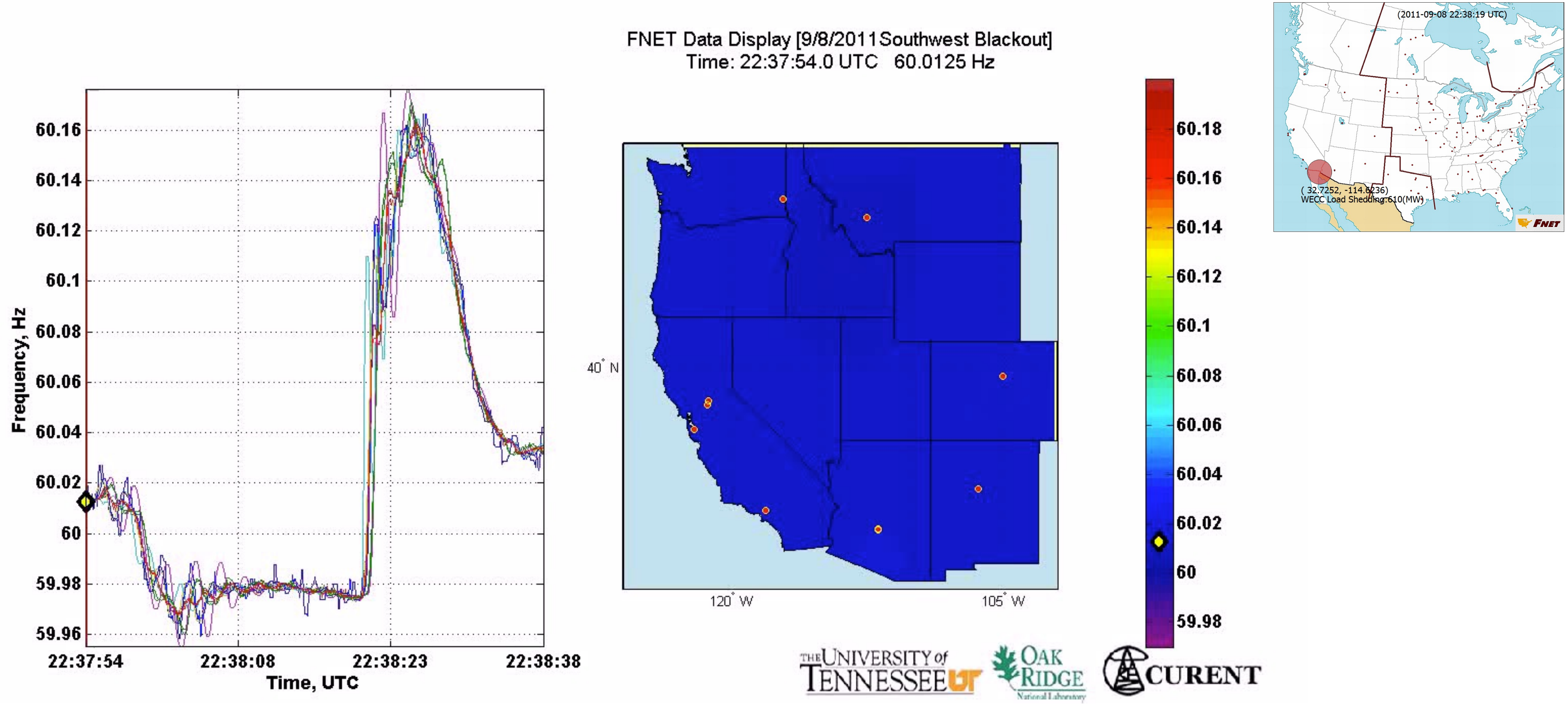
Design Objectives:

- Exploit power electronics capabilities
- Improve Dynamic Performance
- Minimize control effort
- Stability and Robustness

Roadmap to Low Inertia Frequency Control

- Performance Specification and Analysis
- Limits of Virtual Inertia and Droop Control
- Control Design: Frequency Shaping

Power System Performance



Depends on several factors: generators, network, disturbance

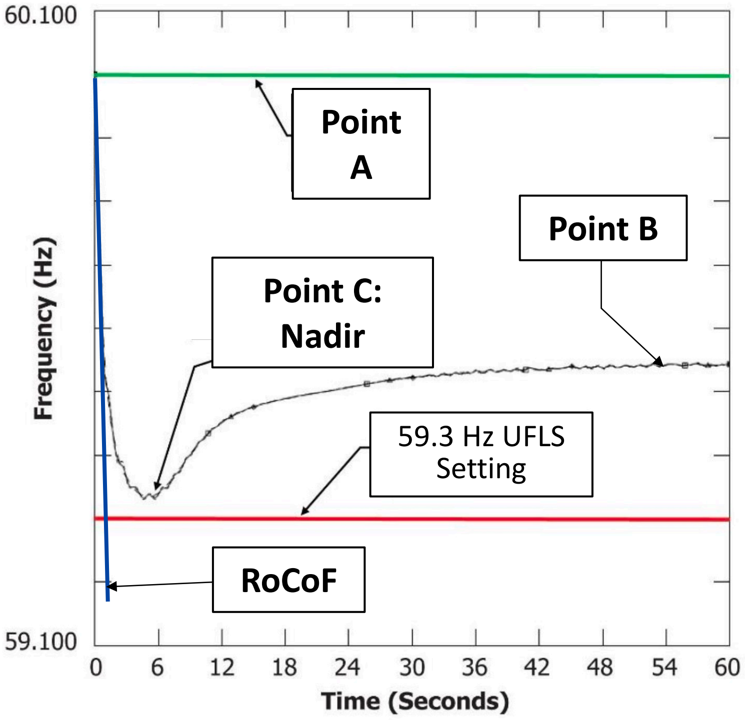
good performance metric must identify the source of the degradation!

Power Engineering Metrics

based on classical control theory…

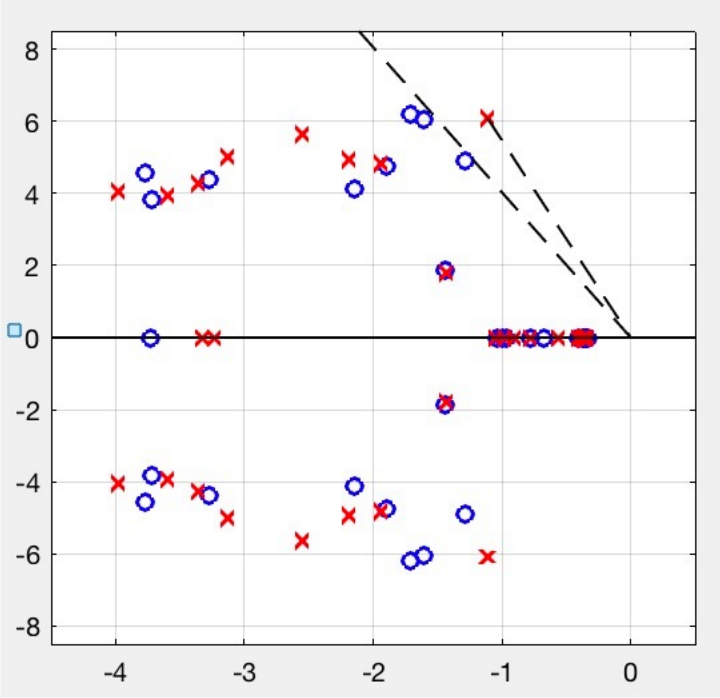
Step Response

Nadir, RoCoF, Steady-State



Eigenvalue Analysis [Verghese '89]

Dom. Eig, Damping Ratios, Part. Fact.

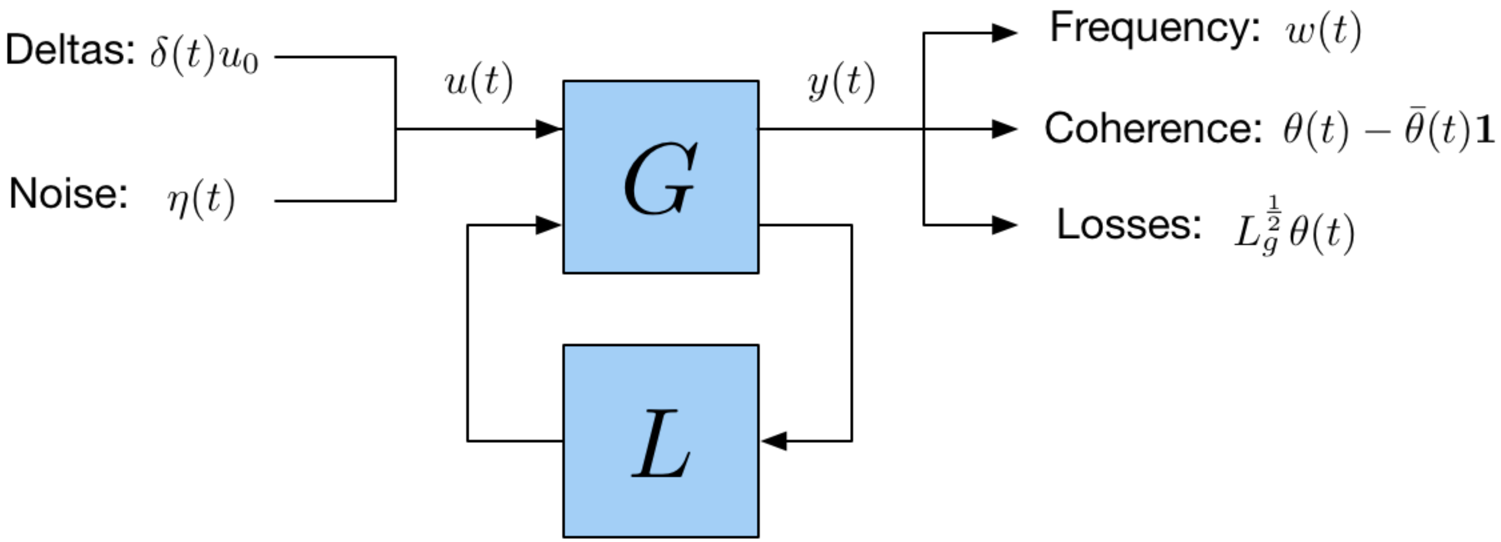


+ Domain specific, capture system degradation

− Relation with cause? Eigenvalue sensitivity? More than one "dominant" eig.?

System Theoretic Metrics

[Tegling… ’15, Poolla… ’15, Grunberg… ’16, Simpson-Porco…‘16, Wu et al ’16, Adreasson ’17, Coletta ‘17…]



$\mathcal{L}_\infty$-norm:

$$||y||_\infty := \sup_{t\geq 0}\max_i |y_i(t)|$$

$\mathcal{L}_2$-norm:

$$||y||_2 := \left(\int_0^\infty y(t)^T y(t)dt\right)^{\frac{1}{2}}$$

\+ Close form solutions, qualitative analysis, computational methods

\- Restrictive assumptions, not direct connection with RoCoF, Nadir, step disturbances

Performance Specification

Frequency Response

Control Effort

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, **IEEE Transactions on Automatic Control, 2020**
[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Performance Specification

Frequency Response

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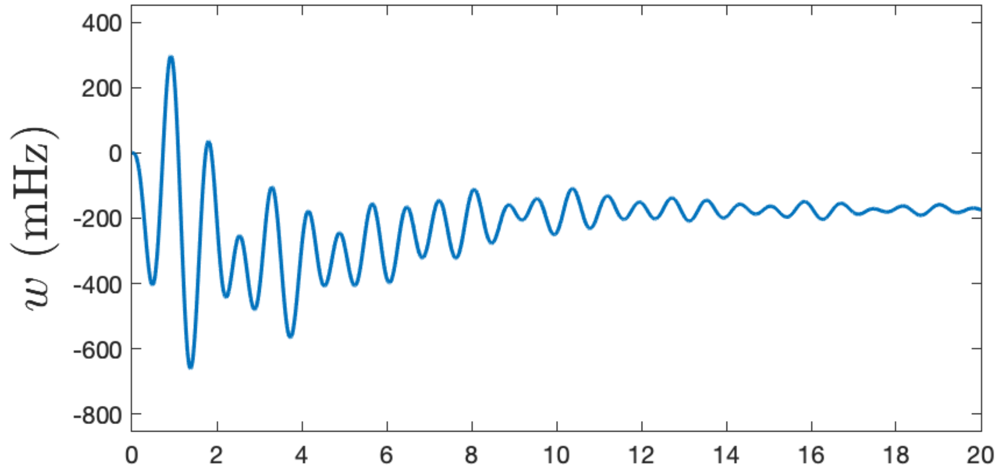
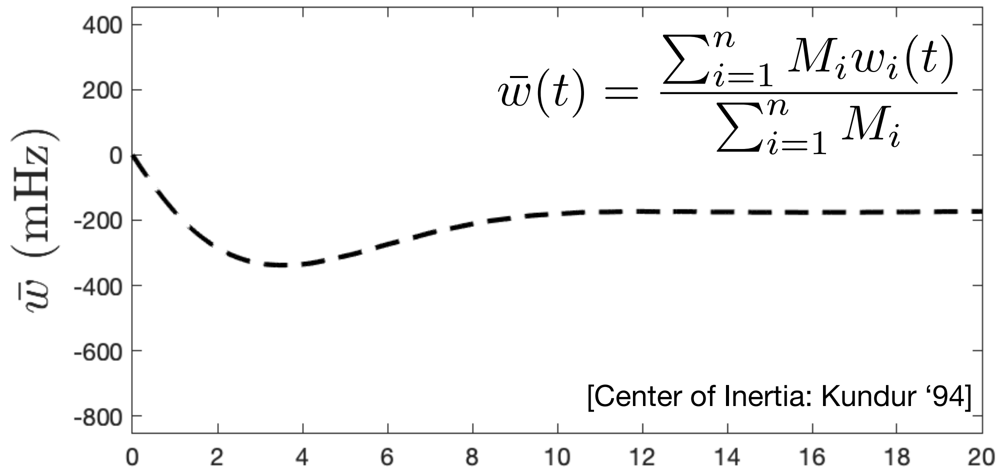
Decomposition of Step Response



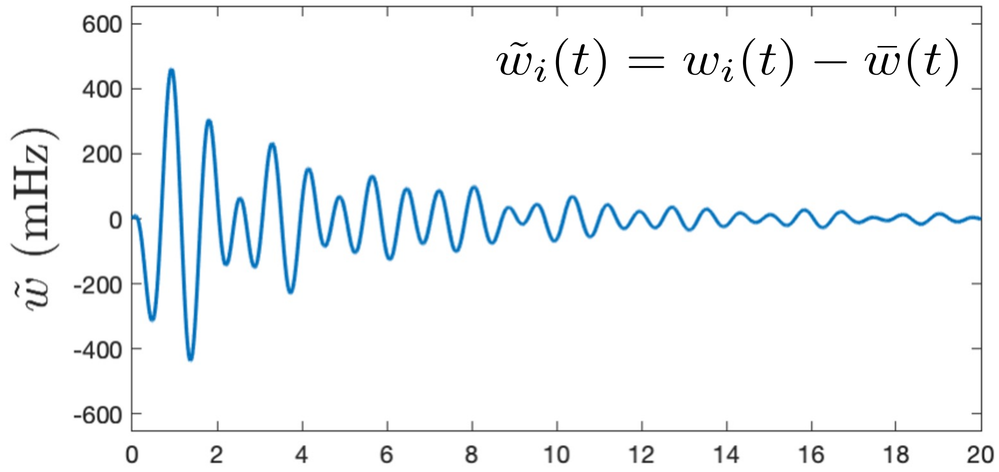
System Frequency



$$\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$$

[Center of Inertia: Kundur ‘94]

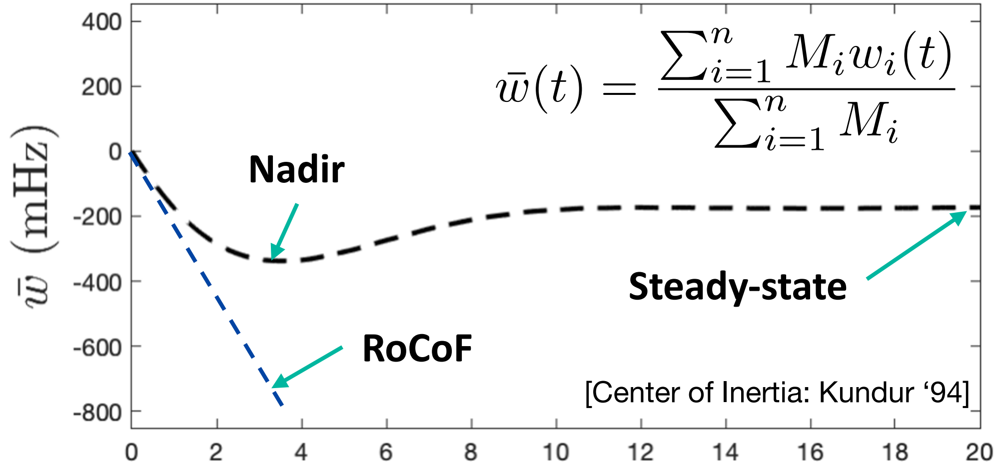
Synchronization Error

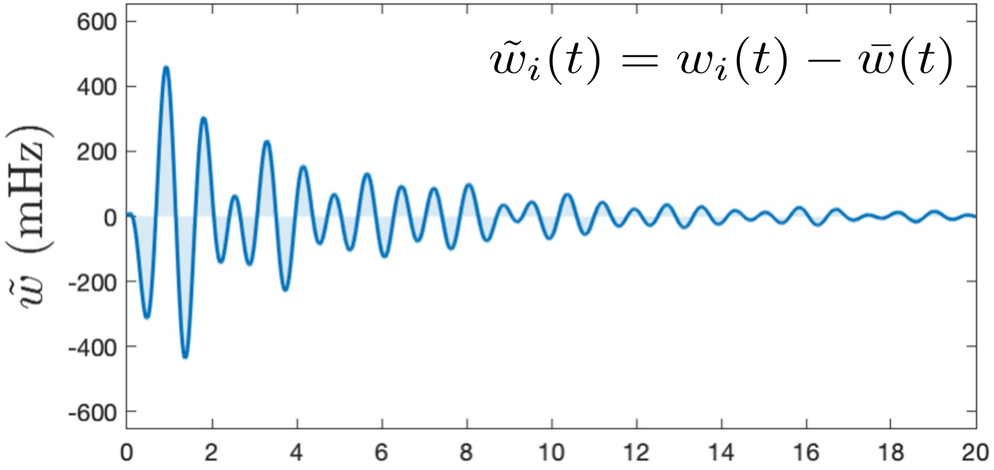


$$\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, **IEEE Transactions on Automatic Control, 2020**
[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Step Disturbance Performance


System Frequency


Deviation from Mean

Nadir

$$||\bar{w}||_\infty := \sup_{t\geq 0}|\bar{w}(t)|$$

RoCoF

$$||\dot{\bar{w}}||_\infty := \sup_{t\geq 0}|\dot{\bar{w}}(t)|$$

Steady-state

$$||\dot{\bar{w}}||_\infty := \sup_{t\geq 0}|\dot{\bar{w}}(t)|$$

Synchronization Cost

$$||\tilde{w}||_2 = \left(\int_0^{+\infty} \sum_{i=1}^{n} \tilde{w}_i^2(t)dt\right)^{\frac{1}{2}}$$

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap,* **IEEE Transactions on Automatic Control, 2020**
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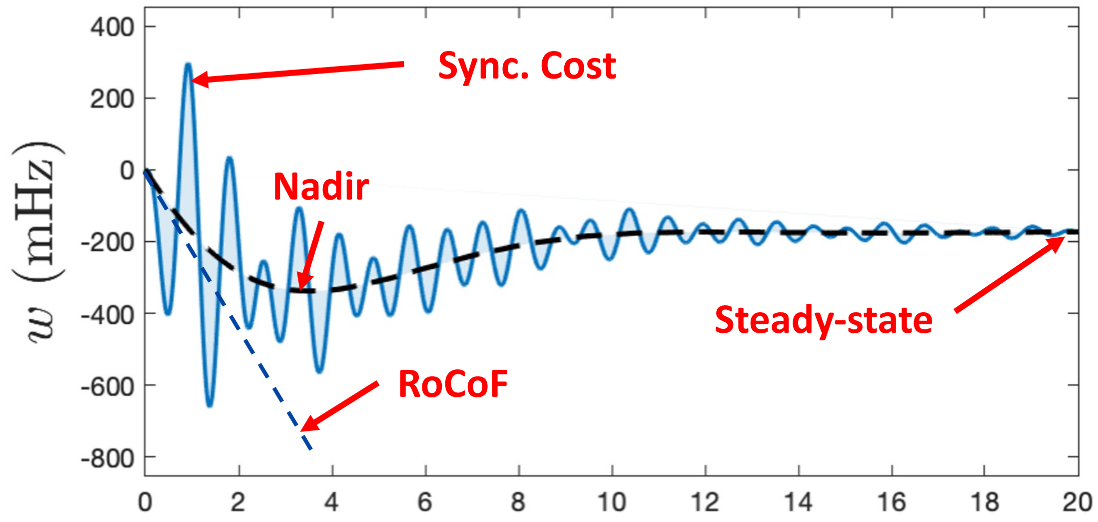
Performance Specification

Frequency Response



System Freq. : $\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort

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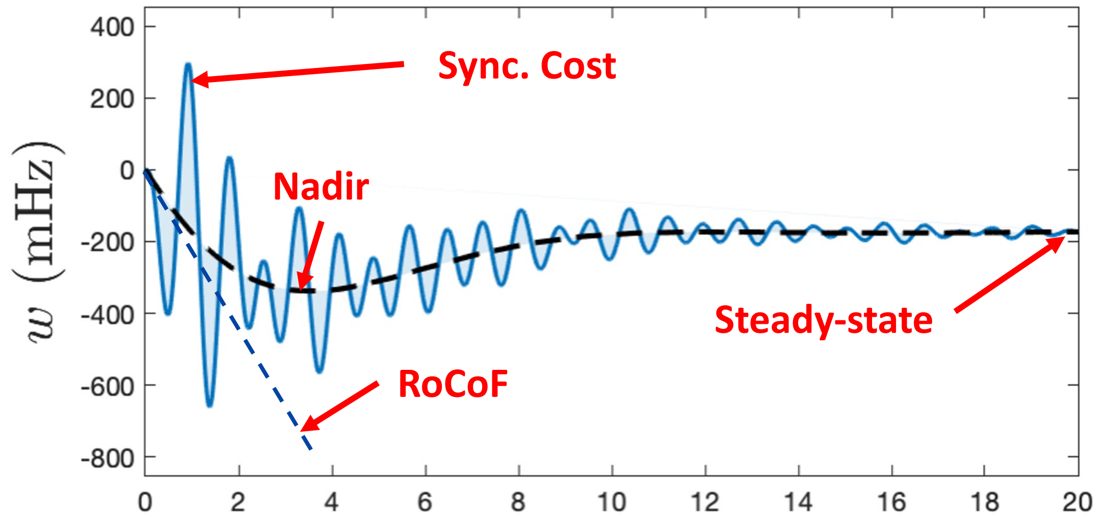
Performance Specification

Frequency Response



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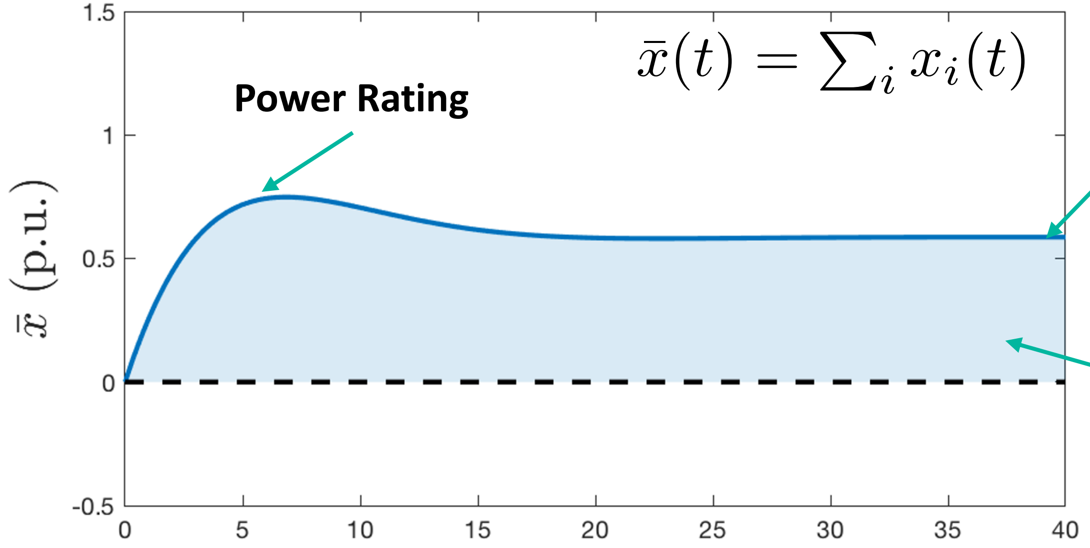
Control Effort

Inverter Power



$\bar{x}(t) = \sum_i x_i(t)$

Power Rating

Steady-State Effort

$|\bar{x}(\infty)| = \sum_i R_{r,i}^{-1} |\bar{w}(\infty)|$

Injected Energy

$\dot{E}(t) = \bar{x}(t)$

Power Rating

$$\|\bar{x}\|_\infty := \sup_{t\geq 0} |\bar{x}(t)|$$

Max Energy

$$\|E\|_\infty := \sup_{t\geq 0} |E(t)|$$

Steady-State Effort

$$|\bar{x}(\infty)| = \sum_i R_{r,i}^{-1} |\bar{w}(\infty)|$$

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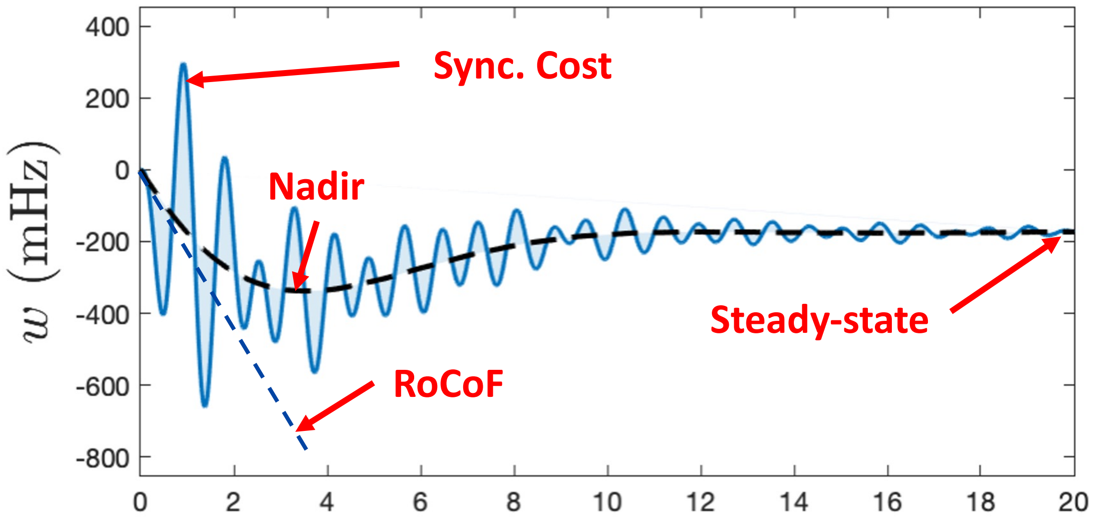
Performance Specification

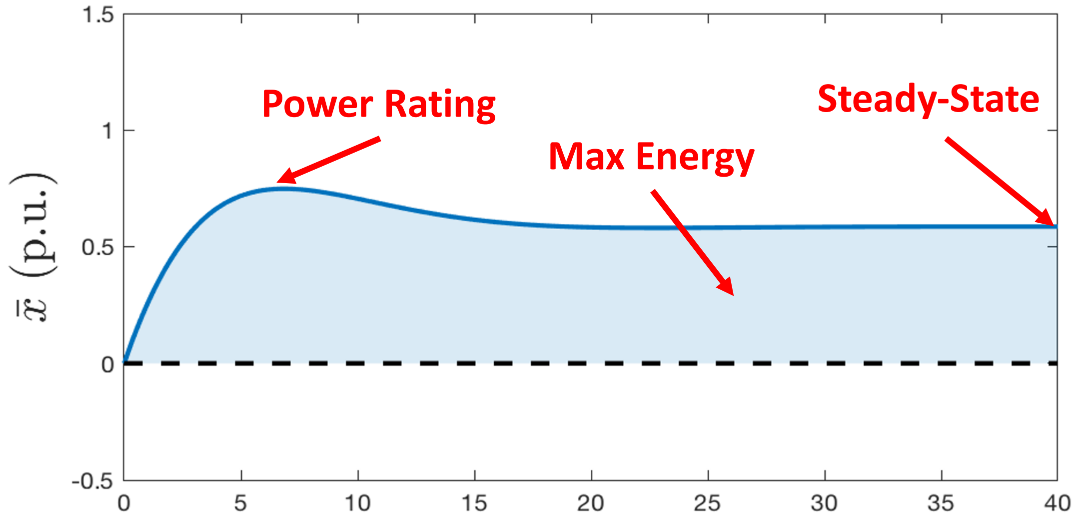
Frequency Response



System Freq. : $\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort



Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

Injected Energy: $\dot{E}(t) = \bar{x}(t)$

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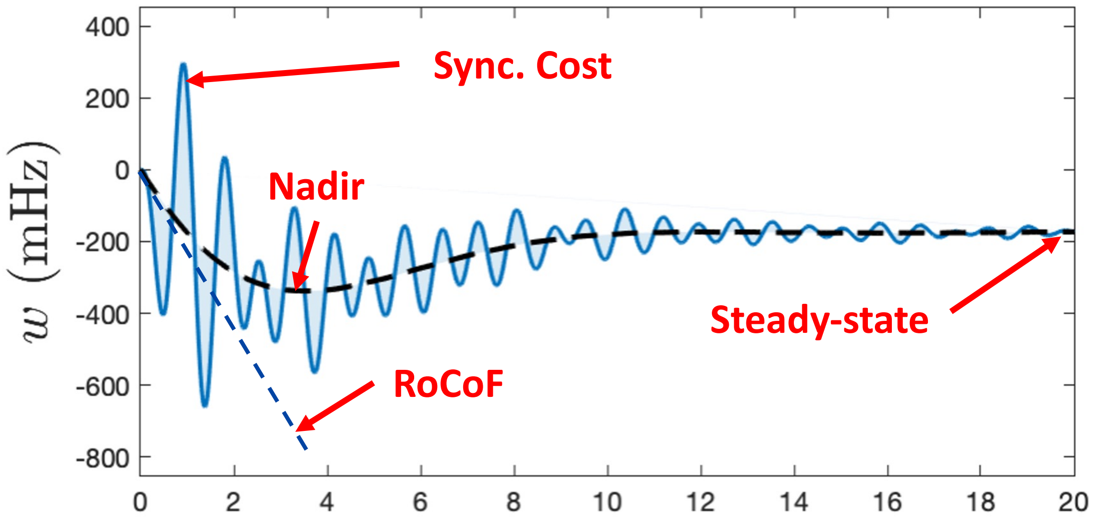
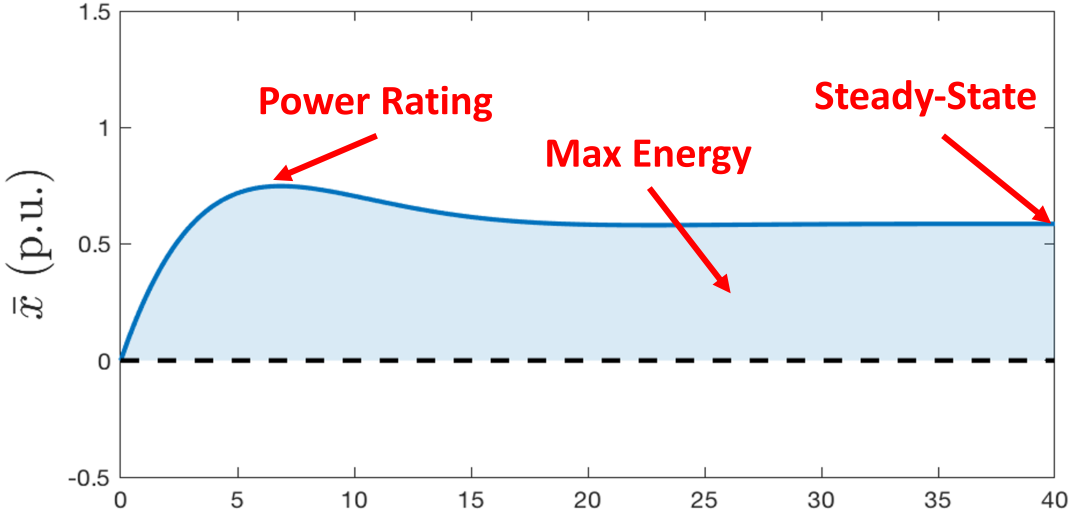
Performance Specification

Frequency Response



Control Effort



System Freq. : $\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

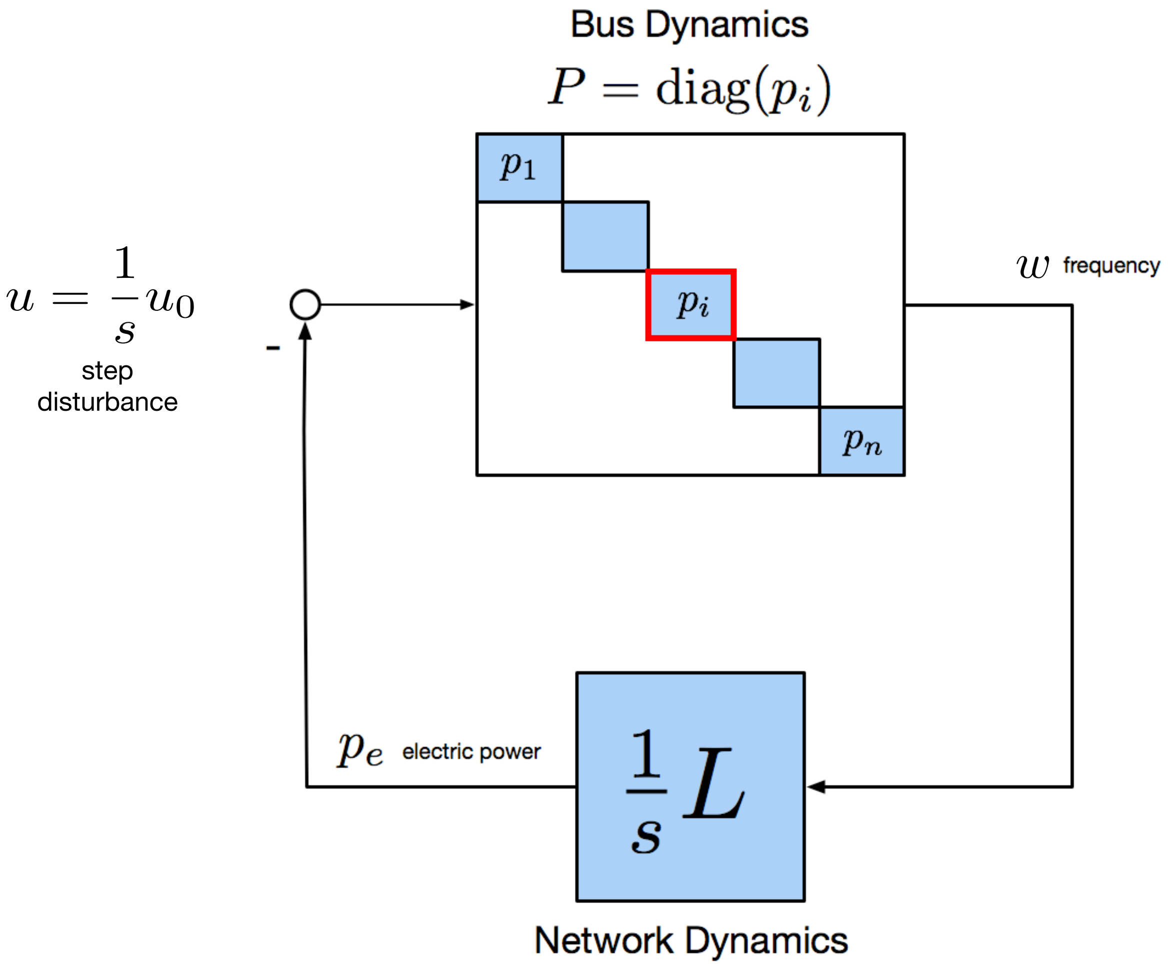
Injected Energy: $\dot{E}(t) = \bar{x}(t)$

Benchmark: Quantify **control ability** to eliminate overshoot in the **Nadir**

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, **IEEE Transactions on Automatic Control, 2020**

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Power Network Model



Step: $u = \frac{1}{s}u_0$

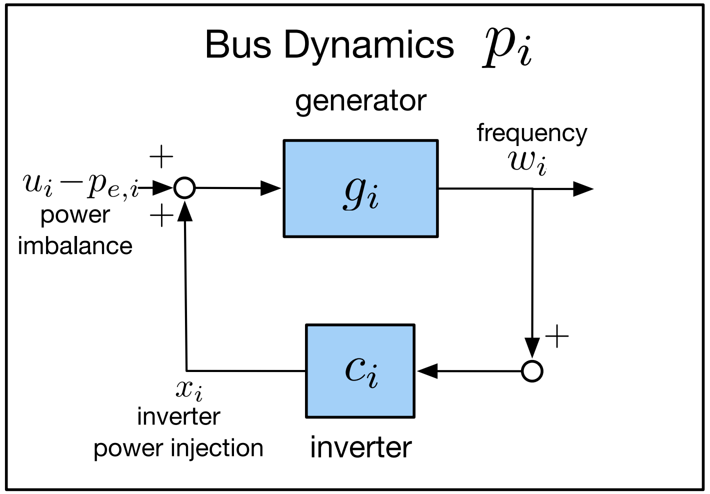
Laplacian Matrix

$$L_{ij} = \begin{cases} -B_{ij} & \text{if } ij \in E \\ \sum_k B_{ik} & \text{if } i = j \\ 0 & \text{o.w.} \end{cases}$$

[Bergen Hill ‘81]

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap,* **IEEE Transactions on Automatic Control, 2020**

Bus Dynamics



Generator: $g_i : (u_i - p_{e,i} + x_i) \mapsto w_i$

Model: Swing Equations + Turbine

$$g_i : \begin{cases} \dot{\theta}_i = w_i \\ M_i \dot{w}_i = -D_i w_i + q_i + (u_i - p_{e,i} + x_i) \\ \tau_i \dot{q}_i = -R_{g,i}^{-1} w_i - q_i \end{cases}$$

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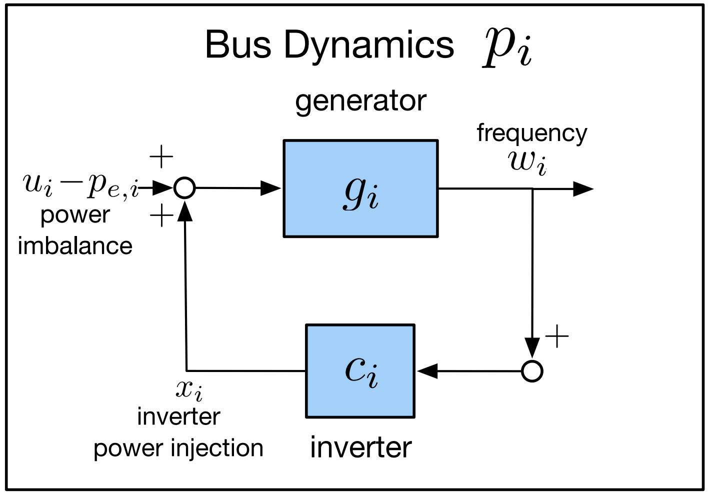

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$$g_i(s) = \frac{\tau_i s + 1}{M_i \tau_i s^2 + (M_i + D_i \tau_i)s + D_i + r_{g,i}^{-1}}$$

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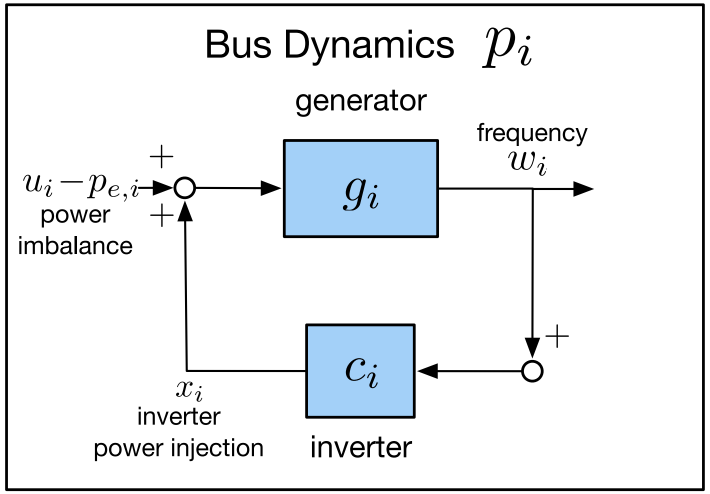
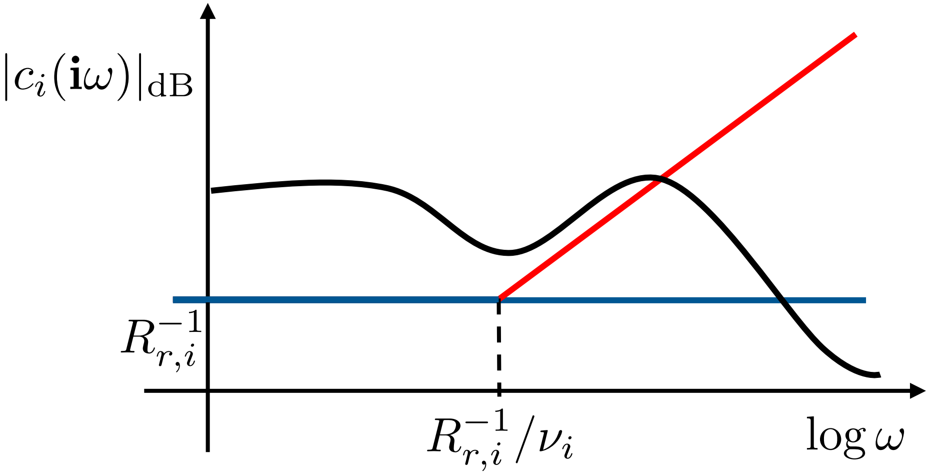
Bus Dynamics





Inverter: $c_i : w_i \mapsto x_i$

Droop Control and **Virtual Inertia**:

$$c_i : \left\{ x_i = -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), \right. \qquad c_i(s) = -(\nu_i s + R_{r,i}^{-1})$$

Closed-loop Bus Dynamics:

$$p_i : \begin{cases} \dot{\theta}_i = w_i \\ (M_i + \nu_i)\dot{w}_i = -(D_i + R_{r,i}^{-1}) w_i + q_i + (u_i - p_{e,i}) \\ \tau_i \dot{q}_i = -q_i - R_{g,i}^{-1} w_i \end{cases}$$

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, **IEEE Transactions on Automatic Control, 2020**

Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i be the *normalized rating*, $f_i = \frac{S_i}{S_{\text{base}}}$, and assume

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

$$c_i(s) = f_i c_0(s)$$

Swing Equations + Turbine

$$g_0(s) = \frac{\tau s+1}{m\tau s^2+(m+d\tau)s+d+r^{-1}}$$

Virtual Inertia

$$c_0(s) = -(\nu s + r_r^{-1})$$

$$M_i = f_i m, \quad D_i = f_i d, \quad R_{g,i} = \frac{1}{f_i} r_g, \quad \tau_i = \tau$$

$$\nu_i = f_i \nu \quad R_{r,i} = \frac{1}{f_i} r_r$$

Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i be the *normalized rating*, $f_i = \frac{S_i}{S_{\text{base}}}$, and assume

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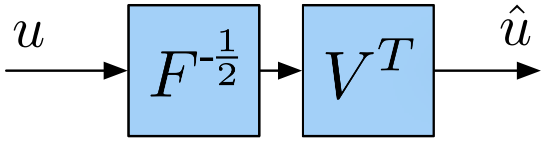
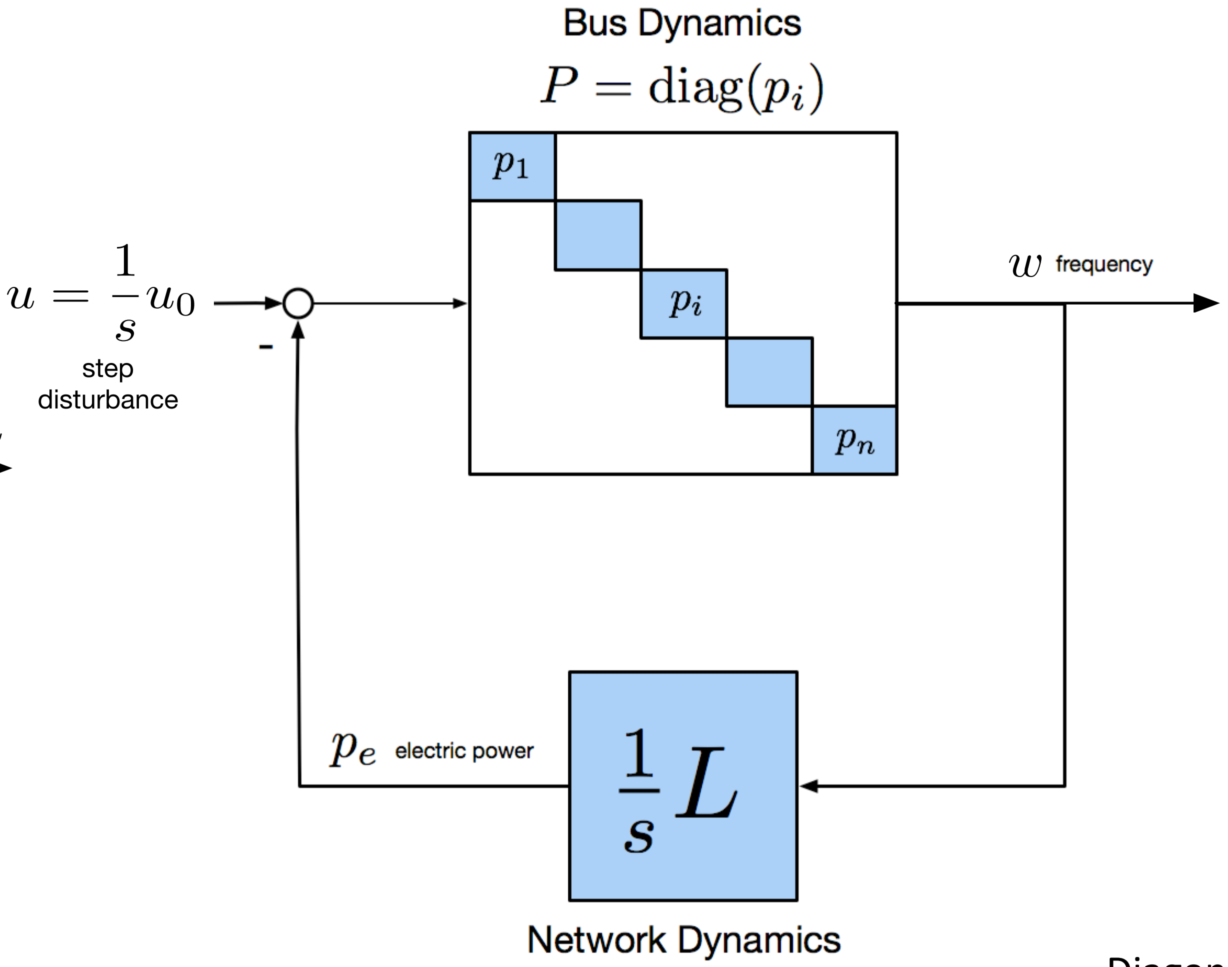
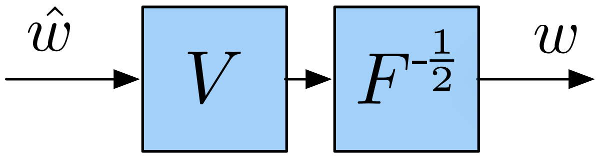
Change of Vars.





Change of Vars.



$$F = \text{diag}(f_i)$$

Coherence [Chow et al ‘93, …]

Diagonalization[Paganini M ‘17 , Guo Low ‘18]

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, **IEEE Transactions on Automatic Control, 2020**

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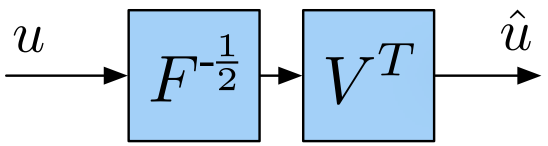
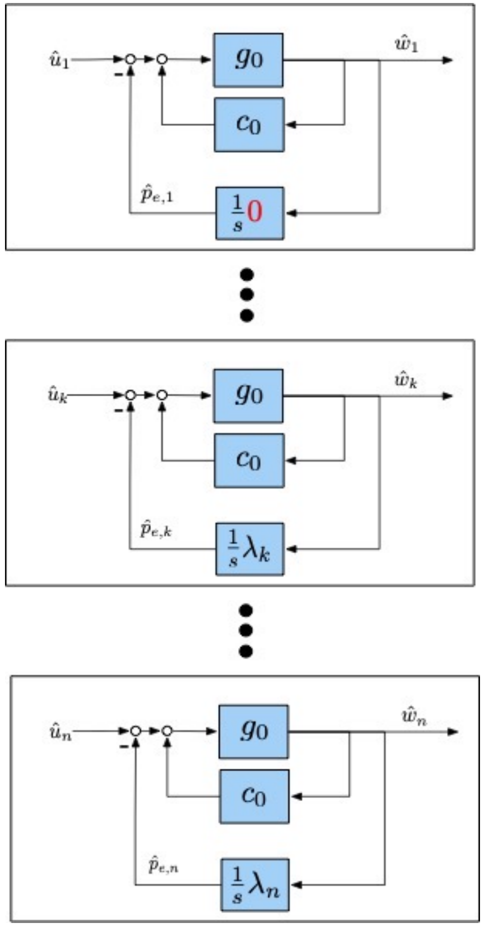


$$F = \text{diag}(f_i)$$

Eigenvalues of: $L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}$

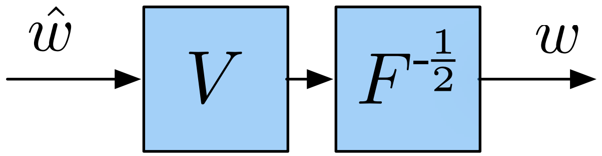
$$0 = \lambda_0 \le \lambda_1 \le \cdots \le \lambda_{n-1}$$



System Frequency

$$\bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

Change of Vars.



Sync Error

$$\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Coherence [Chow et al '93, …]

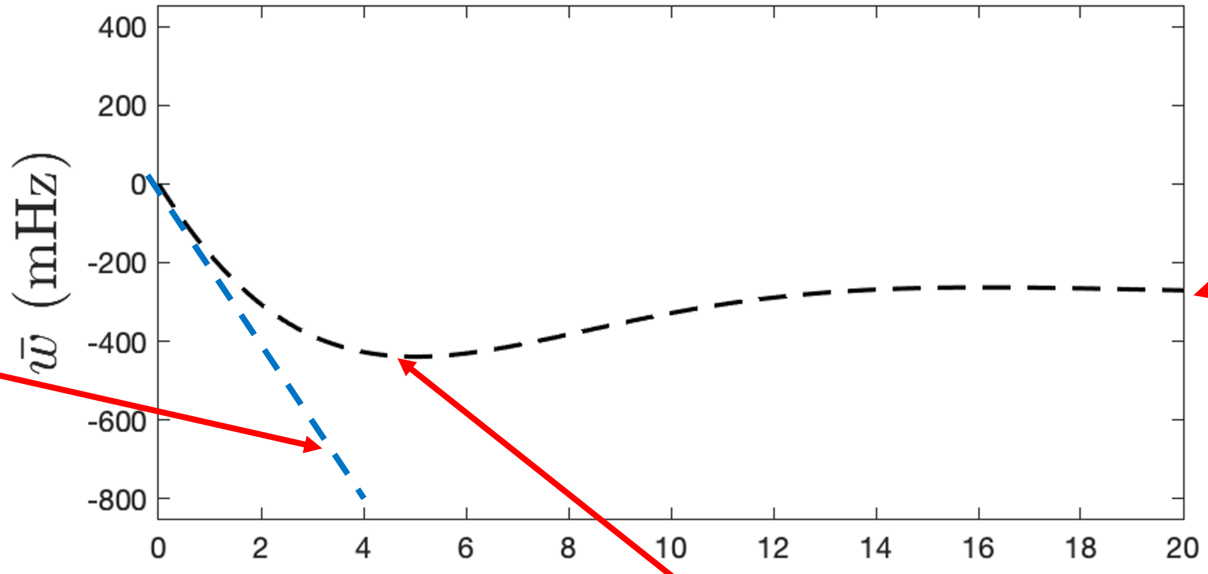
Diagonalization[Paganini M '17 , Guo Low '18]

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap,* **IEEE Transactions on Automatic Control, 2020**

System Frequency – Performance Analysis w/o Inverters

System frequency $\bar{w}$ response:

$$\bar{w}(t) = \frac{\sum_i u_{0i}}{\sum_i f_i}\frac{1}{d+r_g^{-1}}\left[1-e^{-\eta t}\left(\cos(\omega_d t)-\frac{\gamma-\eta}{\omega_d}\sin(\omega_d t)\right)\right]$$



Maximal RoCoF:
initial response.
Inertia appears directly

$$||\bar{w}||_\infty = \frac{|\sum_i u_{0,i}|}{\sum_i f_i}\frac{1}{m}$$

Steady-state.
No dependence on inertia

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i}\frac{1}{d+r_g^{-1}}$$

Nadir at overshoot.
Decreases (mildly)
with inertia

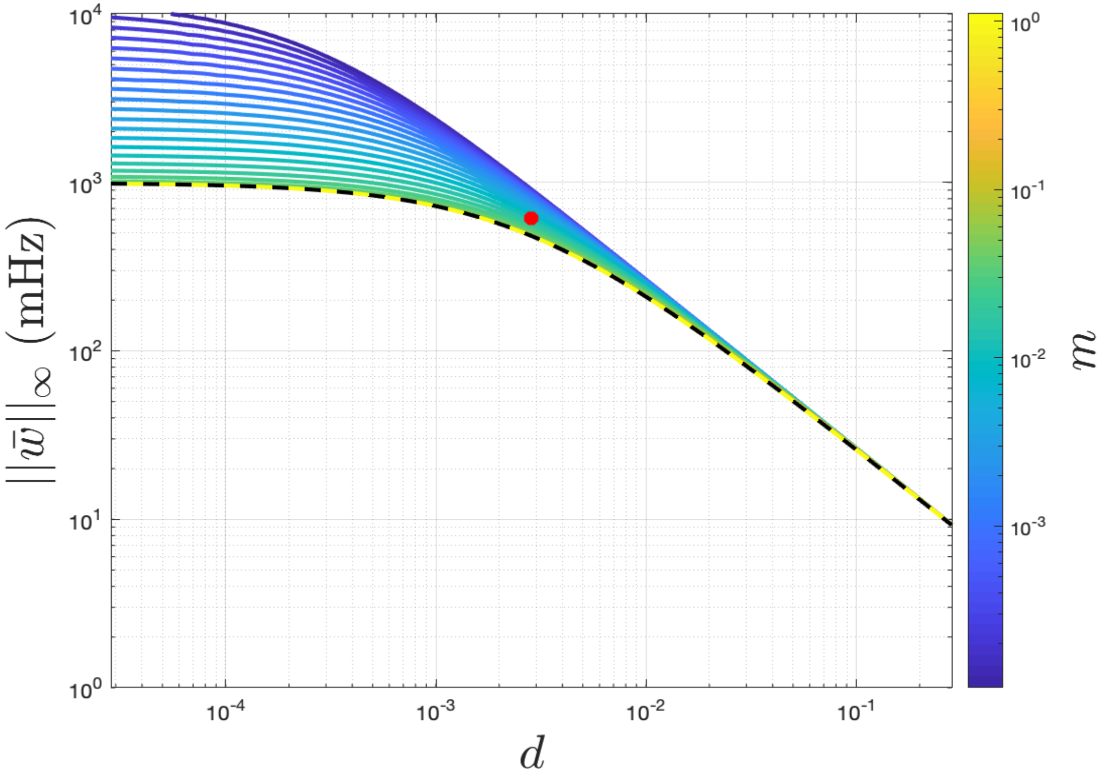
$$||\bar{w}||_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i}\frac{1}{d+r_g^{-1}}\left(1+\sqrt{\frac{\tau r_g^{-1}}{m}}e^{-\frac{\eta}{\omega_d}(\phi+\frac{\pi}{2})}\right)$$

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap,* **IEEE Transactions on Automatic Control, 2020**

System Frequency – Nadir Sensitivity w/o Inverters

Nadir:

$$||\bar{w}||_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{d + r_g^{-1}} \left(1 + \sqrt{\frac{\tau r_g^{-1}}{m}} e^{-\frac{\eta}{\omega_d}(\phi + \frac{\pi}{2})}\right)$$



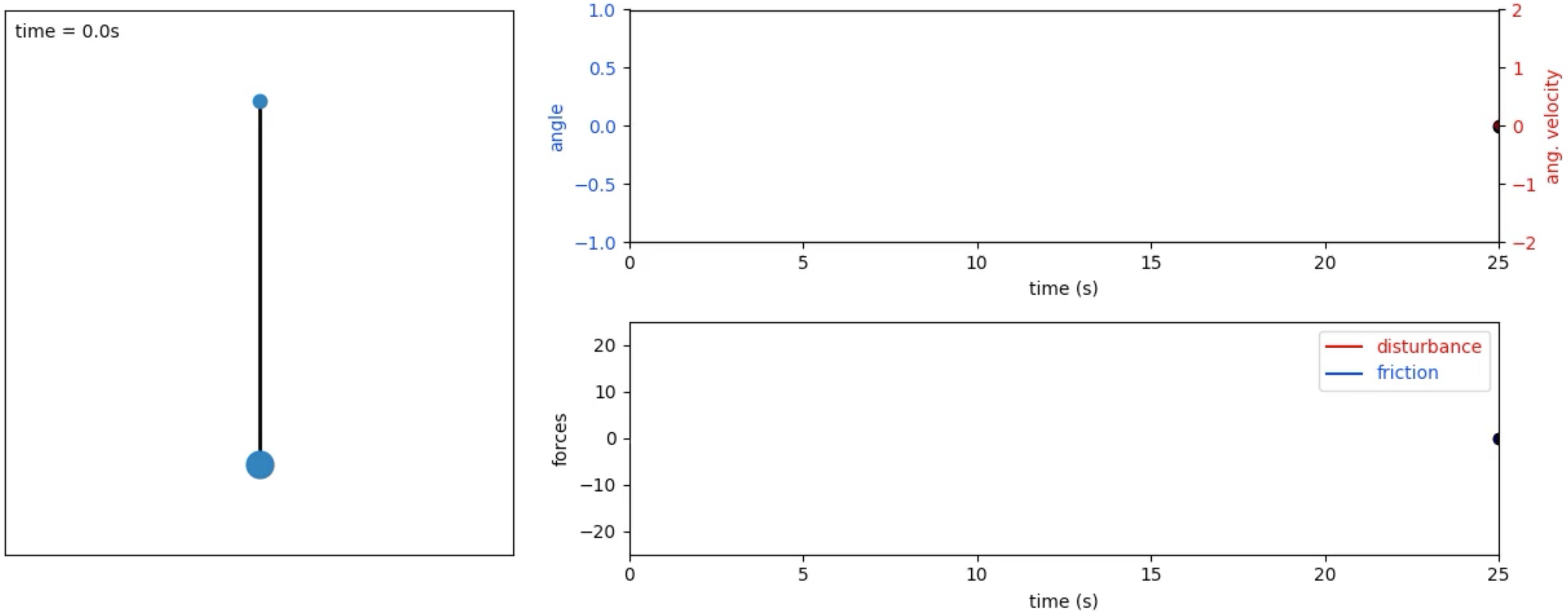
[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap,* **IEEE Transactions on Automatic Control, 2020**

Roadmap to Low Inertia Frequency Control

- Performance Specification and Analysis
- Limits of Virtual Inertia and Droop Control
- Control Design: Frequency Shaping

Control of Low Inertia Pendulum

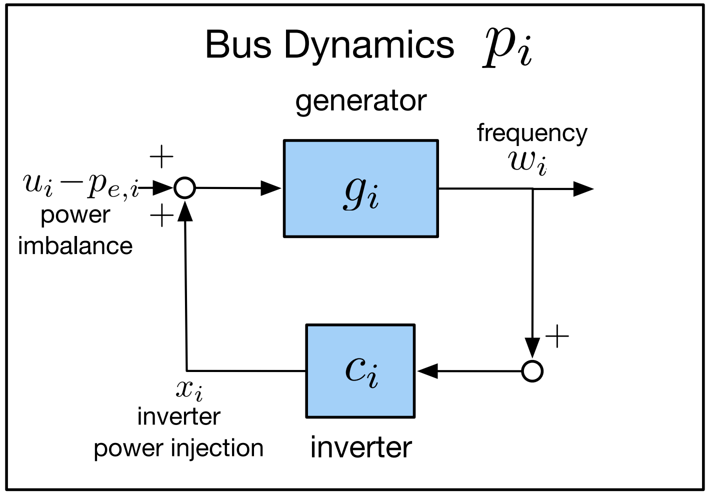
$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f + u$$



Cons: Susceptible to disturbances

Pros: Regains steady-sate faster

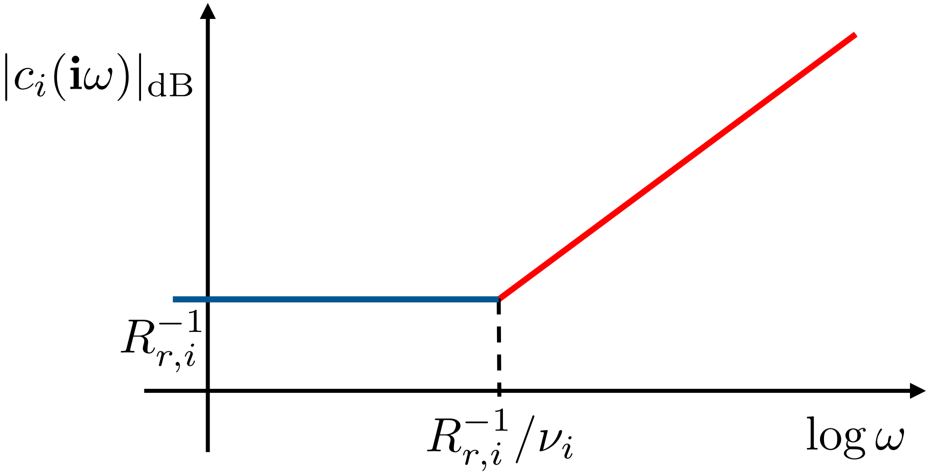
Bus Dynamics /w Virtual Inertia and Droop Control



Inverter: $c_i : w_i \mapsto x_i$

Droop Control and Virtual Inertia:

$$c_i : \left\{ x_i = -(\nu_i \dot{w}_i + R_{r,i}^{-1} w_i), \qquad c_i(s) = -(\nu_i s + R_{r,i}^{-1}) \right.$$



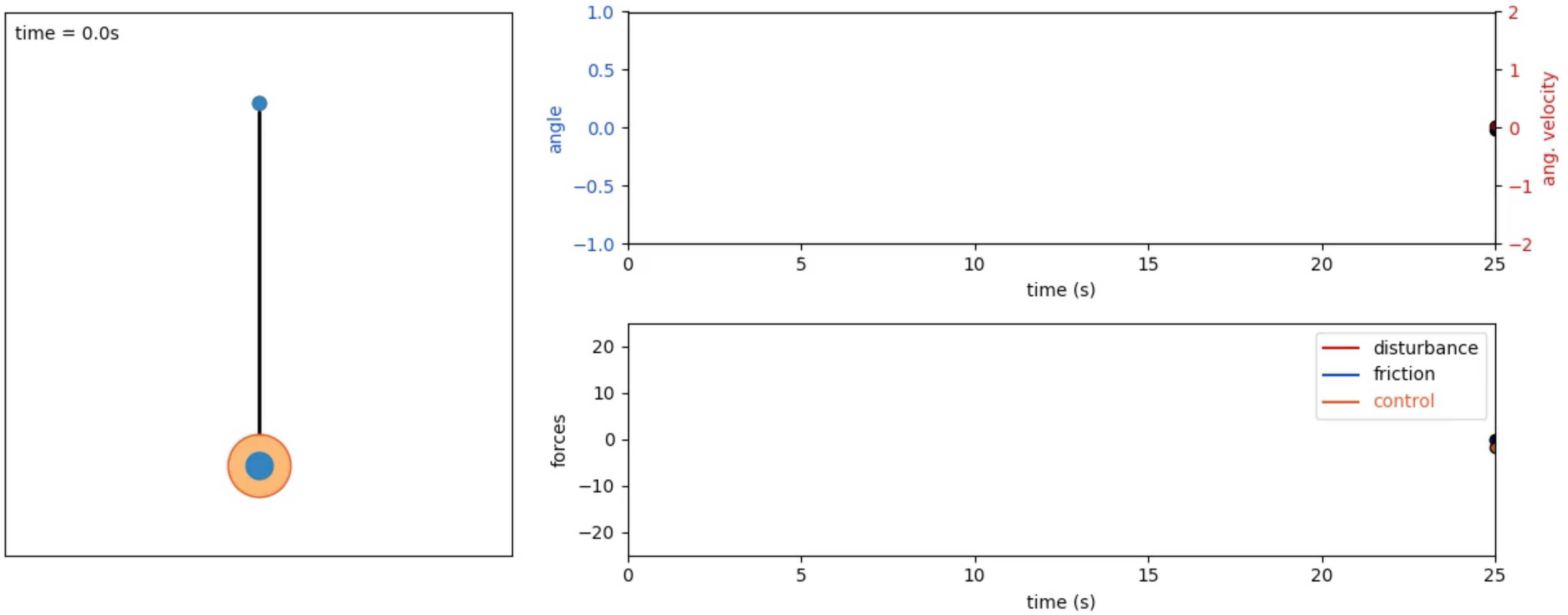
Closed-loop Bus Dynamics:

$$p_i : \begin{cases} \dot{\theta}_i = w_i \\ (M_i + \nu_i)\dot{w}_i = -(D_i + R_{r,i}^{-1})w_i + q_i + (u_i - p_{e,i}) \\ \tau_i \dot{q}_i = -q_i - R_{g,i}^{-1} w_i \end{cases}$$

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap,* **IEEE Transactions on Automatic Control, 2020**

Control of Low Inertia Pendulum

Virtual Mass Control: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - \nu\ddot{\theta}$

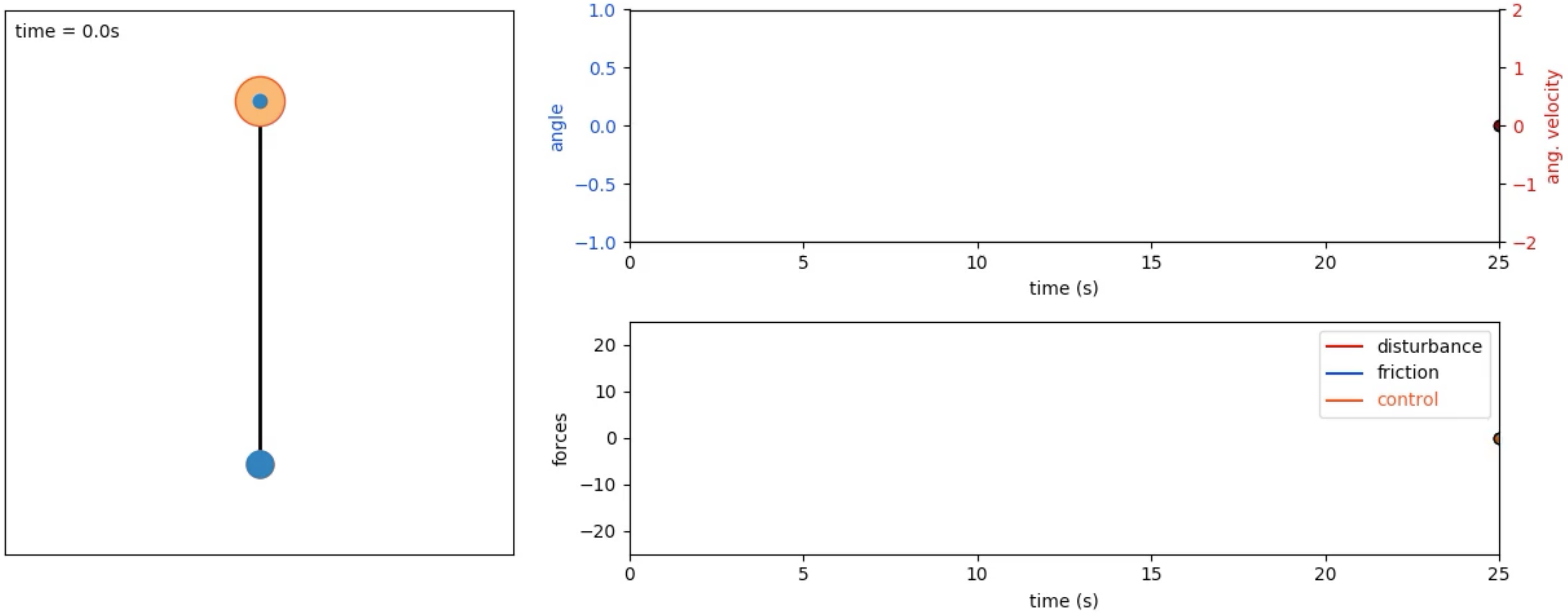


Pros:
Provides disturbance rejection

Cons:
Hard to regain steady-state + **excessive control effort**

Control of Low Inertia Pendulum

Virtual Friction Control: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - r^{-1}\dot{\theta}$



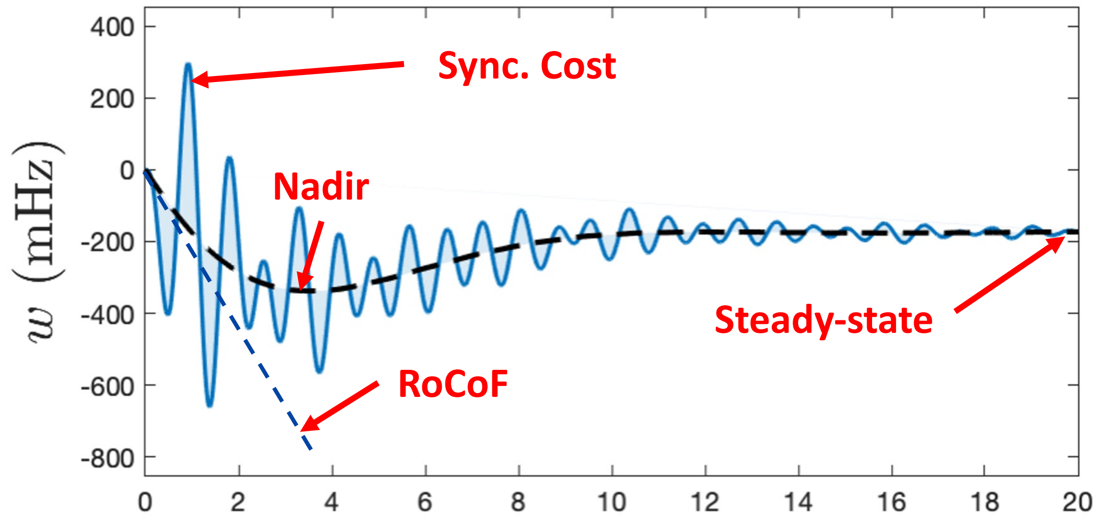
Pros: Provides disturbance rejection, quickly restore steady-state, with reasonable control effort.

Cons?

None, at least for pendulum

Performance Specification

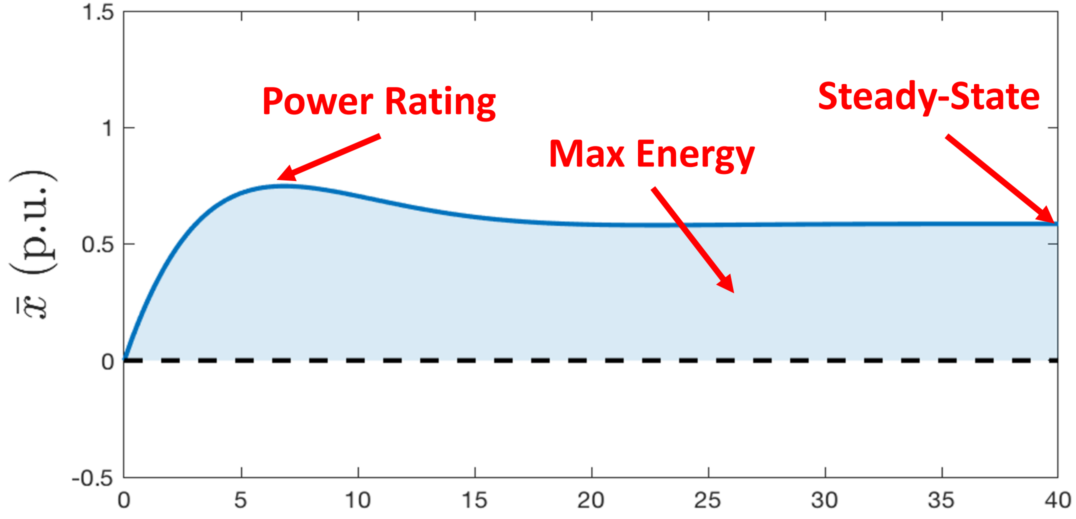
Frequency Response



System Freq. : $\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort



Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

Injected Energy: $\dot{E}(t) = \bar{x}(t)$

Benchmark: Quantify **control ability** to eliminate overshoot in **Nadir**

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[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

System Frequency w/ Virtual Inertia

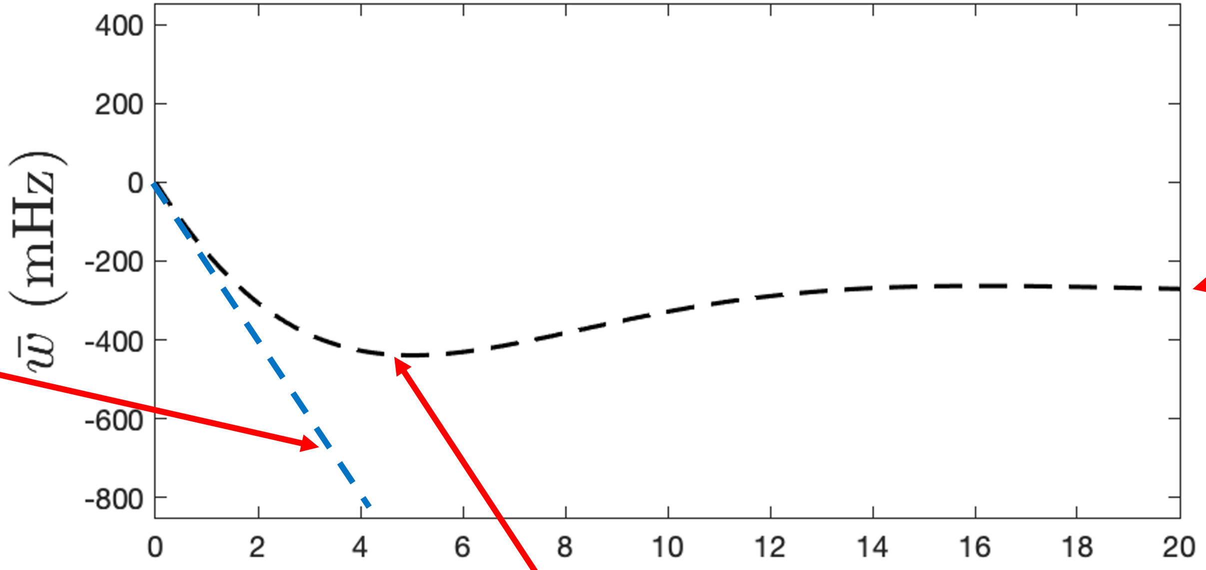
$$c_i : \; x_i = -f_i(\nu \dot{w}_i + r_r^{-1} w_i)$$

Swing Equations with Turbine: 2nd order response, e.g. underdamped

$$\bar{w}(t) = \frac{\sum_i u_{0i}}{\sum_i f_i} \frac{1}{d + r_g^{-1} + r_r^{-1}} \left[1 - e^{-\eta t}\left(\cos(\omega_d t) - \frac{\gamma - \eta}{\omega_d}\sin(\omega_d t)\right)\right]$$

Maximal RoCoF:
initial response.
Inertia appears directly

$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{m + \nu}$$



Steady-state.
No dependence on inertia

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i} \frac{1}{d + r_g^{-1} + r_r^{-1}}$$

Nadir at overshoot.
Decreases (mildly)
with inertia

$$||\bar{w}||_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{d + r_g^{-1} + r_r^{-1}} \left(1 + \sqrt{\frac{\tau r_g^{-1}}{m + \nu}} e^{-\frac{\eta}{\omega_d}(\phi + \frac{\pi}{2})}\right)$$

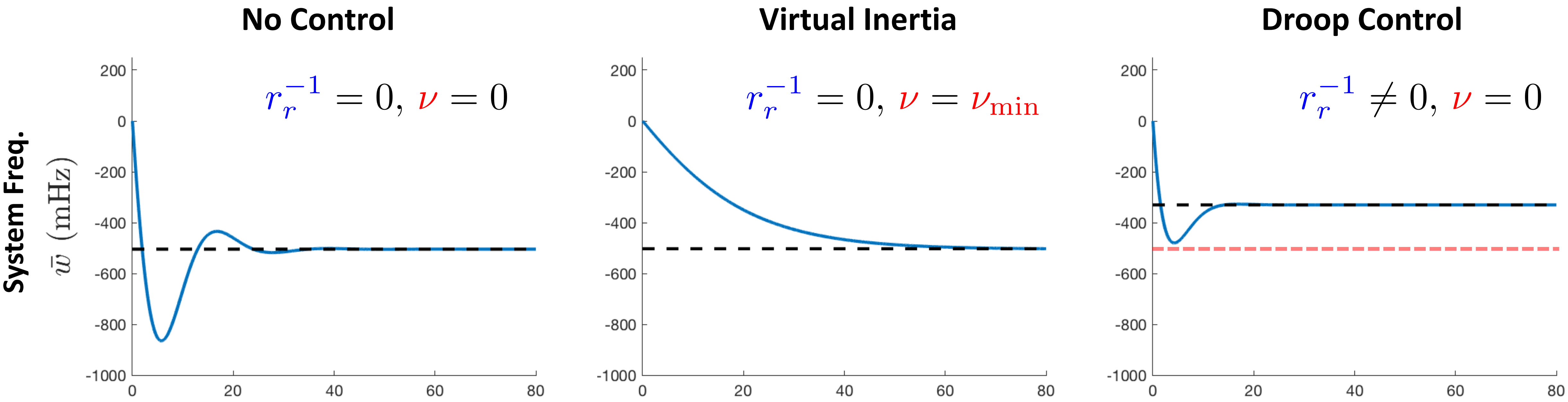
[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

System Frequency w/ Virtual Inertia

$$c_i: \quad x_i = -f_i(\nu \dot{w}_i + r_r^{-1} w_i)$$

Nadir Overshoot Elimination:

$$\nu \geq \nu_{\min} := \tau_g \left(\sqrt{r_g^{-1}} + \sqrt{d + r_g^{-1} + r_r^{-1}} \right)^2 - m$$

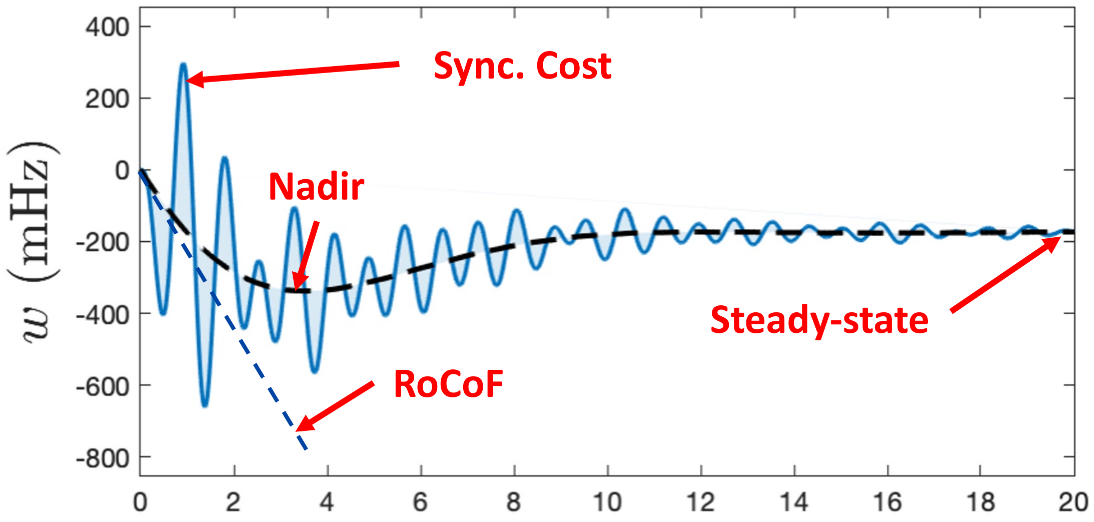


requires $\nu > 0$ in low inertia systems (low m)

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Performance Specification

Frequency Response



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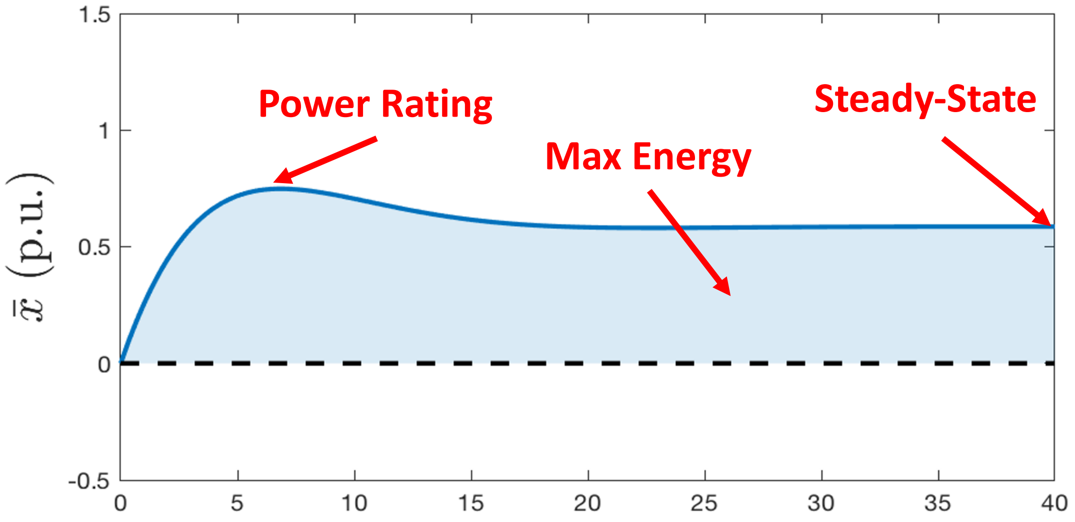
Control Effort



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Control Effort

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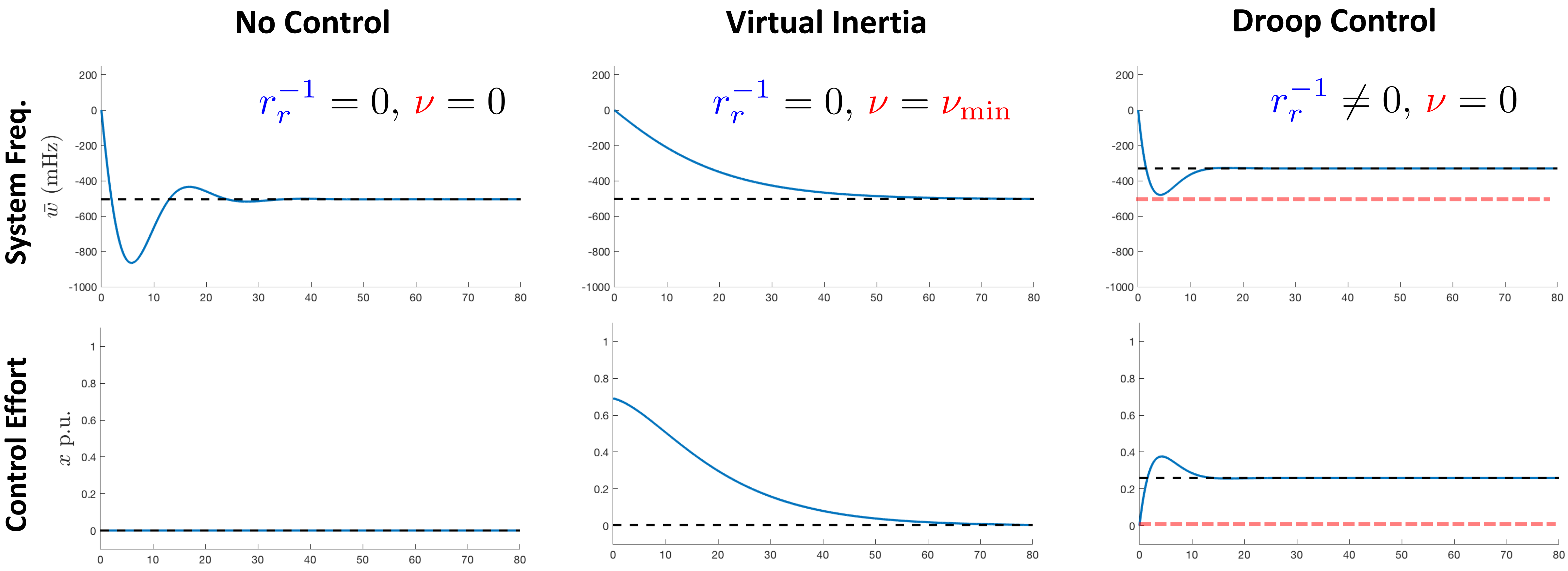
$$\bar{x}(t) = \sum_i x_i(t)$$



No Control

Virtual Inertia

Droop Control

$r_r^{-1} = 0, \; \nu = 0$

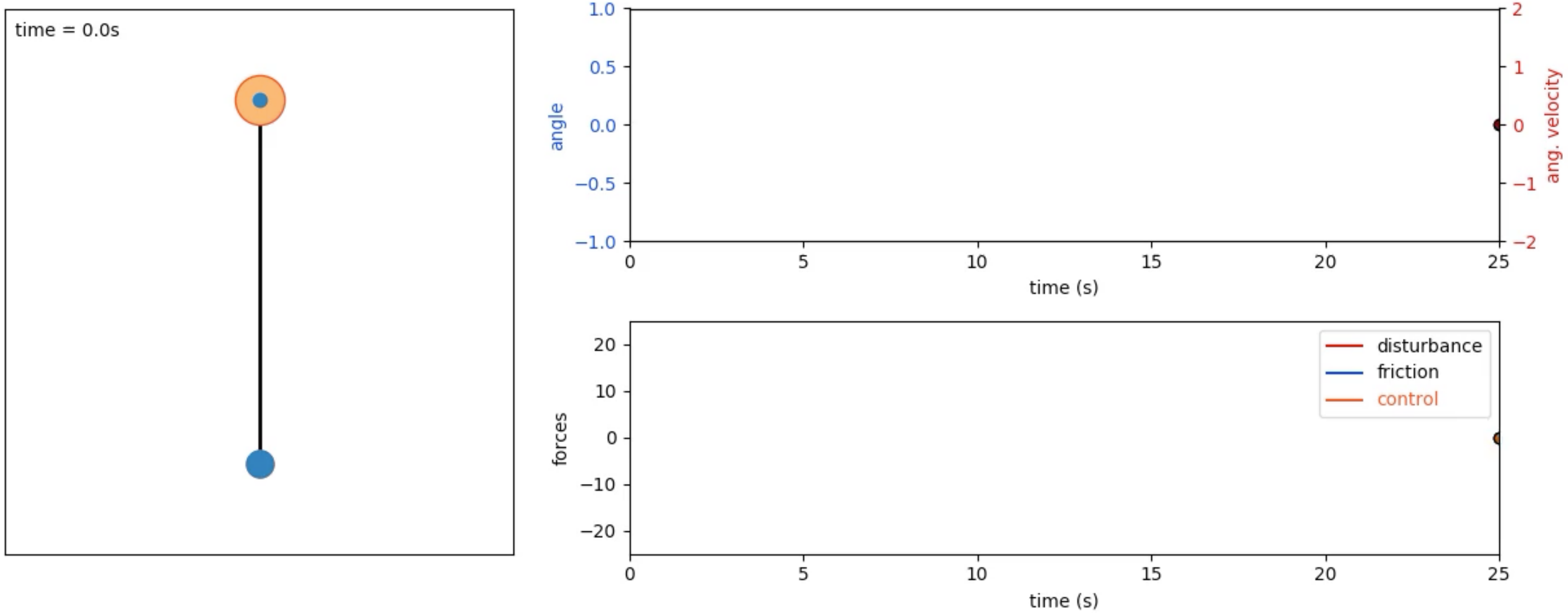
$r_r^{-1} = 0, \; \nu = \nu_{\min}$

$r_r^{-1} \neq 0, \; \nu = 0$

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Control of Low Inertia Pendulum

Virtual Friction Control: $m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f - r^{-1}\dot{\theta}$



Pros: Provides disturbance rejection, quickly restore steady-state, with reasonable control effort.

Cons? Large steady-state effort in power systems

Roadmap to Low Inertia Frequency Control

- Performance Specification and Analysis
- Limits of Virtual Inertia and Droop Control
- Control Design: Frequency Shaping

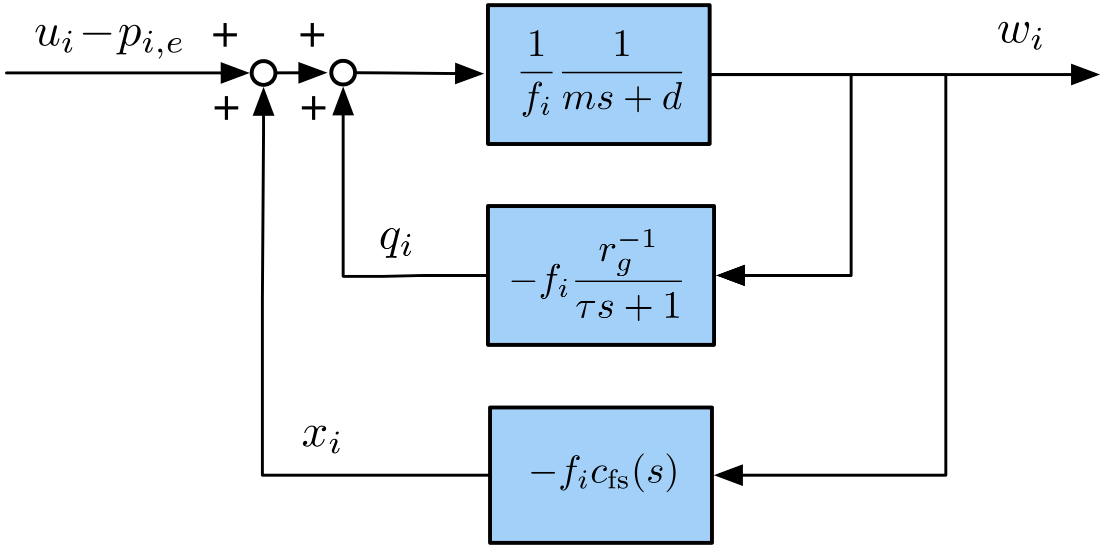
Frequency Shaping

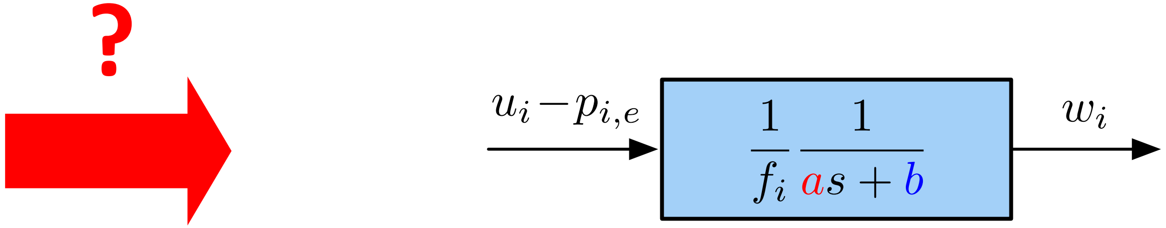
Grid-following Inverters

Grid-forming Inverters

Grid-following Frequency Shaping Control

Key idea: use model matching control (at each bus/area)





Leads to CoI Frequency $\bar{w}$ with:

Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i}\frac{1}{b}$$

RoCoF:

$$\|\dot{\bar{w}}\|_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i}\frac{1}{a}$$

CoI Response:

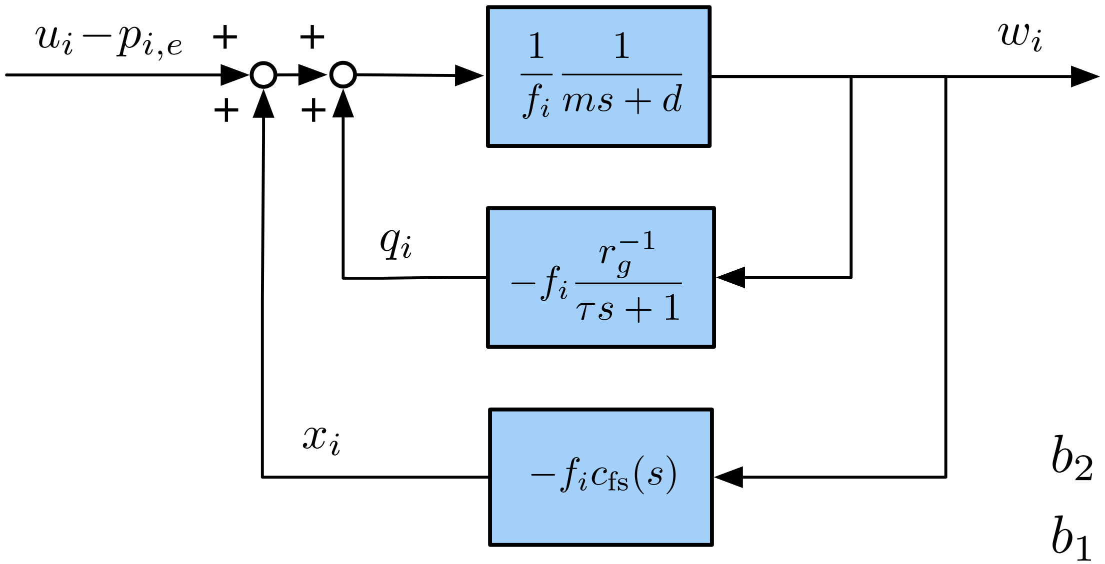
$$\bar{w}(t) = \frac{\sum_i u_{0,i}}{\sum_i f_i}\frac{1}{b}\left(1 - e^{-\frac{b}{a}t}\right)$$

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems, 2021**

Grid-following Frequency Shaping Control

Key idea: use model matching control (at each bus/area)



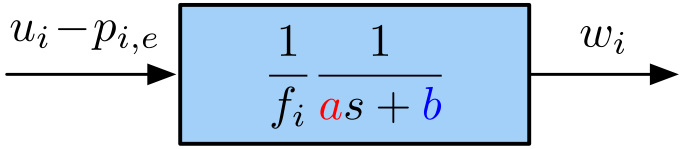
$$c_{\mathrm{fs}}(s) := \frac{b_2 s^2 + b_1 s + b_0}{\tau' s + 1}$$

$$b_2 = \tau(a - m)$$

$$b_1 = (b - d)\tau + a - m$$

$$b_0 = b - r_{\mathrm{g}}^{-1} - d$$

$$\tau' = \tau$$



Leads to CoI Frequency $\bar{w}$ with:

Steady-state:

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RoCoF:

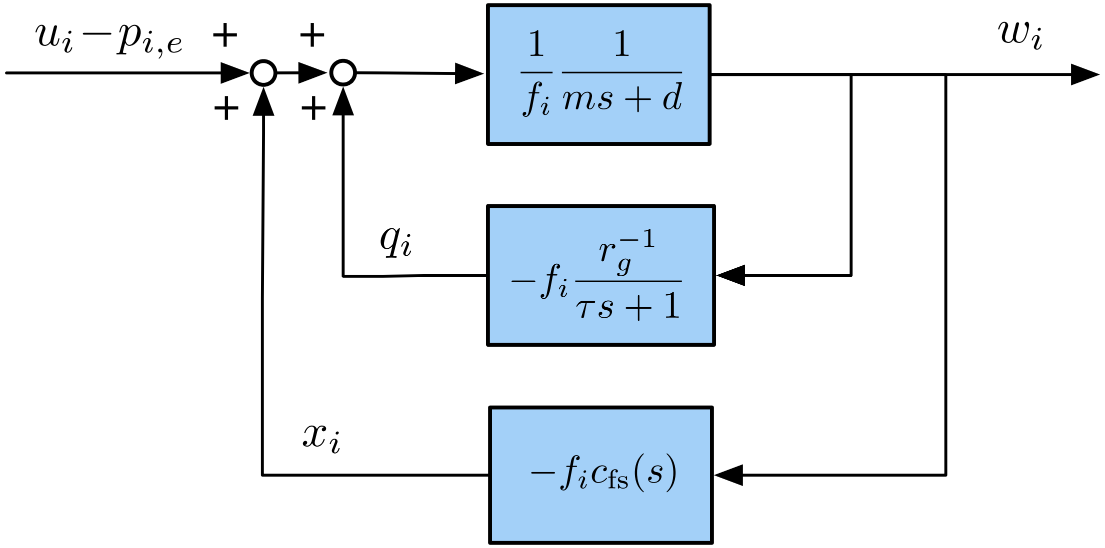
$$\|\dot{\bar{w}}\|_\infty = \frac{\left|\sum_i u_{0i}\right|}{\sum_i f_i}\frac{1}{a}$$

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems, 2021**

Special case: Dynamic Droop (iDroop)

Setting $b_2 = 0$, $b_1 = \nu'\tau'$, and $b_2 = r_r^{-1}$ leads to:



$$c_{\mathrm{fs}}(s) := \frac{b_2 s^2 + b_1 s + b_0}{\tau' s + 1}$$

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

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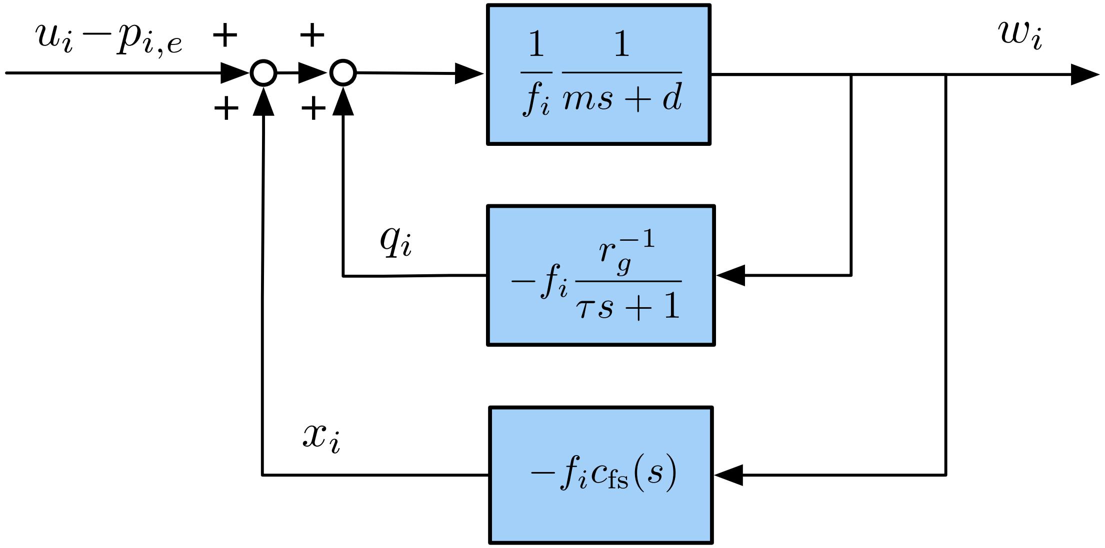


$$c_{\mathrm{fs}}(s) := \frac{\tau'\nu' s + r_r^{-1}}{\tau' s + 1}$$

Inverter Control

$$c_i(s) = -f_i c_{\mathrm{fs}}(s) = -f_i\left(\frac{\tau'\nu' s + r_r^{-1}}{\tau' s + 1}\right)$$

$$c_i : \left\{ \tau'\dot{x}_i = -x_i - f_i\left(r_r^{-1} w_i + \tau'\nu'\dot{w}_i\right)\right.$$

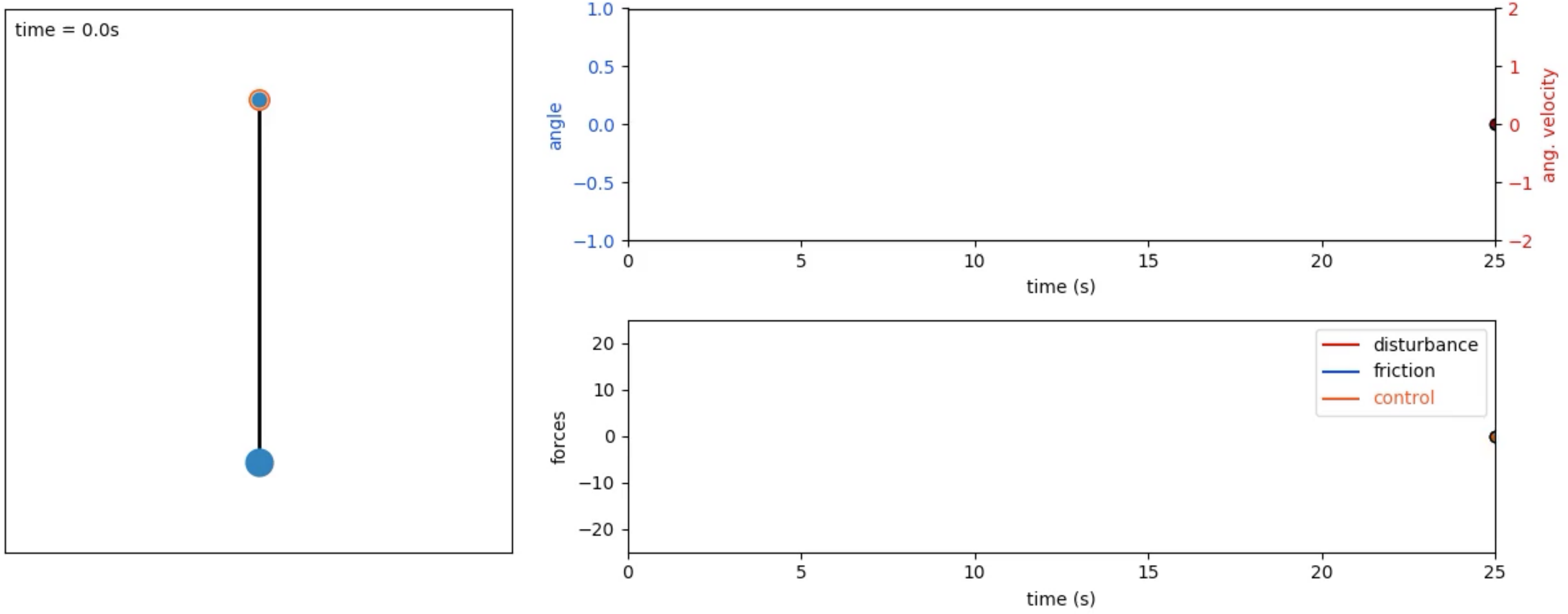
[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Control of Low Inertia Pendulum

Dynamic Friction Control:

$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f + x$$

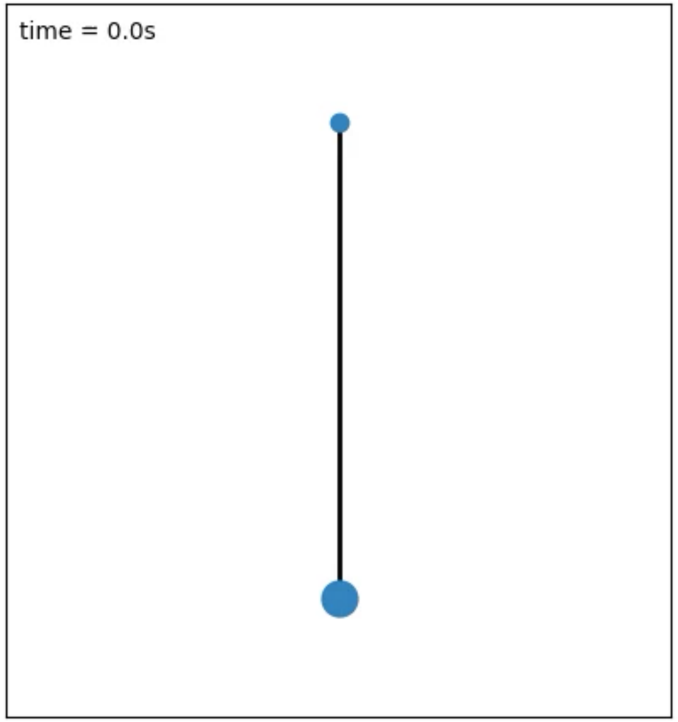
$$\tau'\dot{x} = -x - (r_r^{-1}\dot{\theta} + \tau'\nu'\ddot{\theta})$$



[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

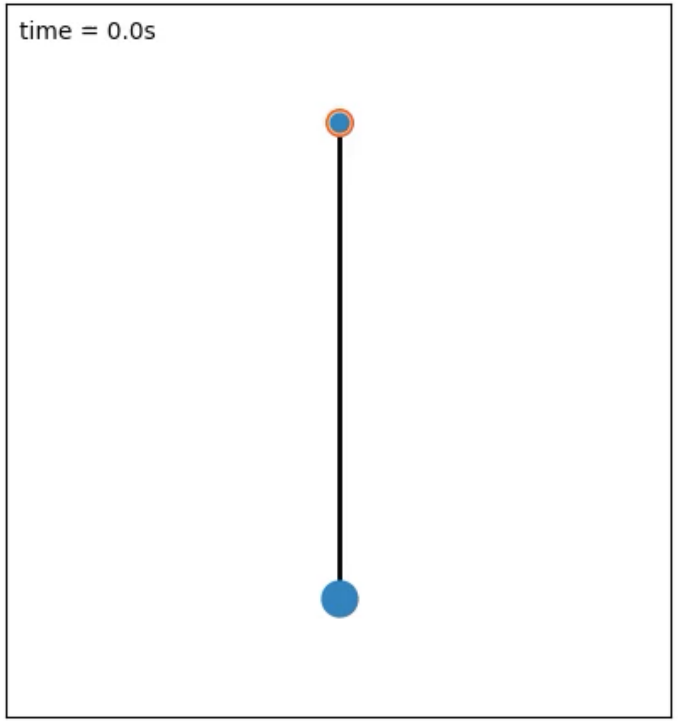
Control of Low Inertia Pendulum

No Control



$$m\ddot{\theta} = -d\dot{\theta} - mg\sin\theta + f$$

Dynamic Friction Control



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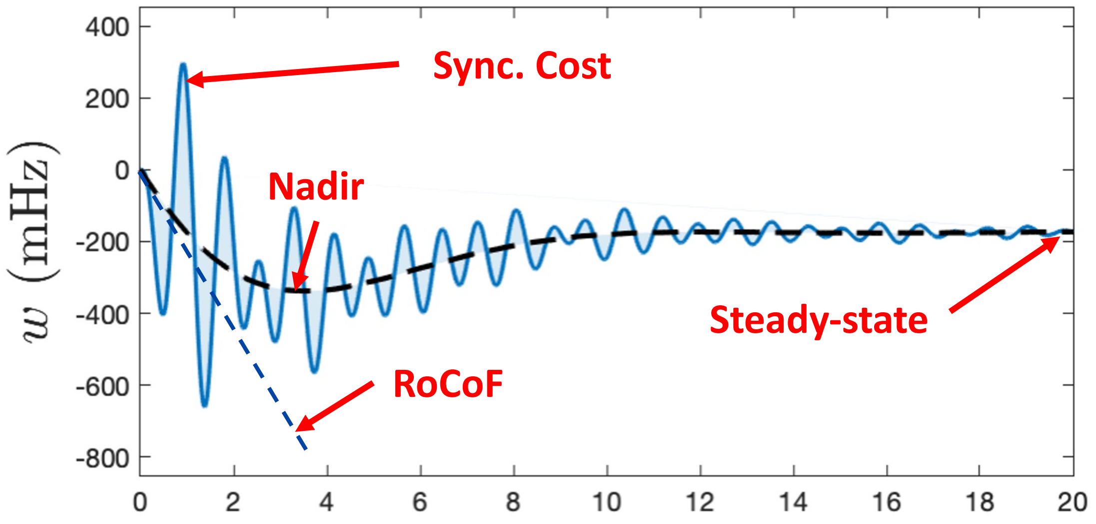
Performance Specification

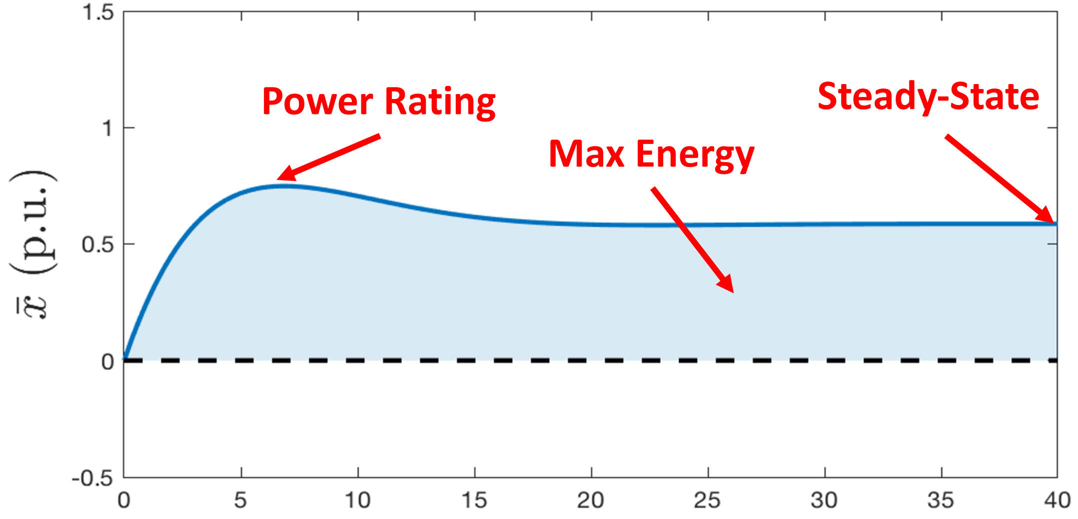
Frequency Response



System Freq. : $\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$

Sync. Error : $\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$

Control Effort



Injected Power: $\bar{x}(t) = \sum_i x_i(t)$

Injected Energy: $\dot{E}(t) = \bar{x}(t)$

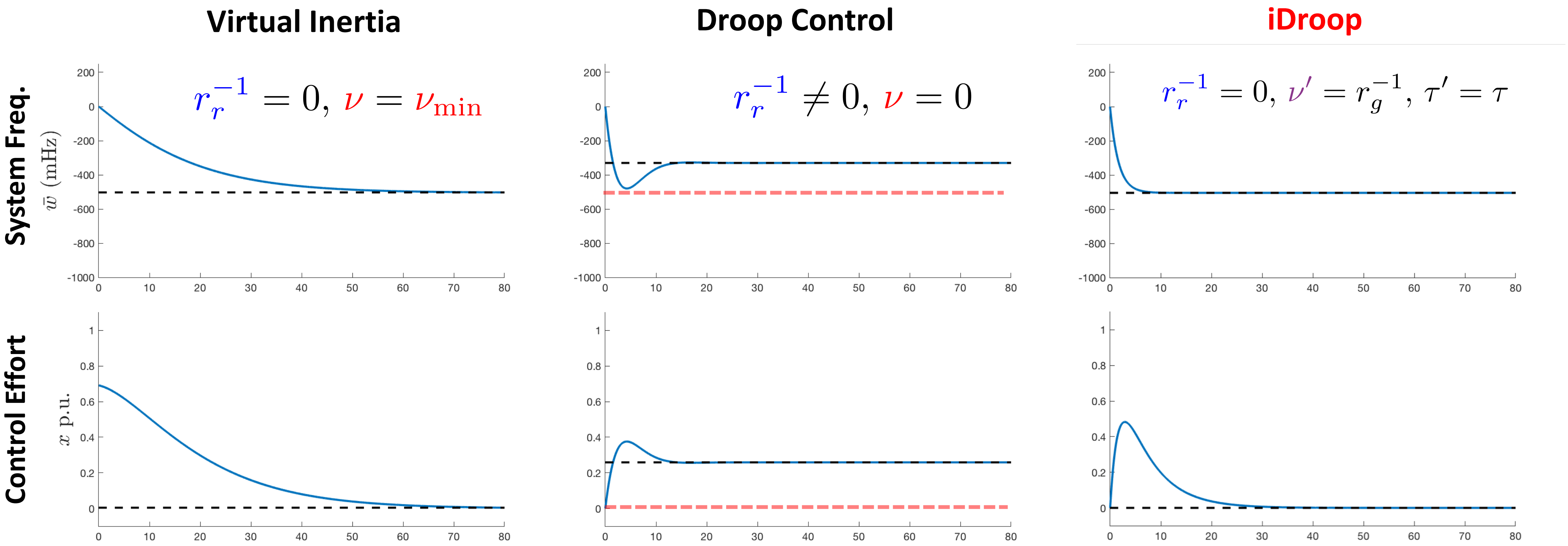
Benchmark: Quantify **control ability** to eliminate overshoot in **Nadir**

[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Frequency Shaping w/ iDroop

$$c_i: \ \tau' \dot{x}_i = -x_i - f_i\left(r_r^{-1} w_i + \tau \nu' \dot{w}_i\right)$$

Whenever $\nu' = r_r^{-1} + r_g^{-1}$ **and** $\tau' = \tau$

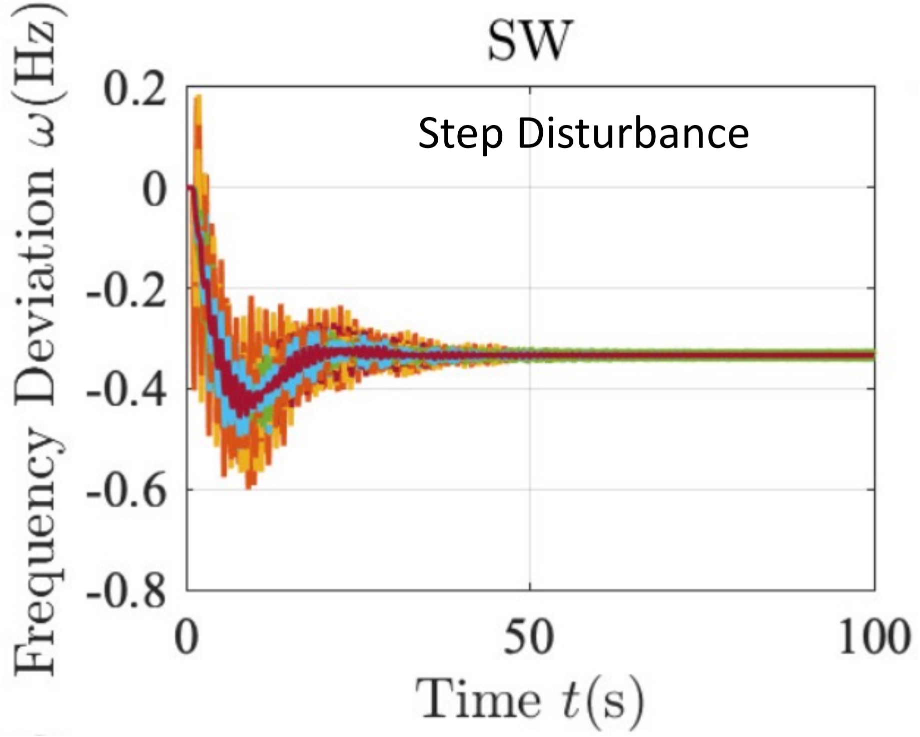


[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

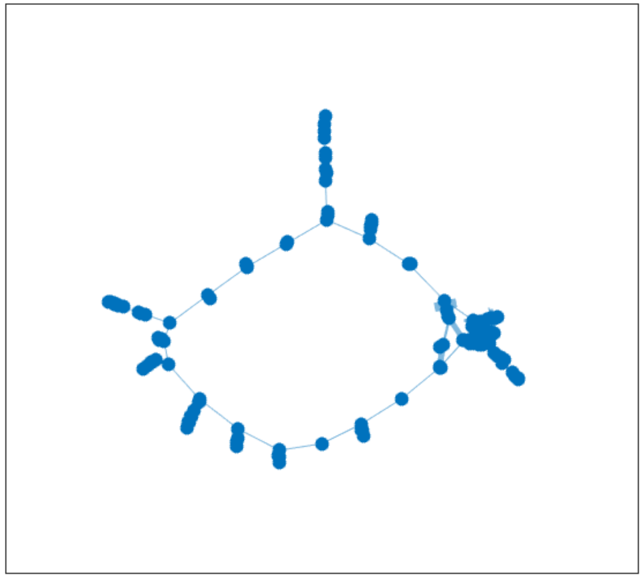
Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

Base Case: No Inverter Control



Icelandic Grid



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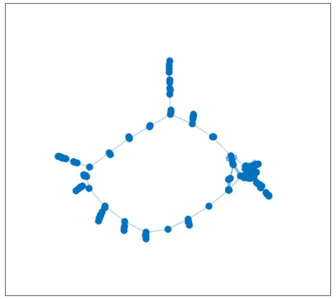
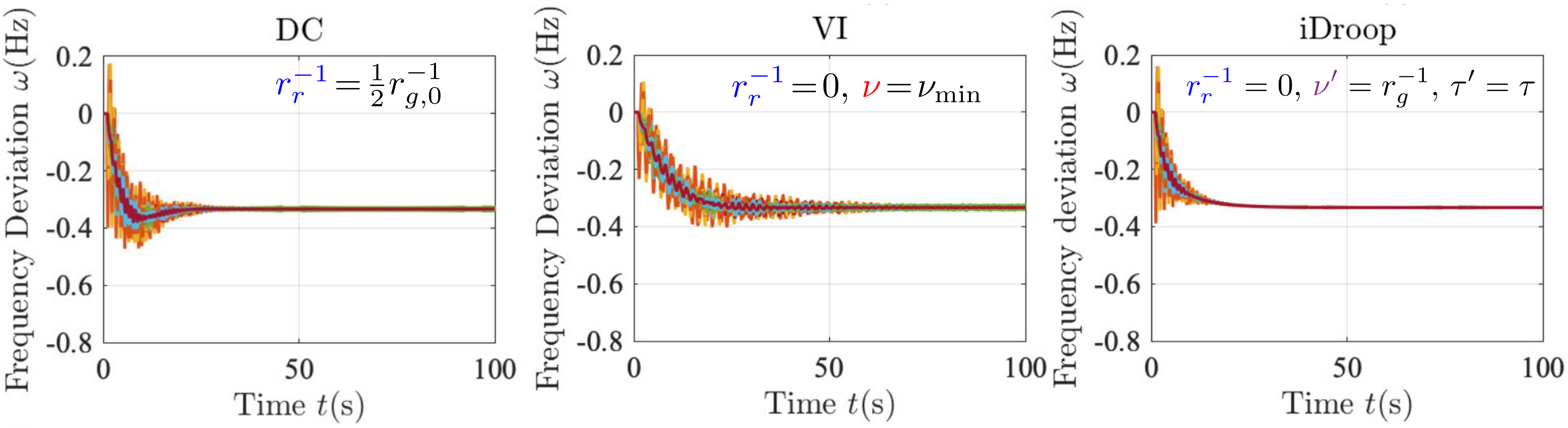
Icelandic Grid



Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)
- Droop equally set for inverters in all cases
- Virtual inertia tuned for **critically damped response** $\nu = \nu_{min}$
- **iDroop** tuned for Frequency Shaping $\nu' = r_g^{-1} + r_r^{-1}$ and $\tau' = \tau$



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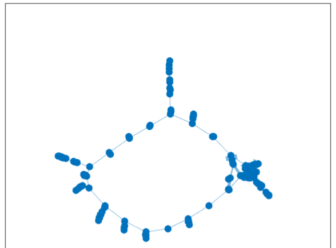
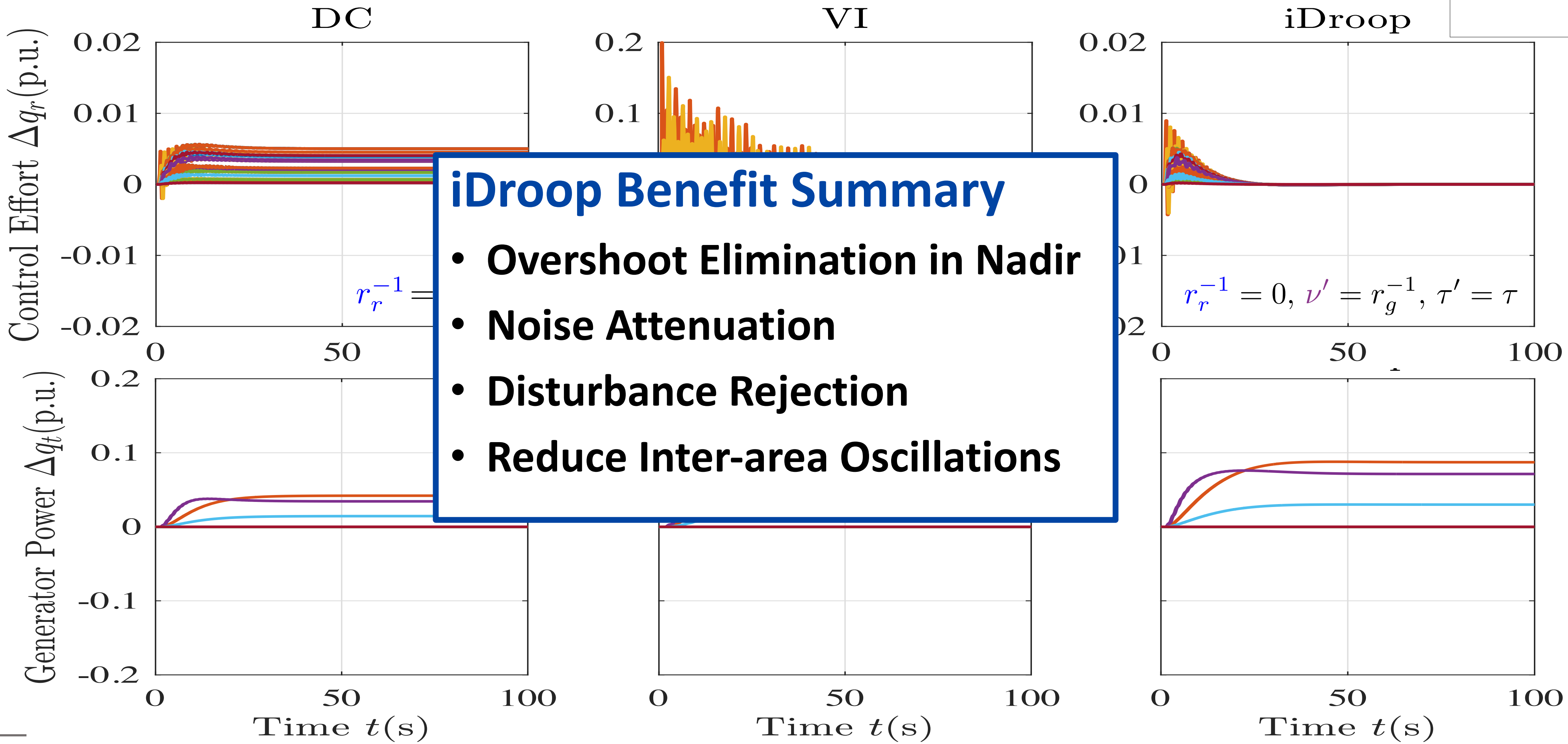




iDroop Benefit Summary

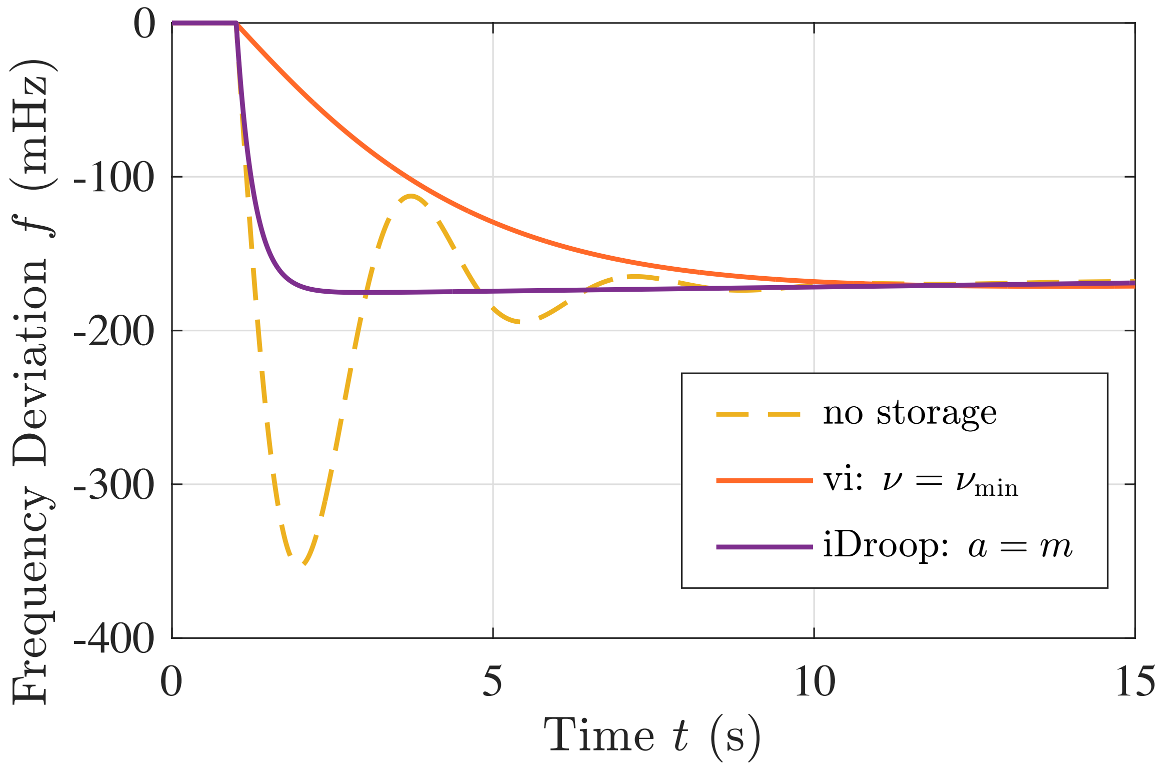
- **Overshoot Elimination in Nadir**
- **Noise Attenuation**
- **Disturbance Rejection**
- **Reduce Inter-area Oscillations**

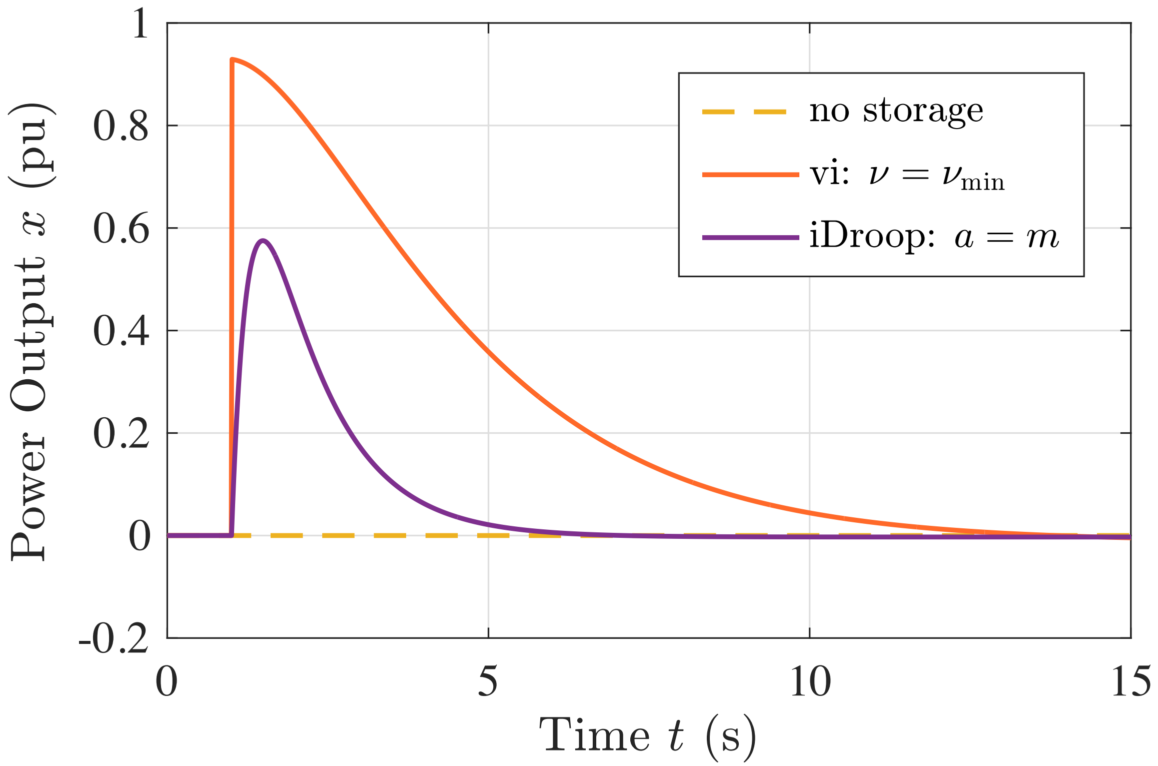
[TAC 21] Jiang, Pates, M, *Dynamic droop control in low inertia power systems*, **IEEE Transactions on Automatic Control, 2021**

Trading off Control Effort and RoCoF

$$b_2 = \tau\,(a - m)$$
$$b_1 = (b - d)\tau + a - m$$
$$b_0 = b - r_{\mathrm{g}}^{-1} - d$$
$$\tau' = \tau$$

$$c_{\mathrm{fs}}(s) := \frac{b_2 s^2 + b_1 s + b_0}{\tau' s + 1}$$





[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems, 2021**

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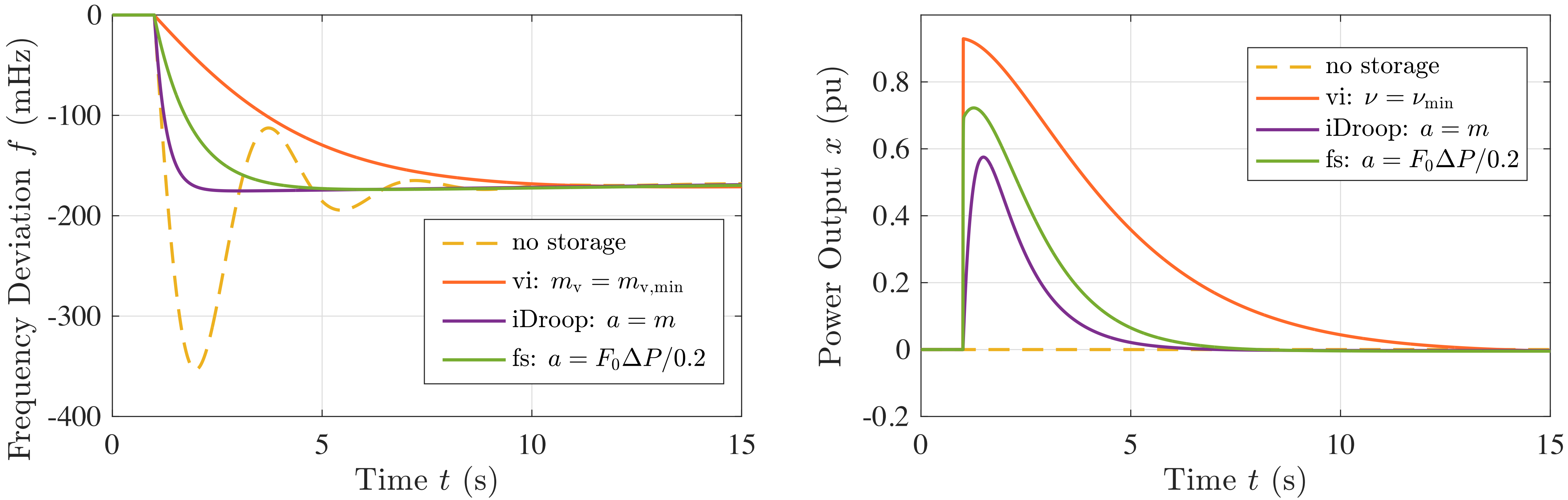
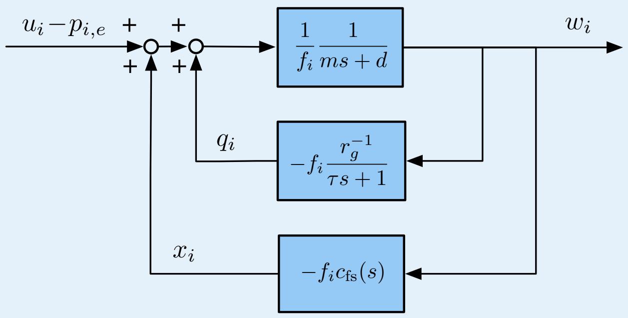


[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems, 2021**

Frequency Shaping

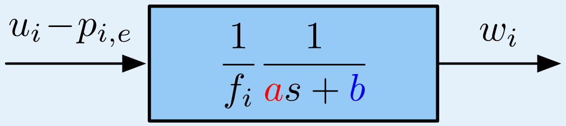
Grid-following Inverters: At each bus/area…



$b_2 = \tau\,(a - m)$ $\quad b_0 = b - r_{\mathrm{g}}^{-1} - d$

$b_1 = (b - d)\tau + a - m$ $\quad \tau' = \tau$

$$c_{\mathrm{fs}}(s) := \frac{b_2 s^2 + b_1 s + b_0}{\tau' s + 1}$$



Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i}\frac{1}{b}$$

RoCoF:

$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i}\frac{1}{a}$$

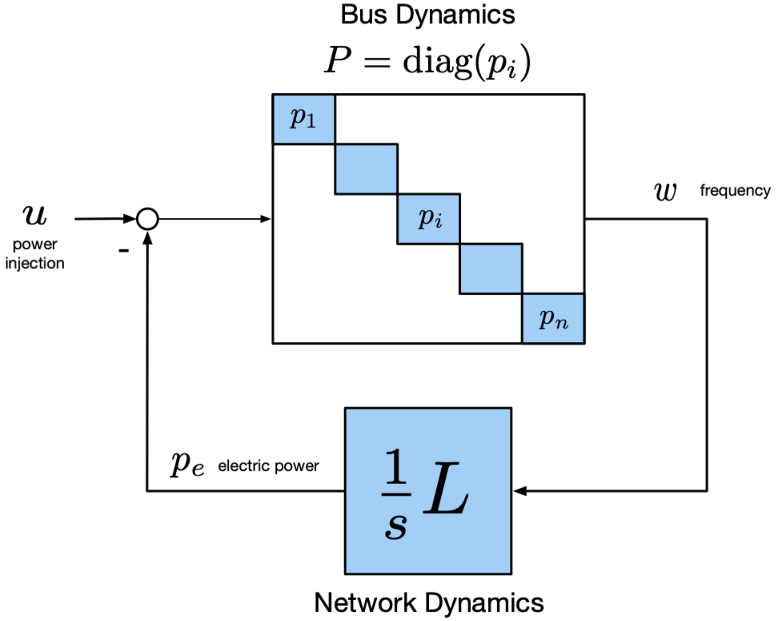
Grid-forming Inverters

Center of Inertia /w Grid Forming Inverters

Center of Inertia Freq.

$$p_i(s) = \frac{1}{f_i} p_0(s)$$

$$\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i} \quad \Longleftrightarrow \quad \bar{w}(s) = p_0(s) \frac{1}{\sum_{i=1}^{n} f_i} \left(\sum_{i=1}^{n} u_i(s) \right)$$



Problem: No longer valid grid-forming inverters

Yet…

$$\bar{w}(s) = p_0(s) \frac{1}{\sum_{i=1}^{n} f_i} \left(\sum_{i=1}^{n} u_i(s) \right)$$

[TAC 22] Min, M, *Coherence and concentration in tightly connected networks*, **IEEE Transactions on Automatic Control**, 2022, under review

Center of Inertia /w Grid Forming Inverters

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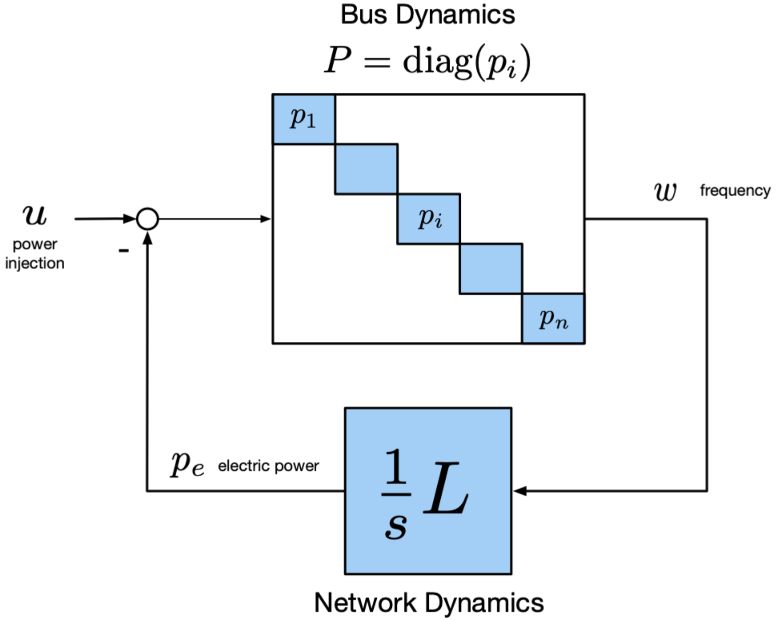
$$\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i}$$

↔

$$\bar{w}(s) = p_0(s) \frac{1}{\sum_{i=1}^{n} f_i} \left(\sum_{i=1}^{n} u_i(s) \right)$$



Problem: No longer valid grid-forming inverters

Yet…

$$\bar{w}(s) = \frac{1}{p_0^{-1}(s)} \frac{1}{\sum_{i=1}^{n} f_i} \left(\sum_{i=1}^{n} u_i(s) \right)$$

[TAC 22] Min, M, *Coherence and concentration in tightly connected networks*, **IEEE Transactions on Automatic Control**, 2022, under review

Center of Inertia /w Grid Forming Inverters

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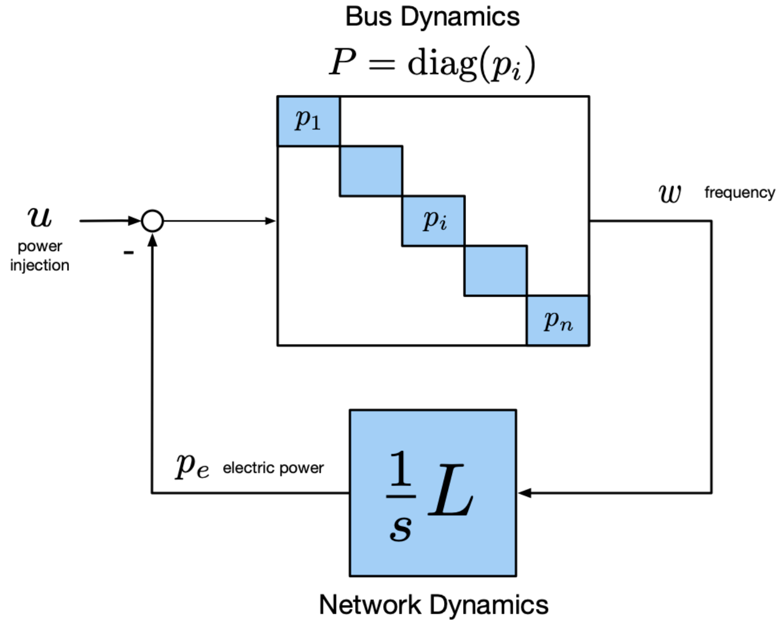


Problem: **No longer valid grid-forming inverters**

Yet…

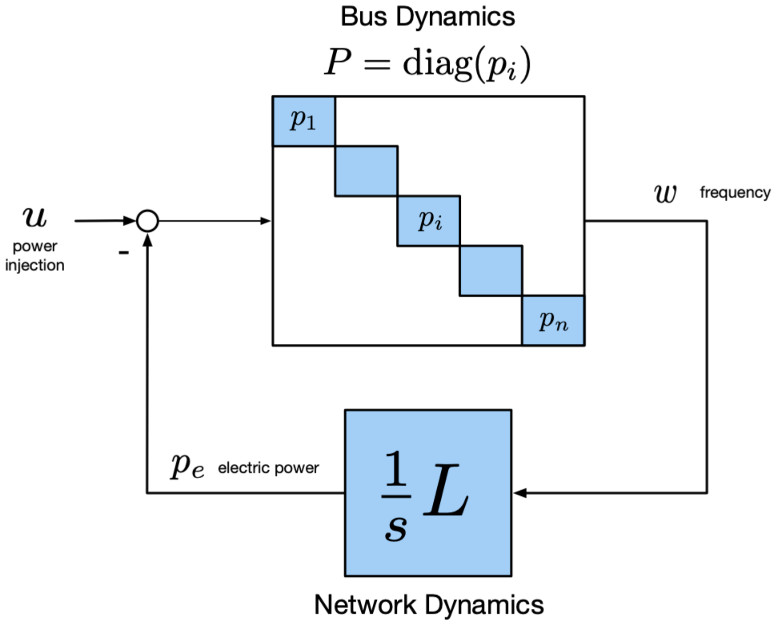
$$\bar{w}(s) = \frac{1}{\sum_{i=1}^{n} f_i \, p_0^{-1}(s)} \left(\sum_{i=1}^{n} u_i(s) \right)$$

[TAC 22] Min, M, *Coherence and concentration in tightly connected networks*, **IEEE Transactions on Automatic Control**, 2022, under review

Center of Inertia /w Grid Forming Inverters

Center of Inertia Freq.

$$\bar{w}(t) = \frac{\sum_{i=1}^{n} M_i w_i(t)}{\sum_{i=1}^{n} M_i} \xLeftrightarrow{p_i(s) = \frac{1}{f_i} p_0(s)} \bar{w}(s) = p_0(s) \frac{1}{\sum_{i=1}^{n} f_i} \left(\sum_{i=1}^{n} u_i(s) \right)$$



Problem: **No longer valid grid-forming inverters**

Yet…

$$\bar{w}(s) = \left(\sum_{i=1}^{n} p_i^{-1}(s) \right)^{-1} \left(\sum_{i=1}^{n} u_i(s) \right)$$

…provides a good approx. of aggregate response!

[TAC 22] Min, M, *Coherence and concentration in tightly connected networks*, **IEEE Transactions on Automatic Control**, 2022, under review

Generalized Center of Inertia

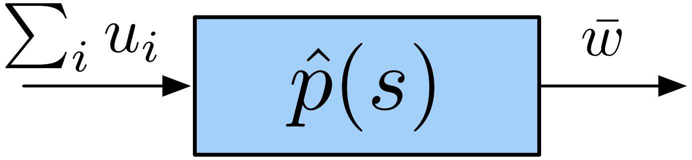
Define the **Generalized CoI Frequency** by:

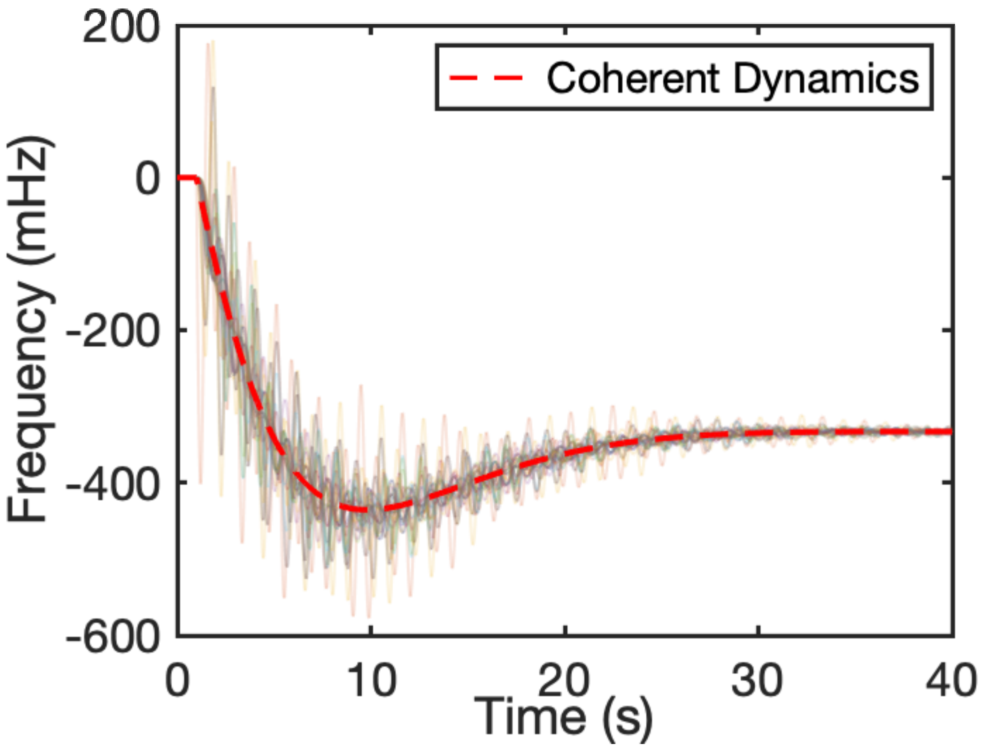
$$\bar{w}(s) = \hat{p}(s)\left(\sum_{i=1}^{n} u_i(s)\right)$$

where…

$$\hat{p}(s) = \left(\sum_{i=1}^{n} p_i^{-1}(s)\right)^{-1}$$

represents the ***coherent dynamics.***





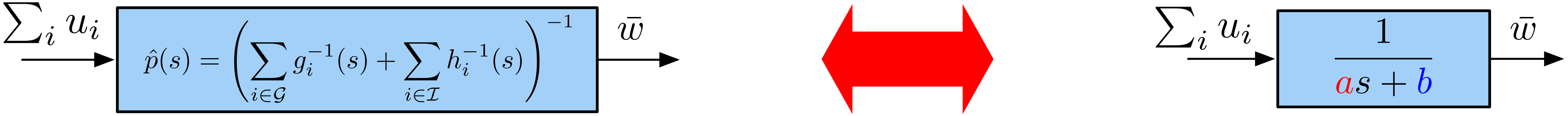
Coherent Dynamics:

- Representation of aggregate response
- Accuracy of approximation:
  - is frequency dependent
  - increases with network connectivity
- Provides excellent template for reduced order models (via balance-truncations)
- More details in [TAC 22]

[TAC 22] Min, M, *Coherence and concentration in tightly connected networks*, **IEEE Transactions on Automatic Control**, 2022, under review

Grid-forming Frequency Shaping Control

Key idea: use model matching control on coherent dynamics



$$\hat{p}(s) = \left(\sum_{i\in\mathcal{G}} g_i^{-1}(s) + \sum_{i\in\mathcal{I}} h_i^{-1}(s)\right)^{-1}$$

$$\frac{1}{as+b}$$

Generation:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s+1}}, \quad i \in \mathcal{G}$$

Inverters:

$$h_i(s) = \frac{1}{m_i s + d_i + c_i(s)}, \quad i \in \mathcal{I}$$

$$a := \sum_{i\in\mathcal{G}} m_i + \sum_{i\in\mathcal{I}} m_i$$

$$b := \sum_{i\in\mathcal{G}} (d_i + r_i^{-1}) + \sum_{i\in\mathcal{I}} d_i$$

$$\sum_{i\in\mathcal{I}} c_i(s) = \sum_{i\in\mathcal{G}} \frac{r_i^{-1}\tau_i s}{\tau_i s + 1}$$

RoCoF:

$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_{0i}|}{a}$$

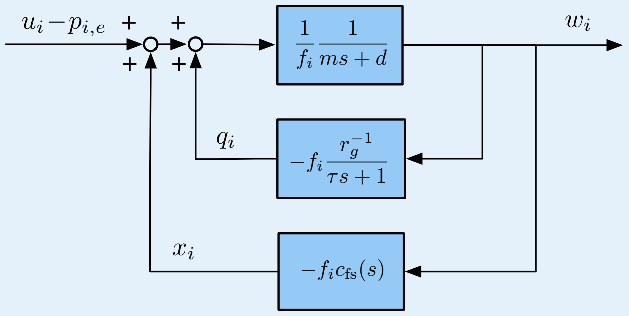
Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{b}$$

[L-CSS 21] Jiang, Bernstein, Vorobev, M, *Grid-forming frequency shaping control*, **IEEE Control Systems Letters**, 2021

Frequency Shaping

Grid-following Inverters: At each bus/area…



$$c_{\mathrm{fs}}(s) := \frac{b_2 s^2 + b_1 s + b_0}{\tau' s + 1}$$

$$b_2 = \tau\,(a - m) \qquad b_0 = b - r_{\mathrm{g}}^{-1} - d$$

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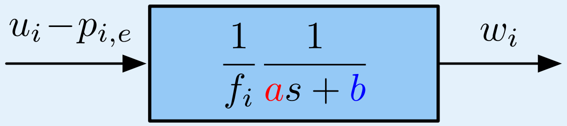

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RoCoF:

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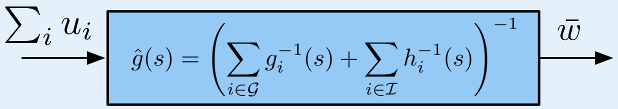
Grid-forming Inverters



$$h_i(s) = \frac{1}{m_i s + d_i + c_i(s)}$$

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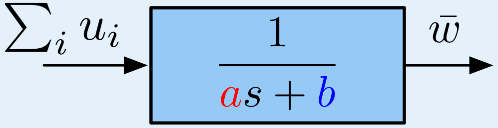

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$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{b}$$

RoCoF:

$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_{0i}|}{a}$$

Summary

- Developed dynamic performance metrics that are analytically tractable and give insight on the effect of inertia, damping, network, etc.
  - To bridge the theory-practice gap, we cover: heterogeneous machines, step response and stochastic metrics
  - Generalized notion of CoI: based on a novel coherence analysis
- Take away messages
  - Role of inertia less dramatic than in conventional wisdom. **Lighter systems are also faster to control.** Short term damping d is a more crucial parameter.
  - Flexibility in the control design provides more opportunities:
    - **Dynamic Droop Control** (iDroop) can cancel Nadir, Sync. Cost, and attenuate frequency variance.
    - **Frequency Shaping Control** can further trade off between RoCoF and Control Effort
- What's missing
  - Stability in the faster timescales (compatibility btwn. converters and gens. voltage regulation)
  - Performance improvement depends on knowledge on system -> Robust Performance

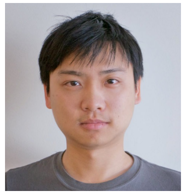
Thanks!

Related Publications:

- Paganini and M, “Global analysis of synchronization performance for power systems: bridging the theory-practice gap,” **IEEE TAC 2020**
- Jiang, Pates, M, “Dynamic Droop Control for Low Inertia Power Systems,” **IEEE TAC 2021**
- Jiang, Cohn, Vorobev, M “Storage-Based Frequency Shaping Control,” **IEEE TPS 2021**
- Min, Paganini, M, “Accurate Reduced Order Models for Coherent Synchronous Generators,” **L-CSS 2020**

Yan Jiang

Johns Hopkins University



Hancheng Min

Johns Hopkins University

Eliza Cohn

Johns Hopkins University

Enrique Mallada

mallada@jhu.edu

http://mallada.ece.jhu.edu

Petr Vorobev

Skoltech
Skolkovo Institute of Science and Technology

Richard Pates

Lund University

Fernando Paganini

ORT
Universidad ORT Uruguay

Backup Slides

Numerical Examples

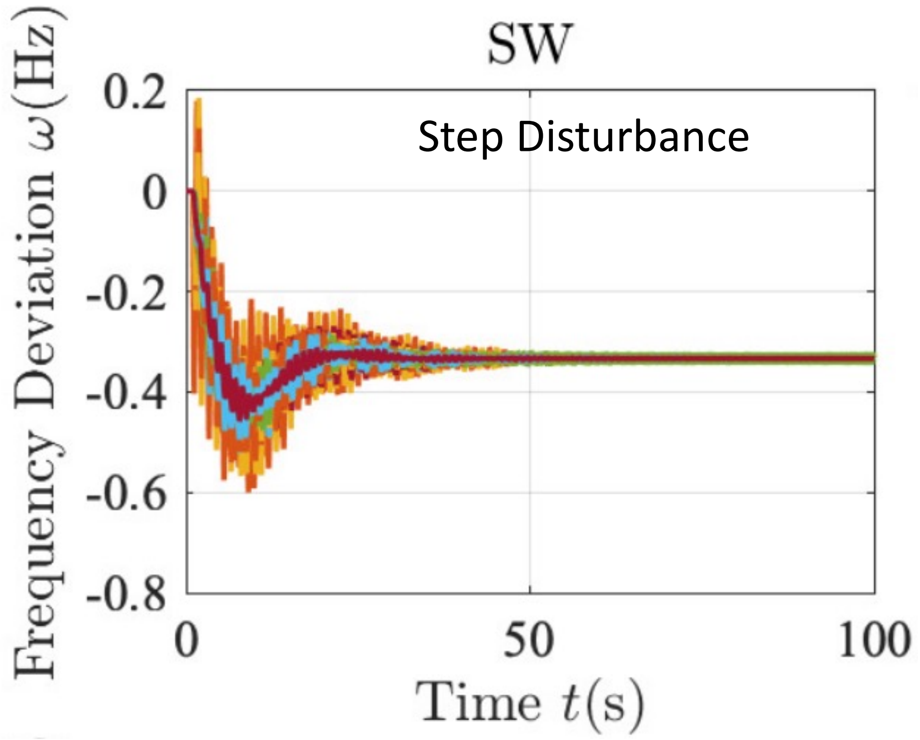
Modal Decomposition

Coherence

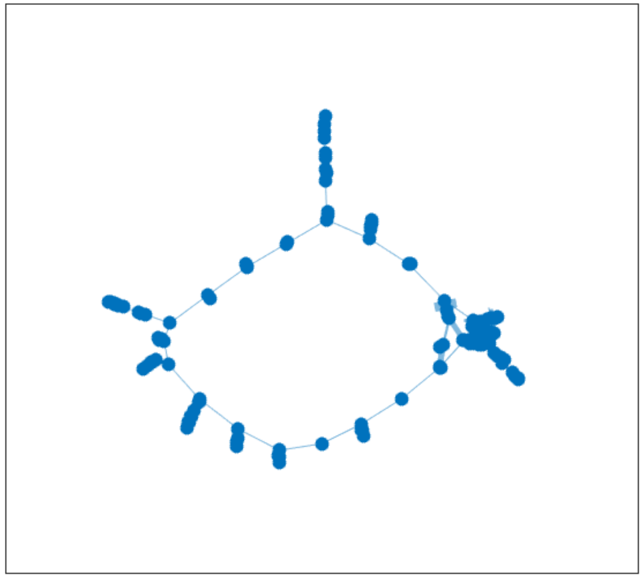
Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

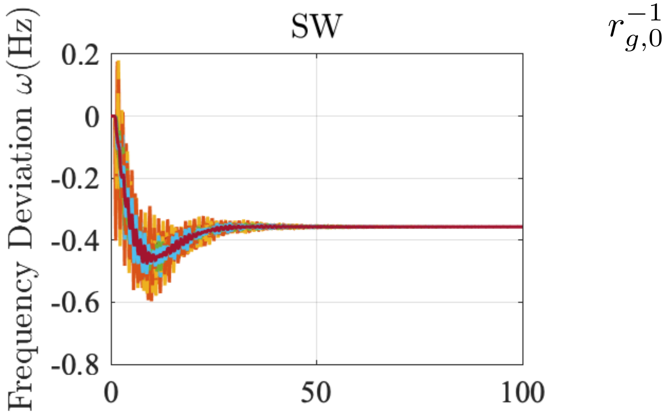
Base Case: No Inverter Control



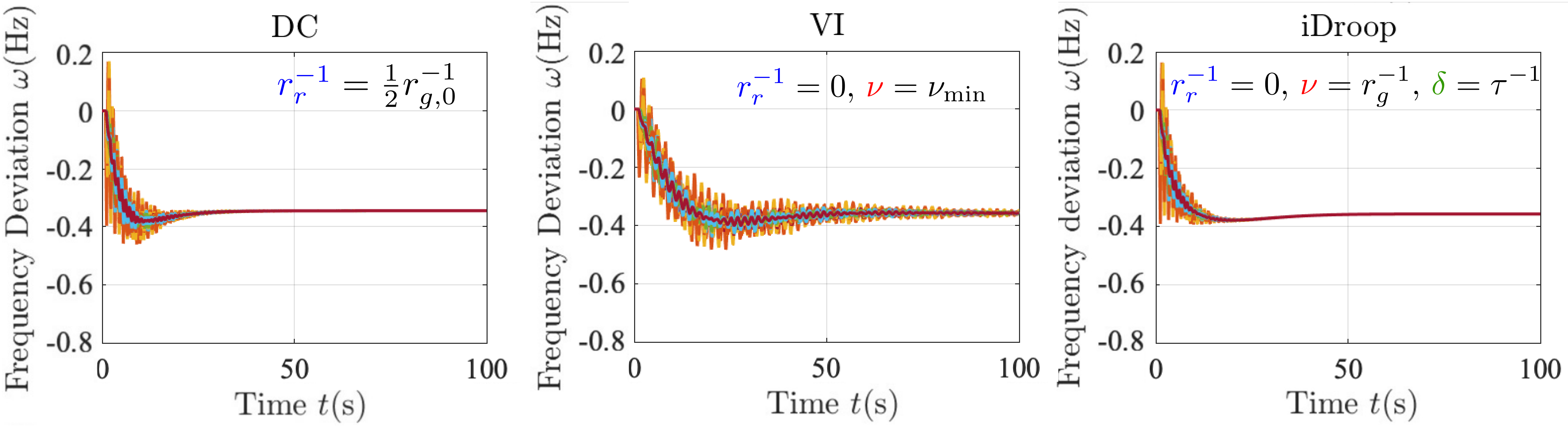
Icelandic Grid



Step Disturbance - Frequency

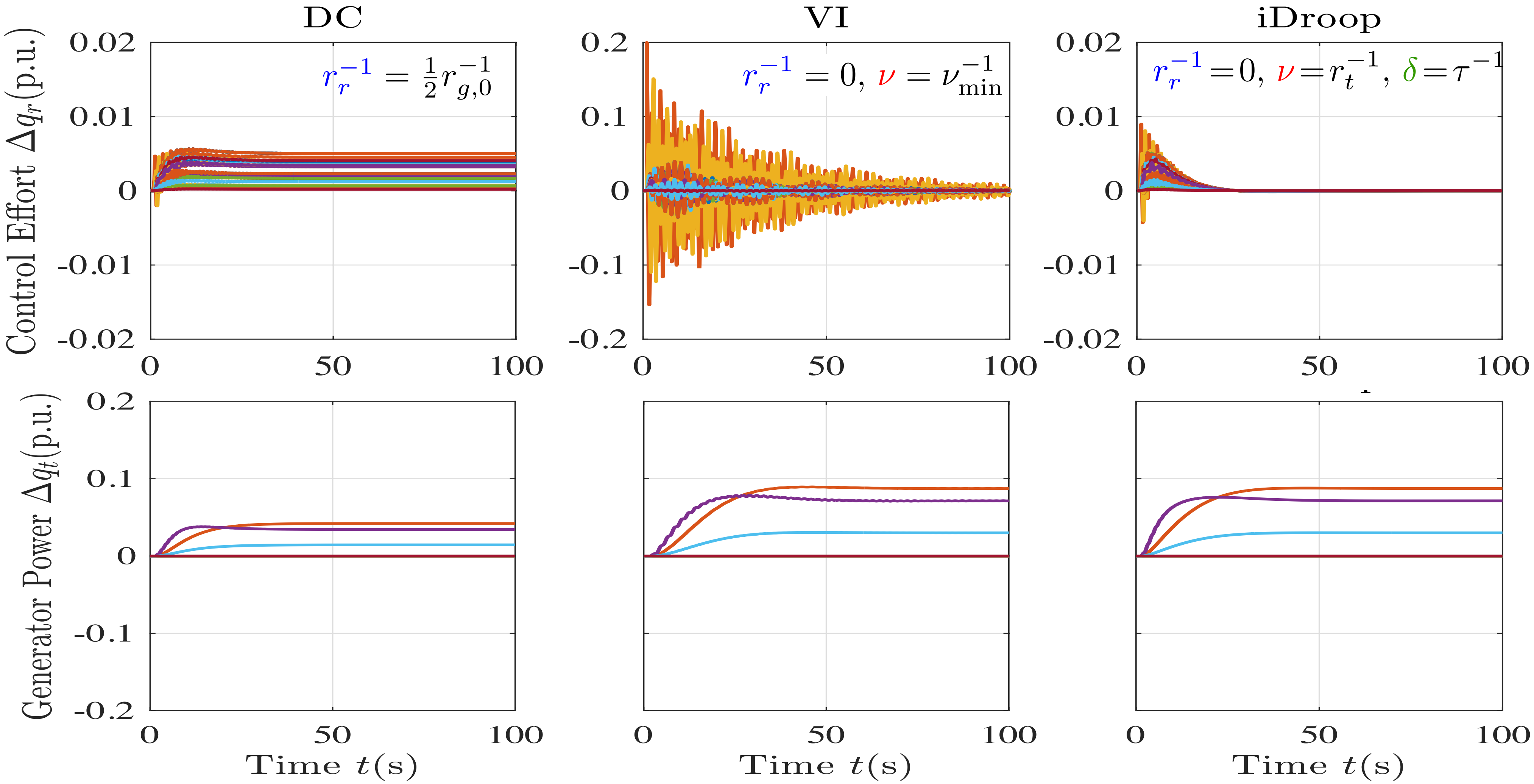


- Droop equally shared between gens. and inverters.
- Virtual inertia tuned for **critically damped response** $\nu = \nu_{min}$
- **iDroop** tuned for Nadir elimination

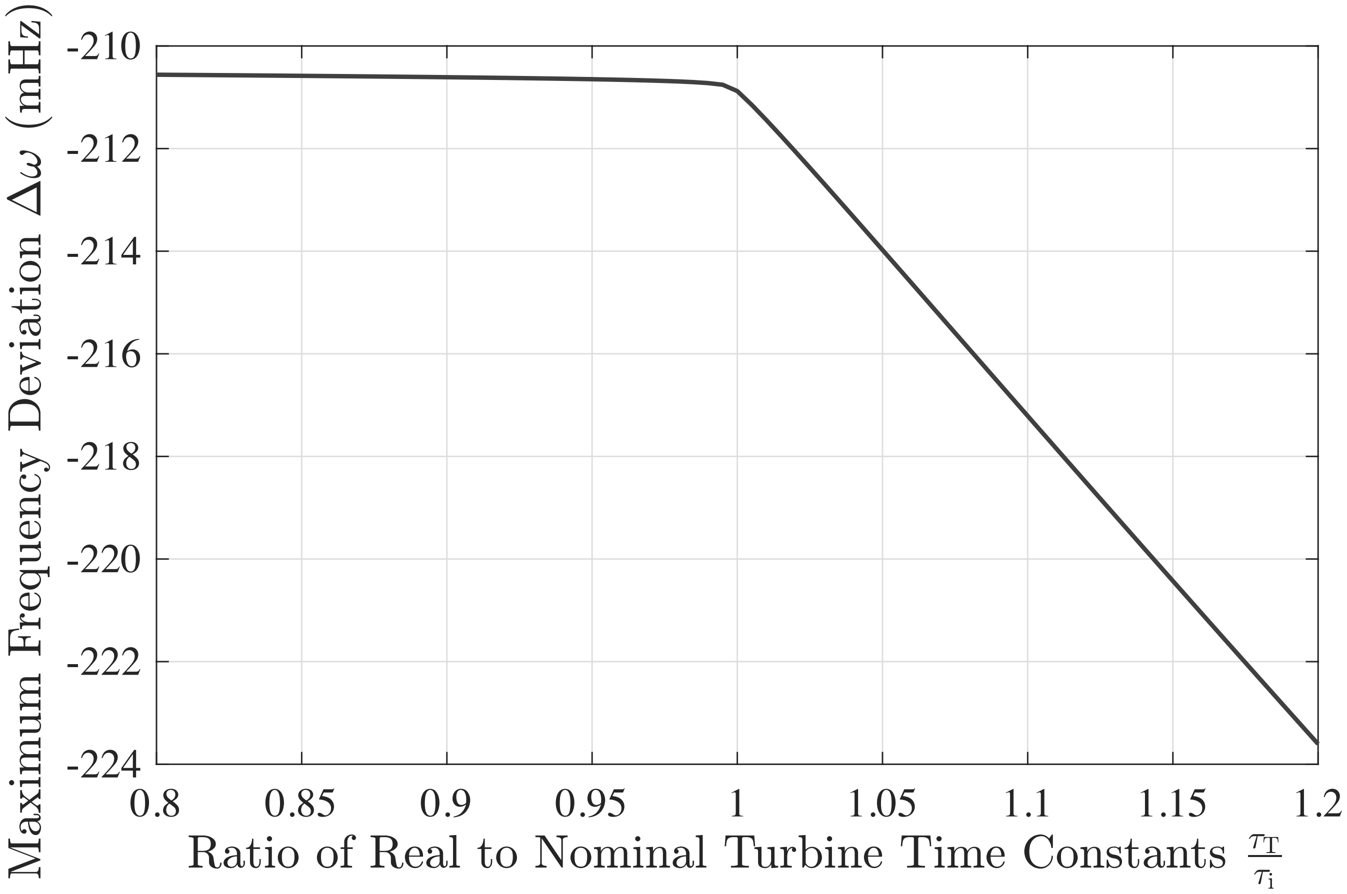


Step Disturbance – Control Effort

Inertia parameters of iDroop and VI are set to achieve zero overshoot.



Parameter Uncertainty



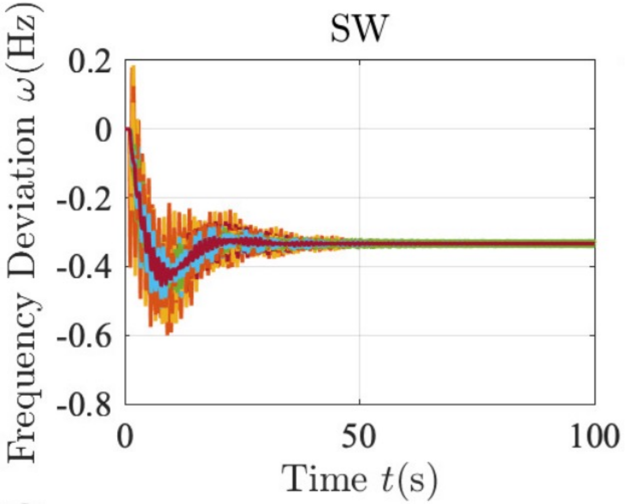
Step Disturbance

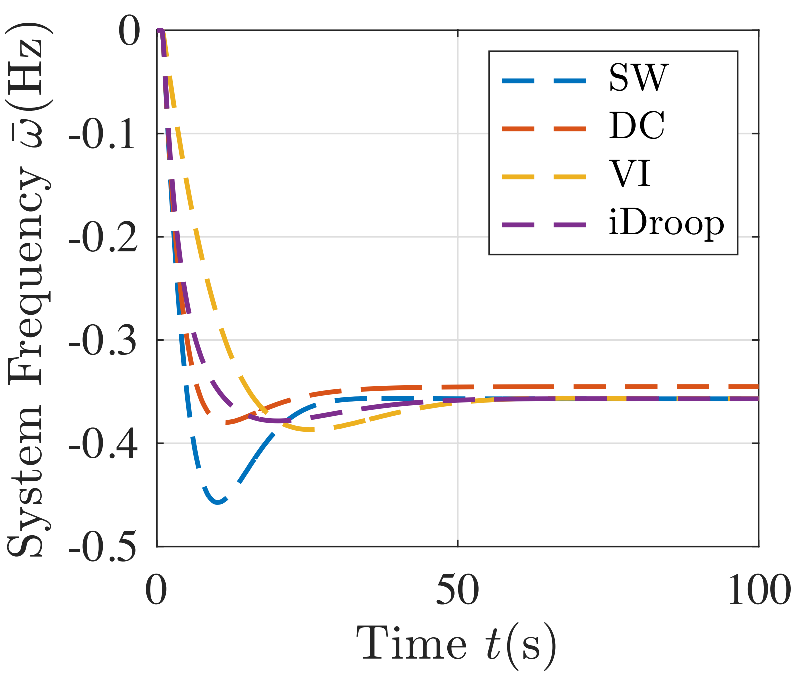
Inertia parameters of iDroop and VI are set to achieve zero overshoot.

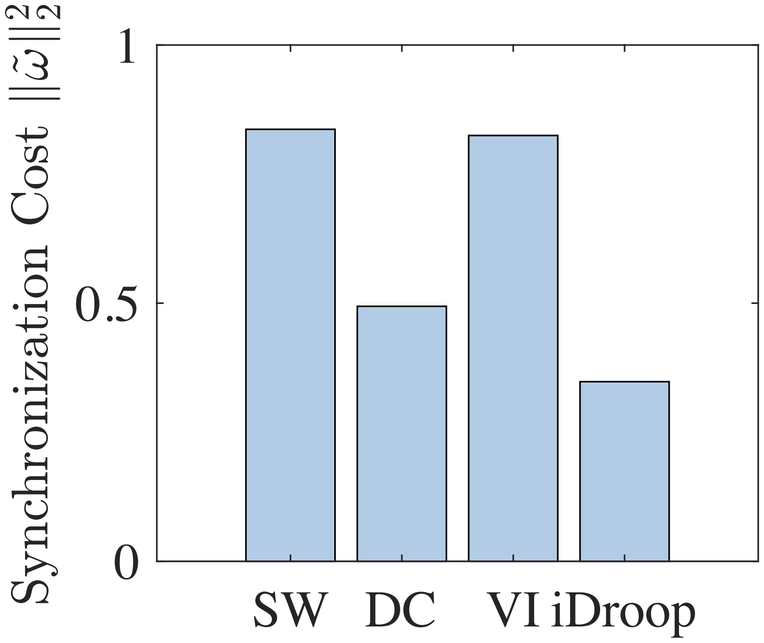
δ^*	0.331
ν^*	0.0054
ν_{VI}	0.035

Zero Nadir Tuning

$$\delta = \tau^{-1}$$
$$\nu = r_r^{-1} + r_g^{-1}$$







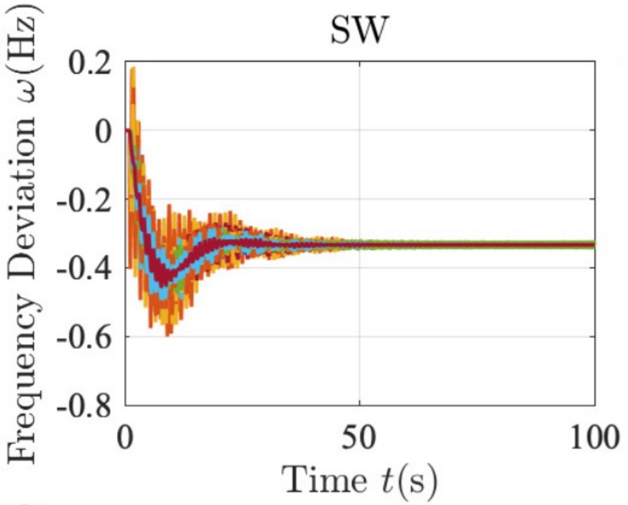
Step Disturbance

Inertia parameters of iDroop and VI are set to achieve zero overshoot.

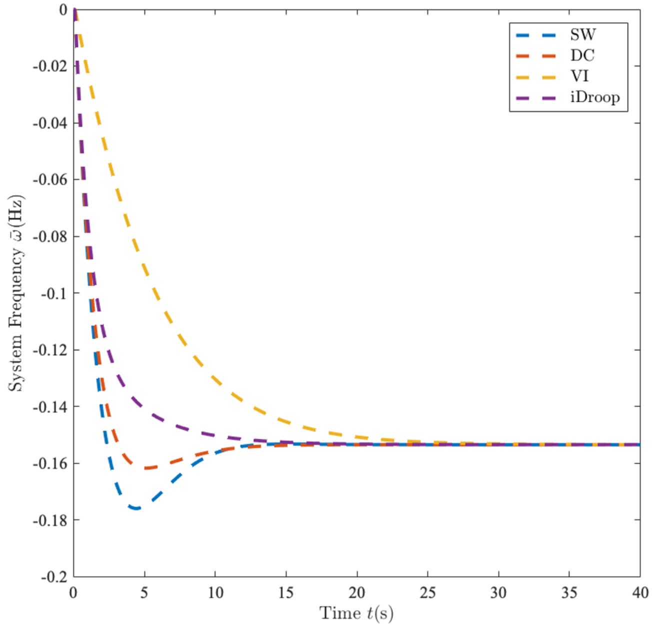
δ^*	0.331
ν^*	0.0054
ν_{VI}	0.035

→

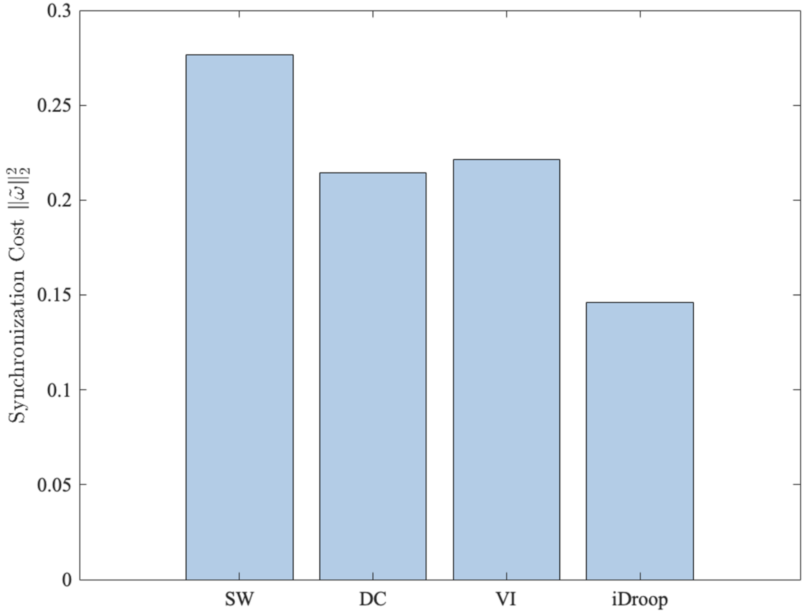
δ^*	0.331
ν^*	0.0108
ν_{VI}	0.070



System Frequency



Synchronization Cost



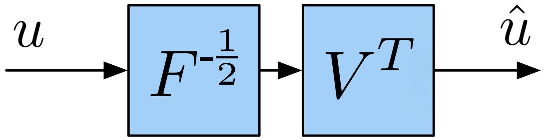
Back

Modal Decomposition for Multi-Rated Machines

Assumption: Let f_i be the machine relative inertia ($f_i = \frac{M_i}{\max_j M_j}$), and

$$\left\{\begin{array}{l} g_i(s) = \frac{1}{f_i} g_0(s) \\ c_i(s) = f_i c_0(s) \end{array}\right.$$

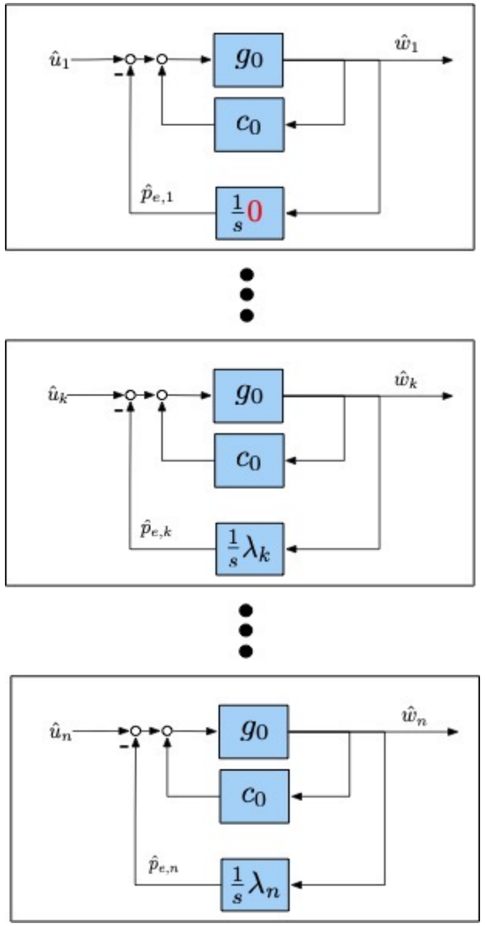
Change of Vars.



$$F = \mathrm{diag}(f_i)$$

Eigenvalues of: $L_F = F^{-\frac{1}{2}} L F^{-\frac{1}{2}}$

$$0 = \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}$$



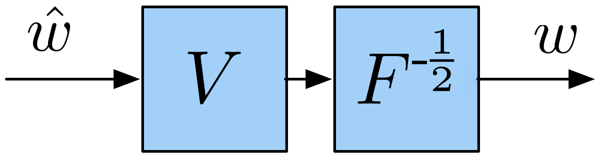
System Frequency

$$\bar{w}(t) = \frac{\sum_{i=1}^n M_i w_i(t)}{\sum_{i=1}^n M_i}$$

Sync Error

$$\tilde{w}_i(t) = w_i(t) - \bar{w}(t)$$

Change of Vars.

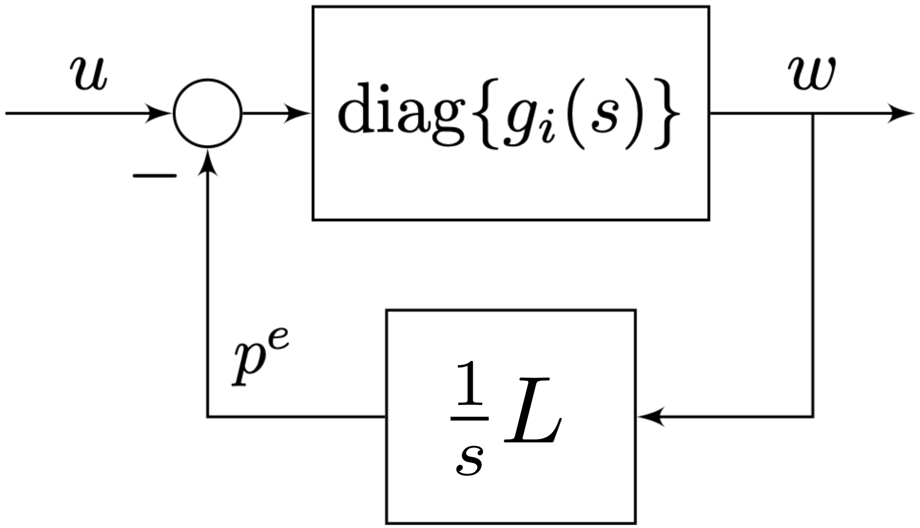


[Paganini M ‘17 , Guo Low 18’]

[TAC 20] Paganini, M, *Global analysis of synchronization performance for power systems: Bridging the theory-practice gap*, **IEEE Transactions on Automatic Control, 2020**

Coherent Dynamics

Block Diagram:



$g_i(s), i = 1, \cdots, n$: Generator dynamics,

$L = [L_{ij}]$,

$L_{ij} = \frac{\partial}{\partial \theta_j} \sum_{k=1}^n |V_i||V_k| b_{ik} \sin(\theta_i - \theta_k)\Big|_{\theta=\theta_0}$

L symmetric, $0 = \lambda_1(L) \leq \lambda_2(L) \leq \cdots \leq \lambda_n(L)$

Generator dynamics:

$$w_i(s) = g_i(s)(u_i(s) - p_i^e(s))$$

$$i = 1, \cdots, n$$

Power flow equation:

$$p^e(s) = \frac{1}{s} L w(s)$$

Characterization of Coherent Dynamics

Assume all generators from a coherent group

$$w_i(s) = \ g_i(s)(u_i(s) - p_i^e(s)) \quad i = 1, \cdots, n$$

Assume generators ”output” identical frequencies

$$w_i(s) = \hat{w}(s)$$

$$g_i^{-1}(s)\hat{w}(s) = u_i(s) - p_i^e(s) \quad i = 1, \cdots, n$$

Sum from 1 to n:

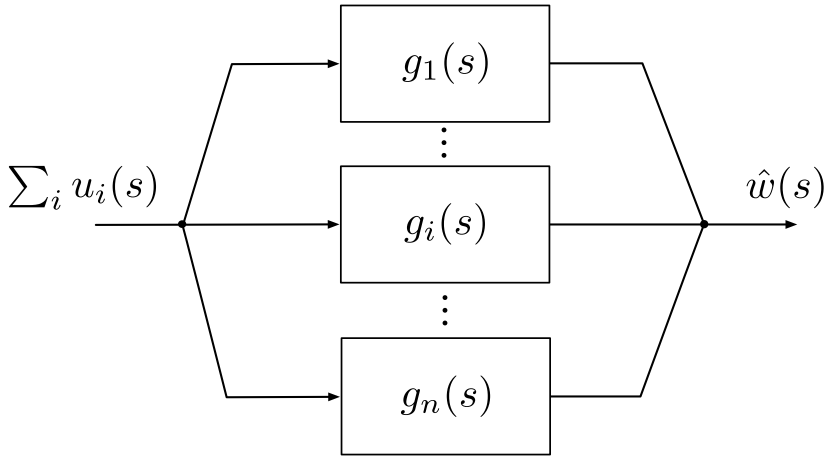
$$\left(\sum_{i=1}^{n} g_i^{-1}(s)\right)\hat{w}(s) = \sum_{i=1}^{n} u_i(s) - \underbrace{\boxed{\sum_{i=1}^{n} p_i^e(s)}}_{=0}$$

Coherent Dynamics:

$$\hat{w}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1} \sum_{i=1}^{n} u_i(s)$$

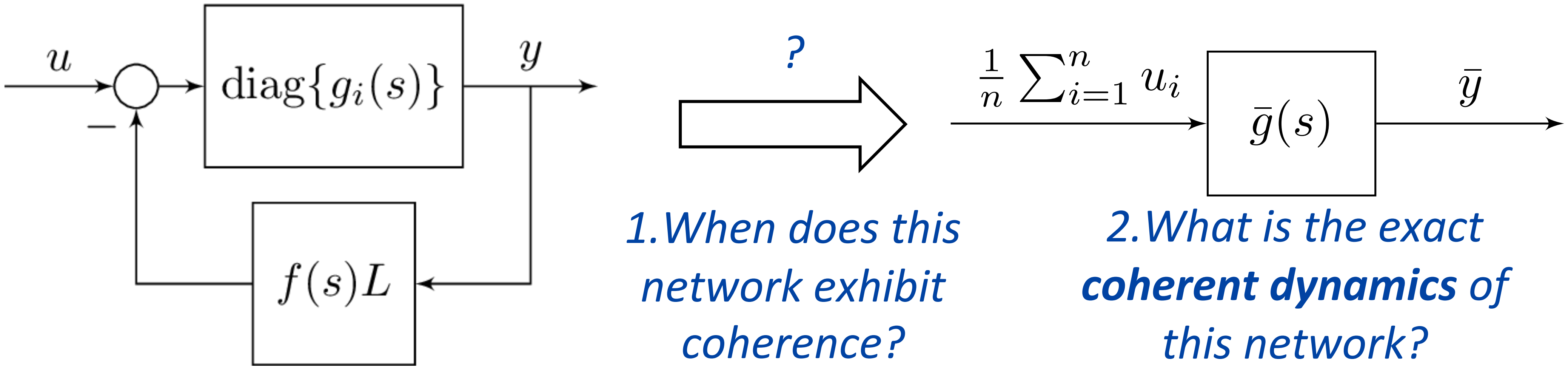
$$\boxed{\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}}$$

parallel impedance formula



Coherence in networked dynamical systems

Block Diagram:



1.When does this network exhibit coherence?

2.What is the exact ***coherent dynamics*** *of this network?*

1. Coherence can be understood as a **low rank** property the **closed-loop transfer matrix**
2. It emerges as the **effective algebraic connectivity** increases
3. The coherent dynamics is given by the **harmonic mean** of nodal dynamics

$$\bar{g}(s) = \left(\frac{1}{n} \sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1}$$

Justification for Previous Derivation

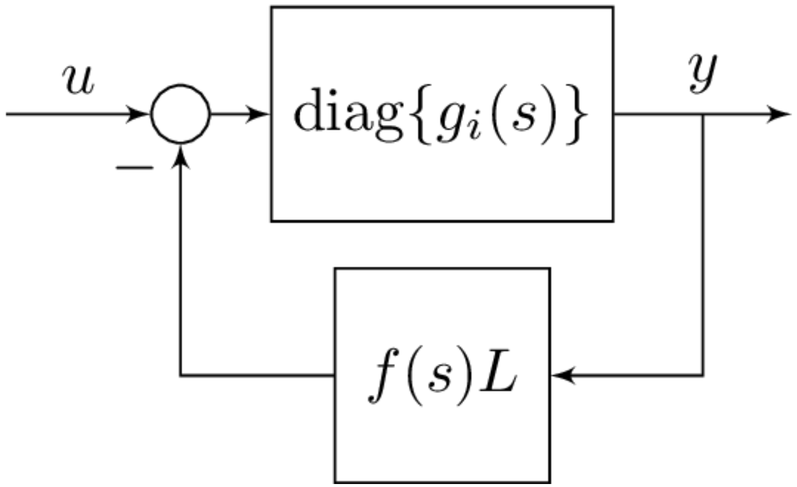
Theorem. *Assume all* $g_i(s)$ *are positive real. Let the transfer matrix from* u *to* w *be* $T(s)$*, then for any* $\eta_0 > 0$*:*

$$\lim_{\lambda_2(L)\to+\infty} \sup_{\eta\in[-\eta_0,\eta_0]} \left\|T(j\eta) - \hat{g}(j\eta)\mathbb{1}\mathbb{1}^T\right\| = 0, \quad j = \sqrt{-1}.$$

- Extension of recent result (CDC '19) on coherence in **networked dynamical systems**
- Convergence on imaginary axis is **related to time-domain response** by Inverse Laplace Transform
- Algebraic connectivity of L is an indicator of **level of coherence**
- $\hat{g}(s)$ accurately represents the aggregate dynamics in the asymptotic sense

Example: Icelandic Power Grid

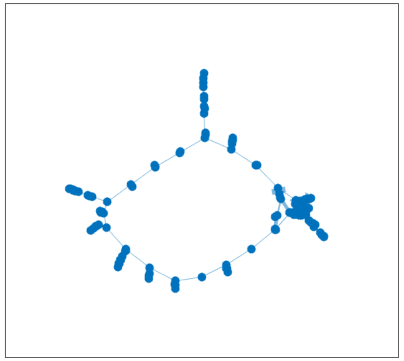
- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)



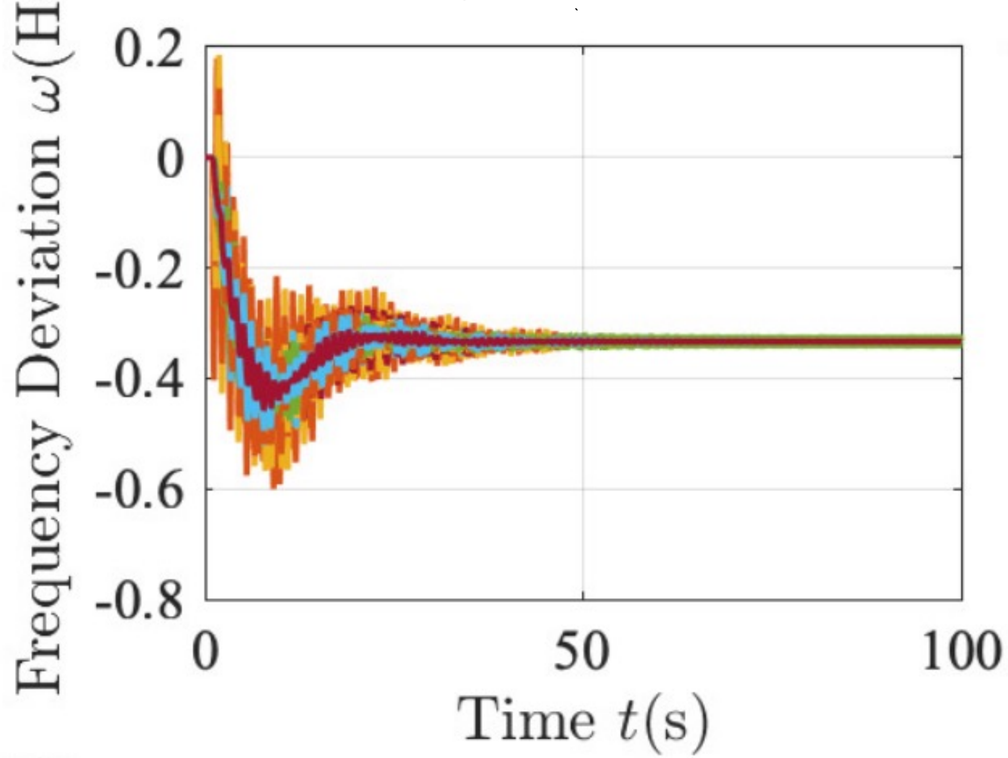
$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s+1}}$$

$$f(s) = \frac{1}{s}$$

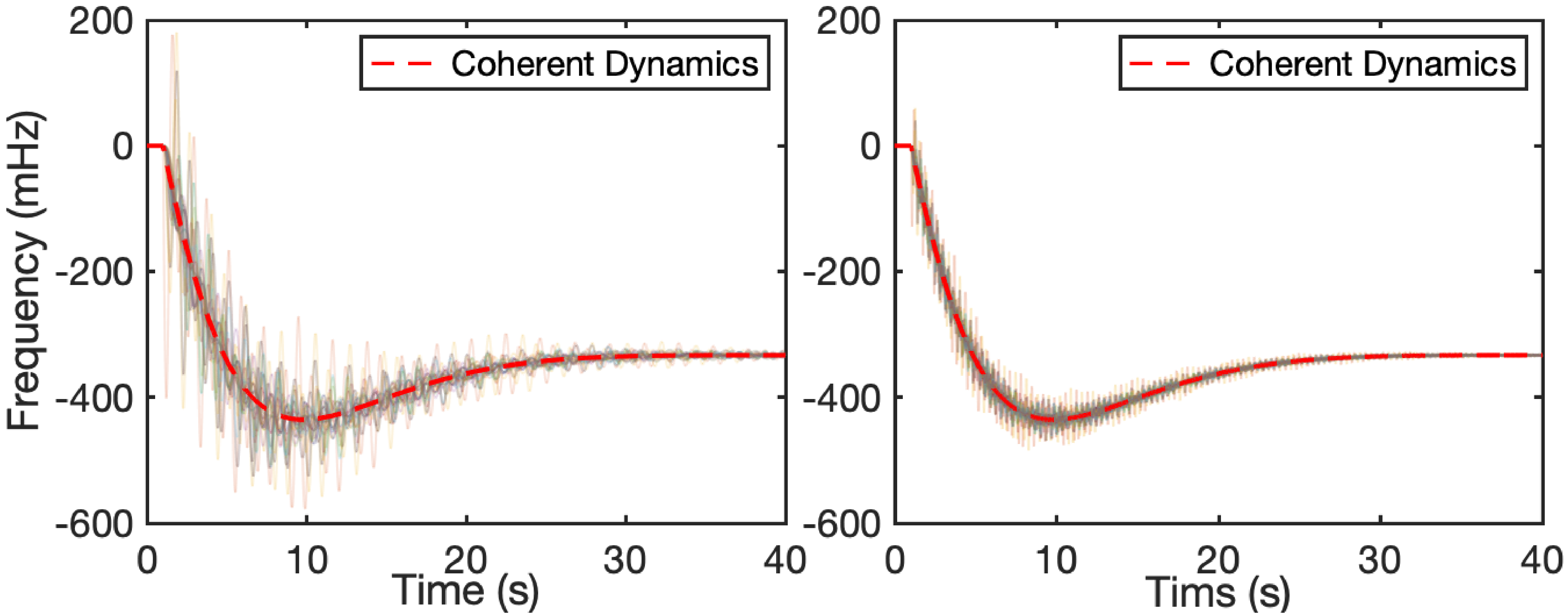
Icelandic Grid



Step Disturbance



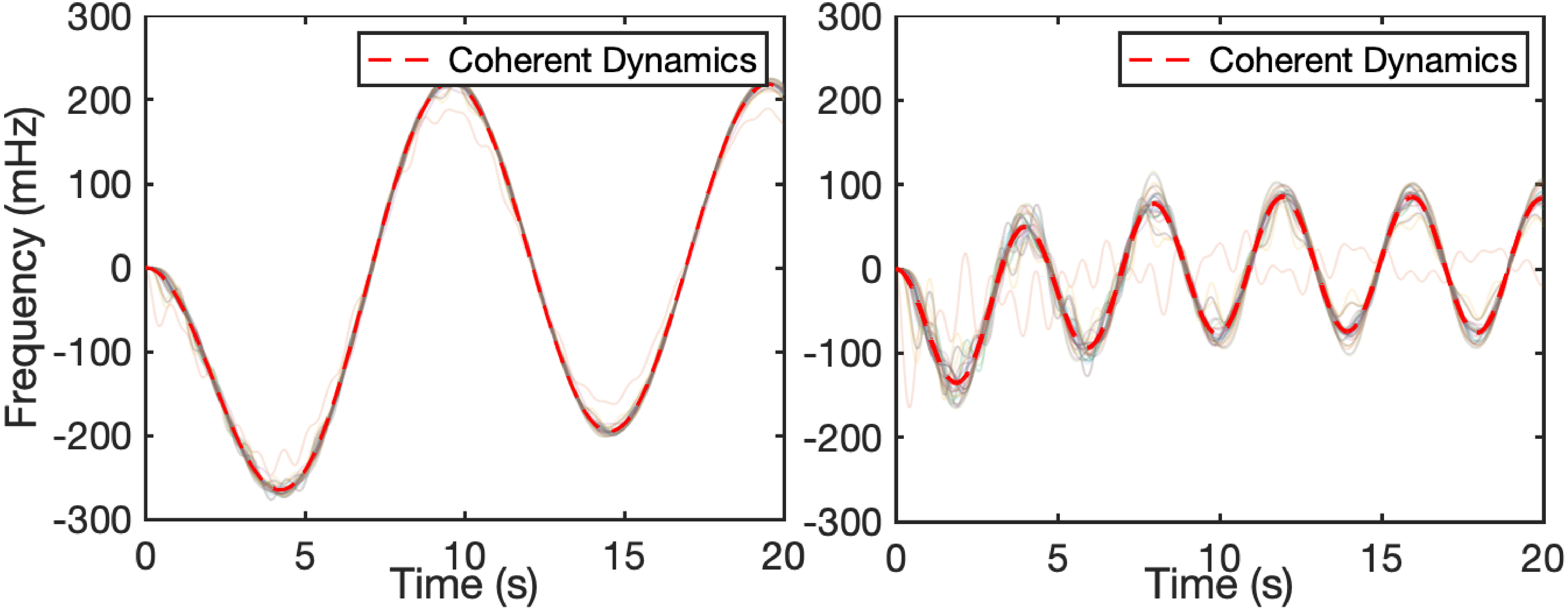
Example: Effect of Network Algebraic Connectivity $\lambda_2(L) \uparrow$



Coherent dynamics acts as a more accurate version of the Center of Inertia (CoI)

Example: Sinusoidal Disturbances: $\sin(\boldsymbol{\omega_d t})$

$\omega_d \uparrow$



Challenges on Aggregating Coherent Generators

For generator dynamics given by a swing model with turbine control:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s+1}}$$

The aggregate dynamics:

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s+1}}}$$

high-order if τ_i are heterogeneous

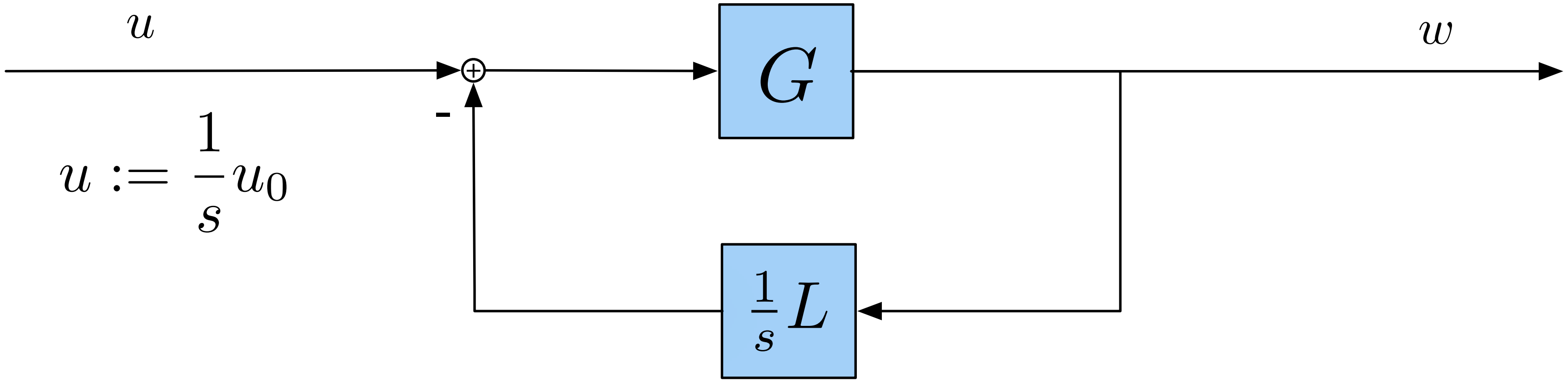
Need to find a low-order approximation of $\hat{g}(s)$

Back

Diagonalization for Step Disturbances

$$u = \frac{1}{s} u_0$$

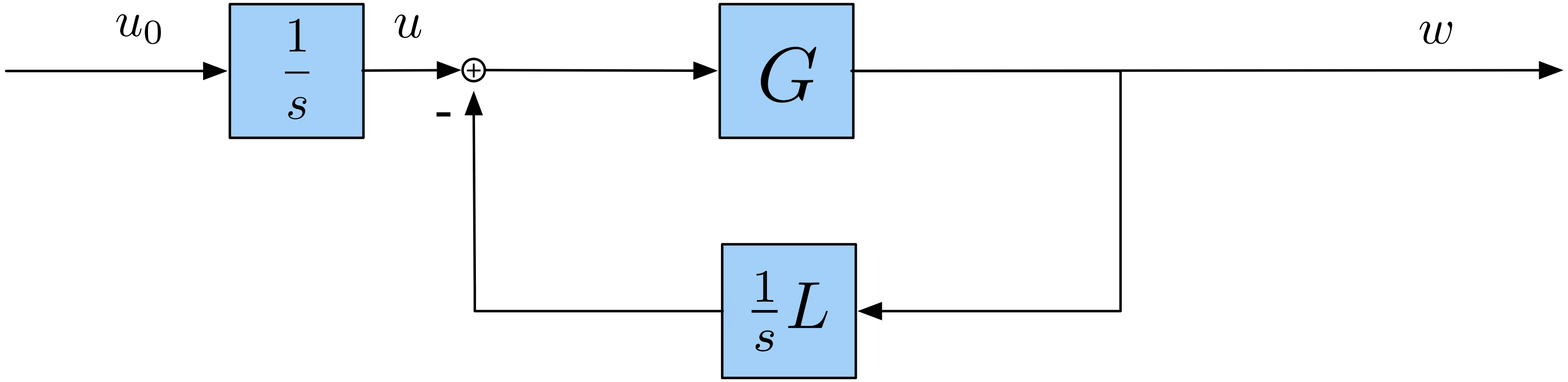
step
disturbance



Diagonalization for Step Disturbances

$$u = \frac{1}{s} u_0$$

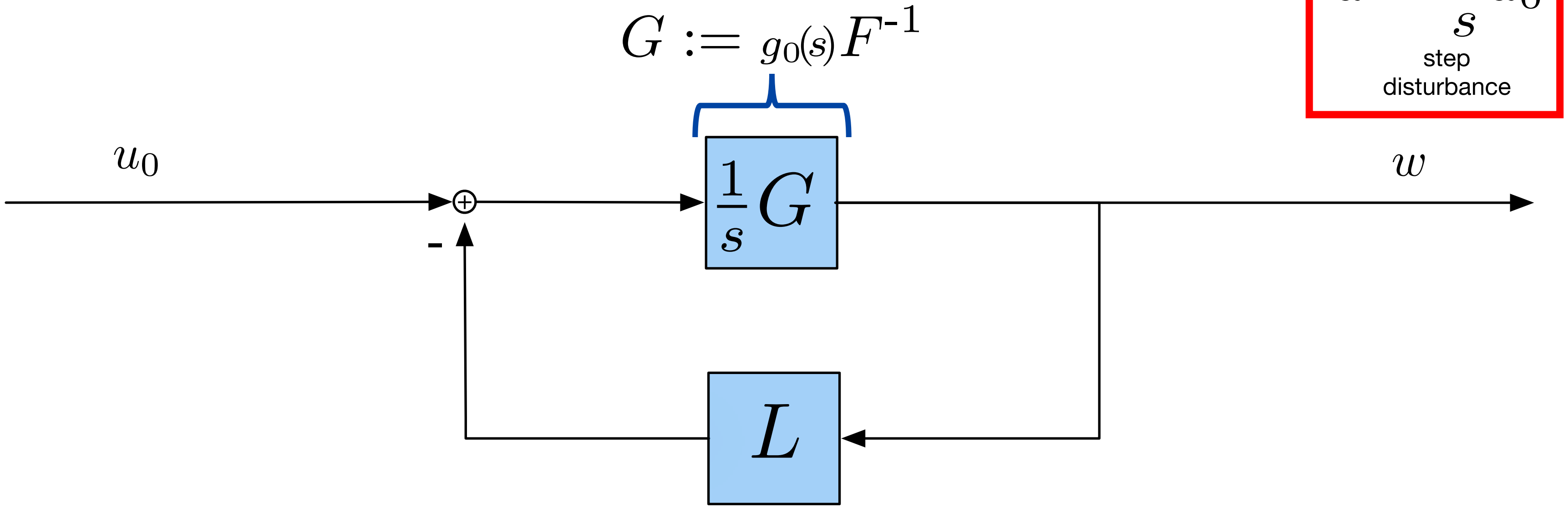
step disturbance



Diagonalization for Step Disturbances

$$u = \frac{1}{s}u_0$$

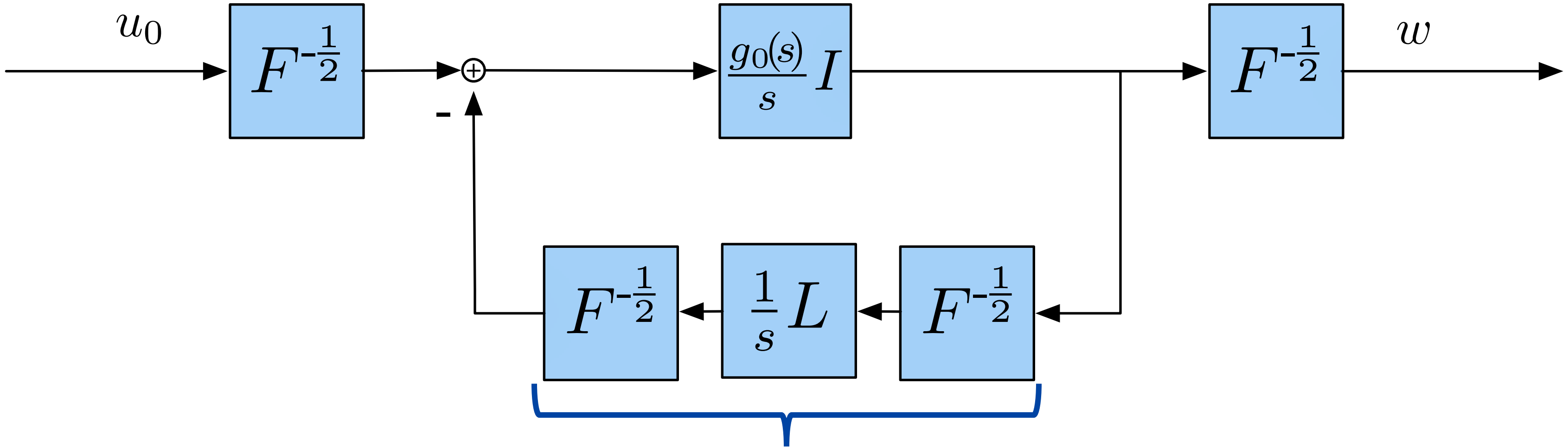
step disturbance



Diagonalization for Step Disturbances

$$u = \frac{1}{s}u_0$$

step disturbance



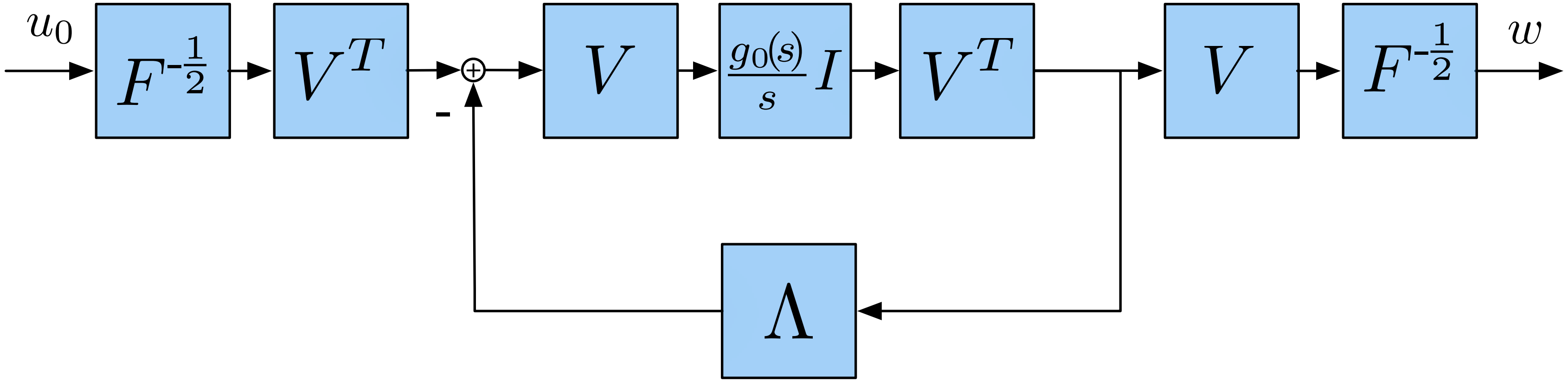
$$F^{-\frac{1}{2}}LF^{-\frac{1}{2}} := L_F = V\Lambda V^T$$

$$\Lambda = \operatorname{diag}(\lambda_i)\,,\ \lambda_0 = 0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}$$

Diagonalization for Step Disturbances

$$u = \frac{1}{s}u_0$$

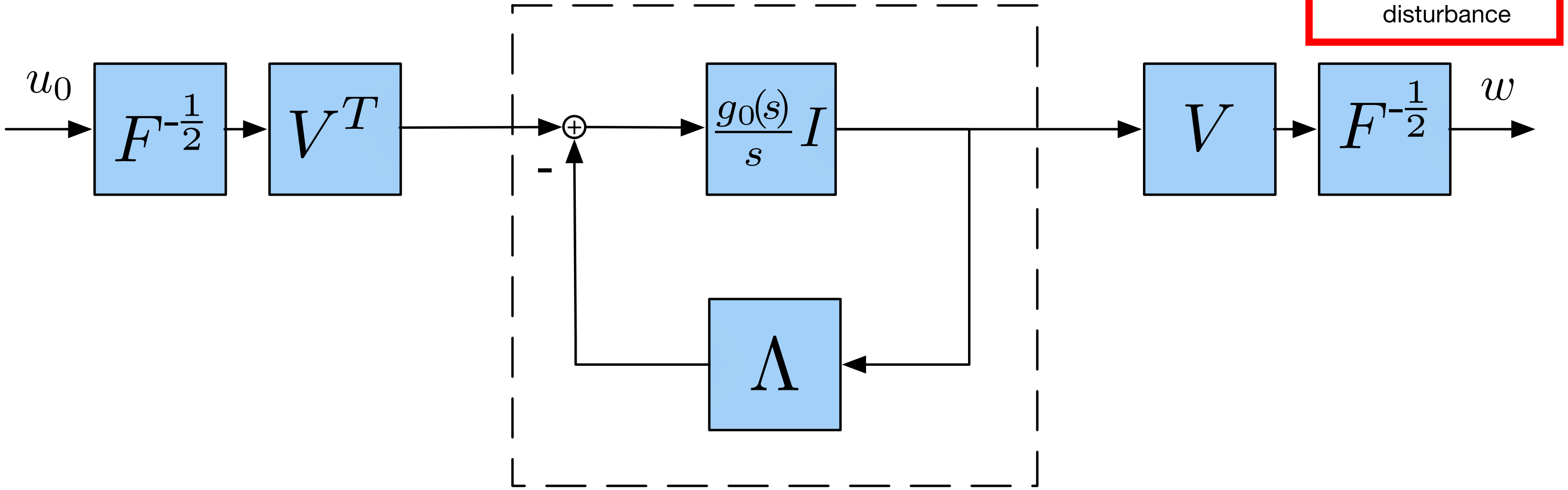
step disturbance



Diagonalization for Step Disturbances

$$u = \frac{1}{s} u_0$$

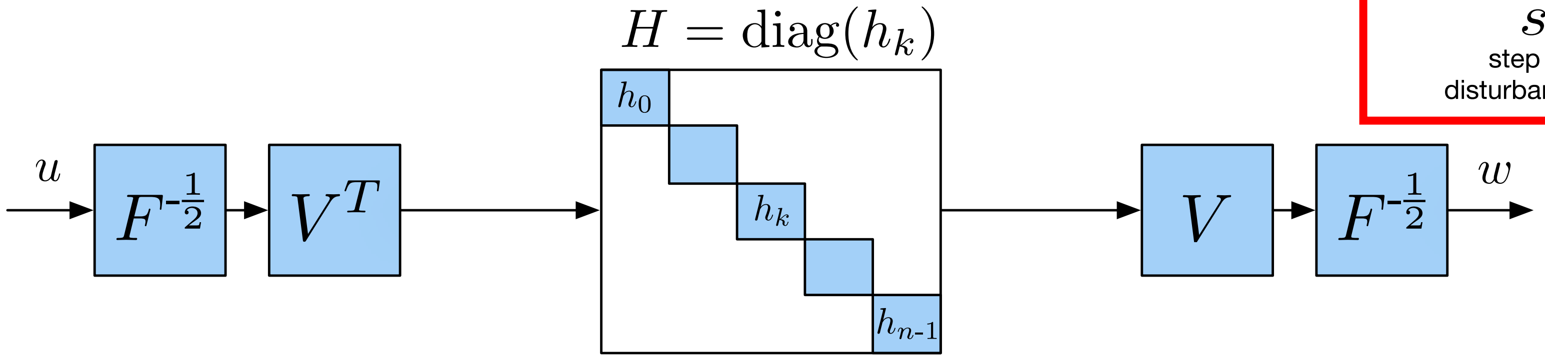
step disturbance



Diagonalization for Step Disturbances

$$u = \frac{1}{s} u_0$$

step disturbance



where $$h_k(s) = \frac{g_0(s)}{s + \lambda_k g_0(s)},$$ with $$\lambda_0 = 0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}$$

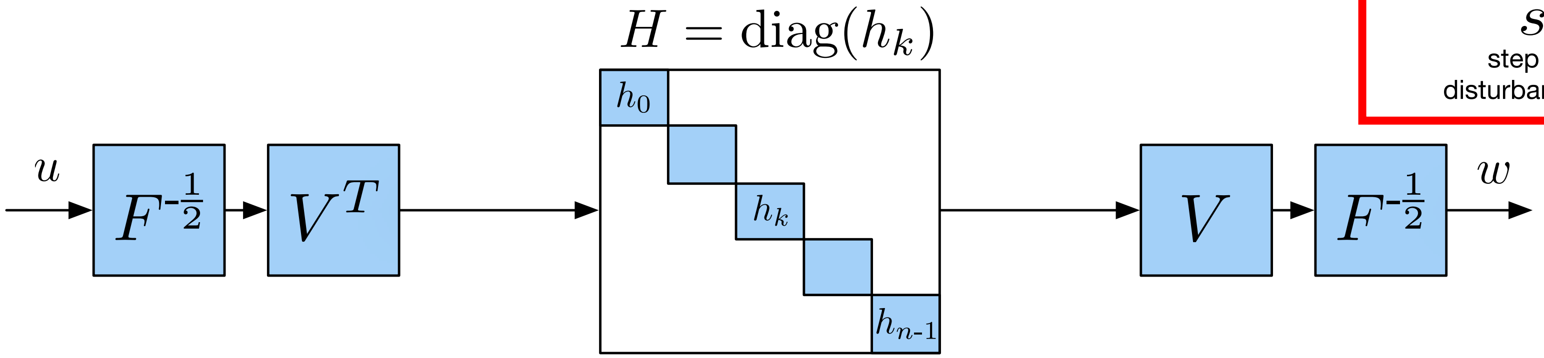
System Frequency:

$$\bar{w}(s) = \overbrace{\frac{1}{\sum_i f_i}}^{\text{Global Response}} h_0(s) \underbrace{1^T u_0}_{\text{Imbalance}}$$

Diagonalization for Step Disturbances

$$u = \frac{1}{s} u_0$$

step disturbance



where $h_k(s) = \dfrac{g_0(s)}{s + \lambda_k g_0(s)}$, with $\lambda_0 = 0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}$

System Frequency:

$$\bar{w}(s) = \overbrace{\frac{1}{\sum_i f_i} h_0(s)}^{\text{Global Response}} \underbrace{1^T u_0}_{\text{Imbalance}}$$

Synchronization Error:

$$\tilde{w}(s) = \underbrace{F^{-\frac{1}{2}} V_\perp \tilde{H}(s) V_\perp^T F^{-\frac{1}{2}}}_{\text{Network Dependent Response}} u_0$$

$$\tilde{H}(s) := \mathrm{diag}(h_1(s), \ldots, h_{n-1}(s))$$

$$V := [v_0 \; V_\perp]$$

Back

Synchronization Cost

$$||\tilde{w}||_2^2 = \int_0^\infty \tilde{w}(t)^T \tilde{w}(t) dt \qquad \tilde{w}(t) := \sum_{k=1}^{n-1} h_k(t) F^{-\frac{1}{2}} v_k v_k^T F^{-\frac{1}{2}} u_0$$

Proposition:

$$||\tilde{w}||_2^2 = z_0^T Y z_0$$

where $Y \in \mathbb{R}^{(n-1)\times(n-1)}$, with elements

$$y_{kl} = \gamma_{kl} \int_0^\infty h_k(t) h_l(t) dt, \quad \Gamma = (\gamma_{kl}) := V_\perp^T F^{-1} V_\perp \quad \text{and} \quad z_0 = V_\perp^T F^{-1} u_0$$

Depends on Generators and Controls.

Depends on Network and Inertia.

Average over step direction: $E_{u_0}\left[||\tilde{w}||_2^2\right]$ **where** $E\left[u_0 u_0^T\right] = \Sigma^u$

Synchronization Cost (II)

$$||\tilde{\omega}||_2^2 = z_0^T Y z_0 \qquad y_{kl} = \gamma_{kl} \int_0^\infty h_k(t) h_l(t) dt$$

Swing Dynamics

$$\int_0^\infty h_k(t) h_l(t) dt = \frac{2d}{m(\lambda_k - \lambda_l)^2 + 2(\lambda_k + \lambda_l) d^2}$$

Homogeneous Case: $F = I \implies \Gamma = I$

$$E_{u_0}\left[||\tilde{\omega}||_2^2\right] = \frac{1}{2d} \sum_{k=1}^{n-1} \frac{1}{\lambda_k}$$

Independent of Inertia!

Heterogeneous case:

- High Inertia: $m \to \infty$ $\quad ||\tilde{\omega}||_2^2 = \sum_{k=1}^{n-1} \frac{\gamma_{kk} z_{0k}^2}{2d\lambda_k}$
- Small Inertia: $m \to 0$ $\quad ||\tilde{\omega}||_2^2 = \sum_{k,l=1}^{n-1} \frac{\gamma_{kl} z_{0k} z_{0l}}{d(\lambda_k + \lambda_l)}$

Inertia has limited effect

Swing Dynamics with Turbines

Homogenous Case:

$$E_{u_0}\left[||\tilde{\omega}||_2^2\right] = \sum_{k=1}^{n-1} ||h_k||^2$$

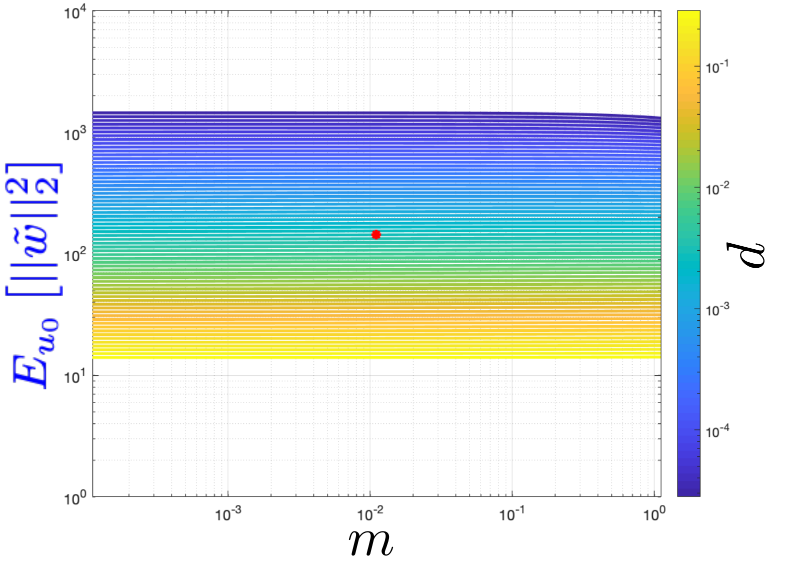
Moderate Improvement w/ Inertia

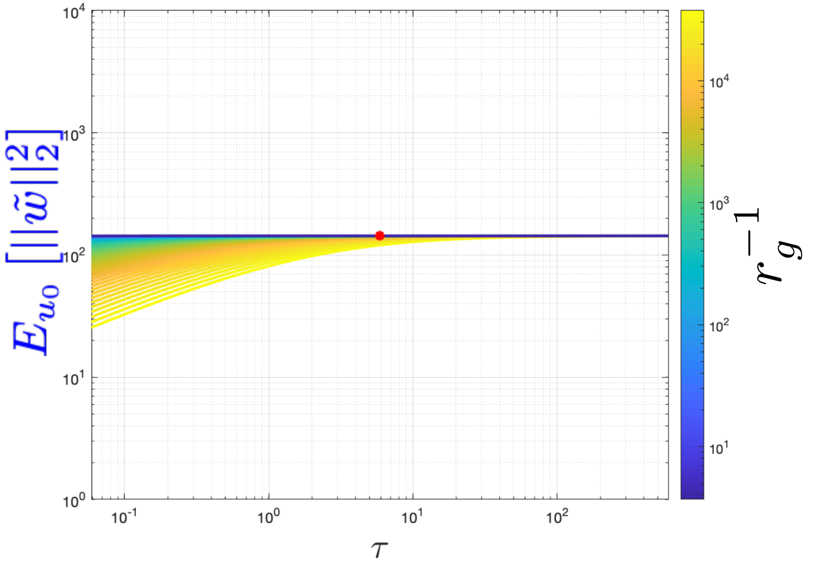
$$\lim_{m \to 0} ||h_k||^2 > \lim_{m \to \infty} ||h_k||^2$$

Heterogeneous Case:

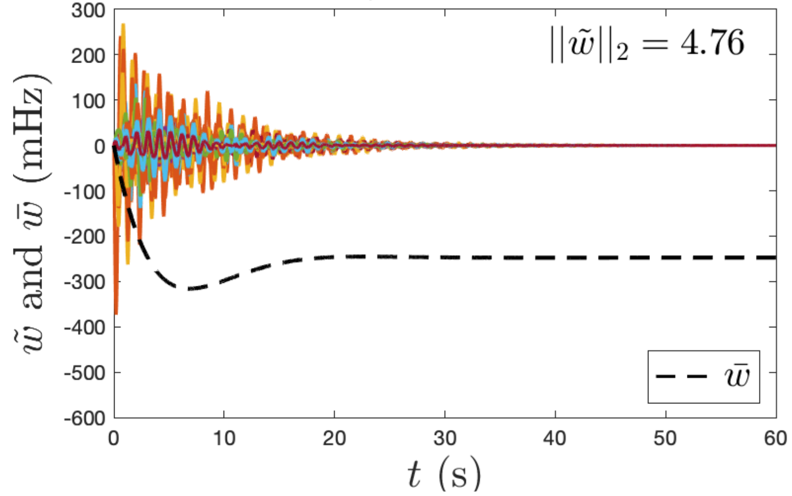
- *More involved expressions…*
- *Limits are always **finite***
- High Inertia: $||\tilde{\omega}||_2^2 = \sum_{k=1}^{n-1} \frac{\gamma_{kk} z_{0k}^2}{2d\lambda_k} \frac{d}{d + r_g^{-1}}$

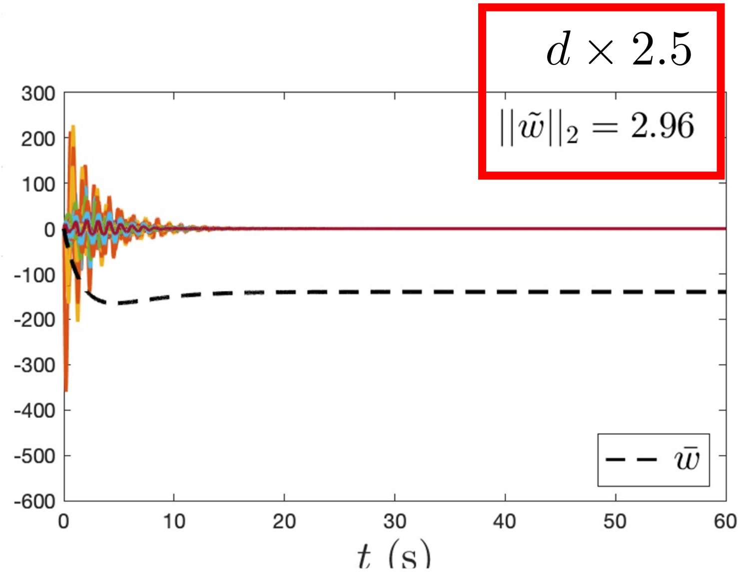
Synchronization Cost (II)





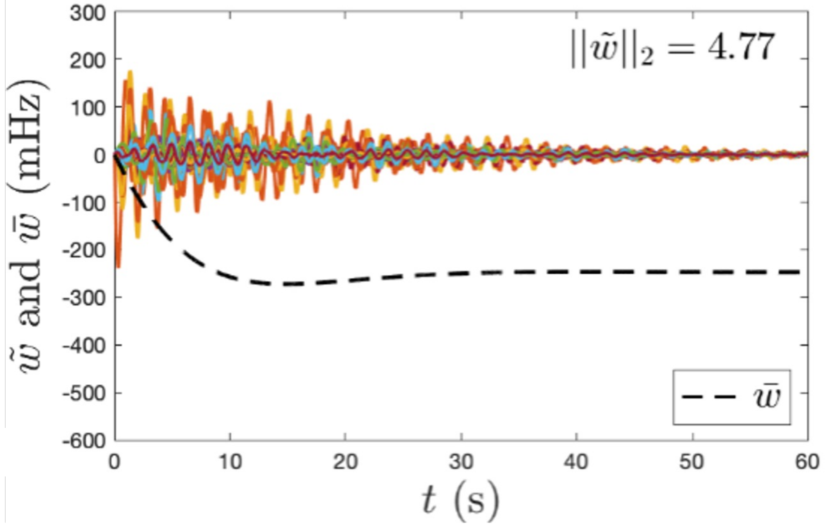
Nominal System



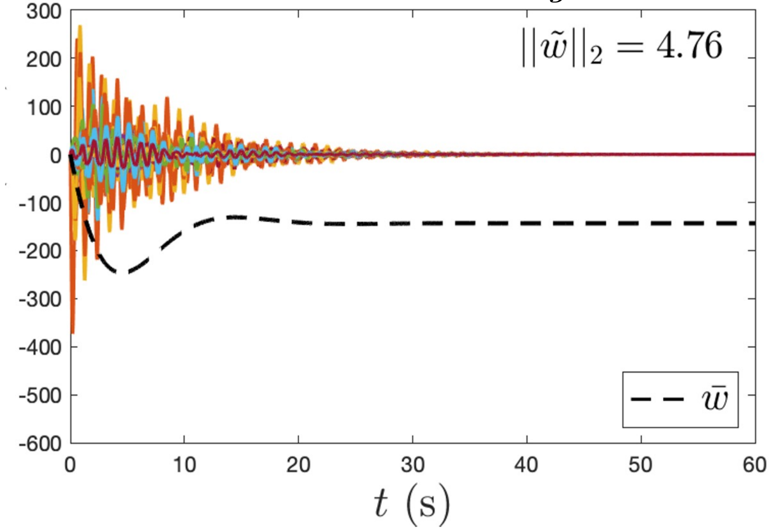

$d \times 2.5$

$\|\tilde{w}\|_2 = 2.96$

$m \times 2.5$



$r_g^{-1} \times 2.5$



Red dots: nominal values