Two-Stage Electricity Market Mechanism with Supply Function Bidding and Storage Degradation

Rajni Kant Bansal, Enrique Mallada, and Patricia Hidalgo-Gonzalez

***Abstract*—We propose novel market mechanisms for a two-stage, multi-interval electricity market that includes storage, generators, and uncertain net demand (defined as demand minus variable renewable generation). Drawing ideas from the supply function equilibrium, we introduce novel market mechanisms, where storage bids cycle depths in the day-ahead and charge-discharge power bids in the real-time market for last-minute adjustments. In the first mechanism, storage submits independent bids in each stage. The system operator clears the market sequentially based on these bids; however, the cumulative dispatch cost temporally couples the storage operator's decisions as they seek to maximize profit across both stages. Although these market mechanisms, under additional model assumptions, result in a unique competitive equilibrium, it may not be feasible for real-time market operations. As an alternative, we propose a *decision-aware* market mechanism, i.e., an independent bid in the day-ahead market and a day-ahead decision-aware bid in the real-time market. We demonstrate that incorporating day-ahead decisions - i.e., a decision-aware market mechanism - into the bidding function offers several advantages. Numerical experiments using New York ISO data show that the proposed mechanism can achieve savings of up to 68% compared to current market practices.**

***Index Terms*—Electricity market, equilibrium analysis, supply function bidding, two-stage settlement, storage degradation.**

I. Introduction

ENERGY storage systems, such as grid-scale lithium-ion batteries, are being considered across the grid for essential services. Recent regulation that allows market participation of emerging technologies, including energy storage, have further increased their adoption in electricity markets [1]–[3]. In particular, several works have investigated the benefits of energy storage for system reliability and power quality, e.g., complementing renewable energy resources, supporting transmission and distribution networks, etc. [4]–[6]. However, determining energy storage's marginal operation costs remains a key challenge in resource schedule and efficient market clearing. Unlike existing fuel-based generators, where marginal costs depend on energy supply, or variable renewable energy resources, with nearly zero operating costs, the degradation incurred due to charge-discharge cycles constitutes the bulk of the operation cost for energy storage [7]–[9].

Recent efforts to develop participation bids while accounting for the operation cost of storage can be broadly classified into two categories. The first approach relies on creating an optimal sequence of charge-discharge energy bids, i.e., price-quantity pairs [10]–[15]. For instance, the work in [10], [11] focuses on maximizing social welfare from a market perspective and propose bidding the fraction of charge-discharge power while revealing its true operating cost to the system operator. Meanwhile, [12] and [13] consider the perspective of individual resource owners to maximize their profit while either assuming or estimating unknown prices in markets. Alternatively, references [14] and [15] use a Bilevel optimization framework, with the energy storage owner as the leader maximizing its profit and the market operator as the follower maximizing social welfare, assuming truthful participation from other market players. However, the resulting market mechanisms based on energy bids require stringent conditions to align with the social optimum, do not consider the impact of decisions of other market players, and may result in incentive-misaligned market outcomes [16].

The second approach relies on developing a charge-discharge cycle depth or state of charge (SoC) based charge-discharge bids to account for SoC-dependent physical characteristics, e.g., storage degradation [17]–[19]. For instance, [17] proposes a SoC segment market model that allows storage to submit bids by SoC segments, while the market operator monitors SoC to update the bids during market clearing. As SoC-dependent bids may lead to non-convex formulations, [18], [19] introduce an alternative strategy using monotonic bids within a joint energy-reserve market clearing. However, these studies offer limited insight into how storage decisions affect the overall market equilibrium. To the best of our knowledge, our earlier work [16] is the first to propose an energy-cycling supply function based on charge-discharge cycle depths and to model the resulting market equilibrium due to the competition between market players. These formulations typically focus on a single stage and ignore how decisions made at one stage impact subsequent stages.

This paper explores mixed market mechanisms for energy storage participation alongside generators and uncertain net demand in a two-stage multi-interval market consisting of a day-ahead market and a real-time market. In this novel mechanism, each storage first submits an energy-cycling function to bid charge-discharge cycle depths as a function of per-cycle prices. Simultaneously, generators bid supply functions to dispatch power as a function of prices in the day-ahead market. The day-ahead market then clears based on the forecast for the next day. In contrast to the day-ahead market, in the real-time market both storage and generators bid power as a function of prices to adjust their day-ahead commitments. The real-time market clears sequentially and optimizes the dispatch decision with an updated forecast for the demand

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in a rolling time horizon window fashion that includes one binding and multiple advisory intervals. This two-stage market mechanism is implemented via a convex optimization problem that utilizes a quadratic cost for dispatching generators and a convex degradation cost function as a combination of the Rainflow cycle counting algorithm with a cycle stress function for storage operation [16], [20], [21].

We consider two participation strategies for storage. The first involves participants submitting independent supply functions in two stages. The system operator clears the market sequentially based on these bids; however, the cumulative dispatch cost temporally couples the storage operator's decisions as they seek to maximize profit across both stages. While this mechanism is guaranteed to have a competitive equilibrium, storage dispatch in the real-time market is locally constrained, requiring a customized iterative algorithm, which may not be feasible for real-time operations. To address this, we propose an alternative where storage incorporates its day-ahead decision in the bid function. Storage first bids an energy-cycling function for the day-ahead dispatch and then submits a real-time supply function that maps dispatch power to prices.

Contributions: The main contributions include:

1) *Uniform price market mechanism:* First, drawing ideas from the supply function equilibrium for generators in the existing market design, we propose a time-varying uniform price market mechanism for storage participation in the day-ahead market, i.e., a uniform per-cycle price for heterogeneous storage units. We then show that the resulting competitive equilibrium in the day-ahead market aligns with the underlying social planner problem, signaling an efficient market design.
2) *Day-Ahead unaware bidding:* Second, under the assumption of uniform prices in the market, we propose an independent participation strategy in two-stage markets, i.e., day-ahead commitment (decision) unaware bidding at the real-time stage. We then characterize the closed-form solution of the competitive equilibrium.
3) *Day-Ahead aware bidding:* Since the closed-form equilibrium analysis in the previous market design requires additional market model assumptions, we then propose an alternative market mechanism, i.e., a day-ahead decision-aware market participation. In this mechanism, each storage unit submits a supply function that reflects its day-ahead commitment in the real-time stage. Our equilibrium analysis shows that competitive equilibrium always exists, and it can be solved using off-the-shelf convex optimization solvers.
4) *Numerical study:* Lastly, we provide a case study for the proposed market mechanism and compare it with the social planner problem as we vary the physical characteristics of energy storage. Our analysis shows that the proposed mechanism operates within the bounds of the benchmark social planner solution and achieves up to 68% savings in the cycling costs.

Organization: The rest of the paper is organized as follows: Section II describes the market model and social planner problem, Section III introduces the market mechanisms for day-ahead and real-time markets, Section IV discusses the day-ahead-unaware market model and uniform price mechanism for the day-ahead market, Section V covers the alternative day-ahead-aware market model and a case study, and Section VI concludes the paper.

Notation: We use $||\cdot||_2^2$ to denote the Euclidean norm and $\langle\cdot,\cdot\rangle$ to denote the inner product. Also, $f(a;b)$ denotes a function of independent variable a with b as a parameter.

II. Market Model

A. Model Preliminaries

Consider a two-stage market consisting of a day-ahead and a real-time market, where a set $\mathcal{G}$ of generators and a set $\mathcal{S}$ of storage units participate to meet uncertain net demand (demand minus renewable generation) over a time horizon $t \in \mathcal{T} := \{1, ..., T\}$.

1) Day-Ahead market: The day-ahead market is cleared based on the forecast for the following day. Specifically, we formulate a two-day optimization horizon, as illustrated in Figure 1. Day 1 consists of T periods as the *binding* period, with the demand forecast denoted by $d^{d,bin} \in \mathbb{R}^T$. Day 2 also comprises of T periods as the *advisory* period, with the associated demand forecast denoted by $d^{d,adv} \in \mathbb{R}^T$.

The optimal decisions made during the binding period are implemented in the market, whereas the optimal solutions for the advisory period may be revised later. We denote the generator dispatch for $j \in \mathcal{G}$ by $(g_j^{d,bin}, g_j^{d,adv}) \in \mathbb{R}^T$ for the binding and advisory periods, respectively. The dispatch of generator j is subject to capacity constraints,

$$\underline{g}_{j,t}^d \le g_{j,t}^{d,bin} \le \overline{g}_{j,t}^d, \quad \underline{g}_{j,t}^d \le g_{j,t}^{d,adv} \le \overline{g}_{j,t}^d, \; t \in \{1,..,T\} \quad (1)$$

where $\underline{g}_{j,t}^d, \overline{g}_{j,t}^d$ denote the minimum and maximum generation limits, respectively. Similarly, the storage dispatch for $s \in \mathcal{S}$ – i.e., discharge (positive) or charge (negative) rate – is denoted by $(u_s^{d,bin}, u_s^{d,adv}) \in \mathbb{R}^T$, and is bounded as:

$$\underline{u}_{s,t}^d \le u_{s,t}^{d,bin} \le \overline{u}_{s,t}^d, \; \underline{u}_{s,t}^d \le u_{s,t}^{d,adv} \le \overline{u}_{s,t}^d, \; t \in \{1,..,T\}. \quad (2)$$

Here $\underline{u}_{s,t}^d, \overline{u}_{s,t}^d$ denote the minimum and maximum storage rate limits, respectively. Furthermore, the stored energy in storage unit s, represented by a normalized state-of-charge (SoC) profile, is given by $(x_s^{d,bin}, x_s^{d,adv}) \in \mathbb{R}^{T+1}$, and evolves over time horizon $\mathcal{T}$ as,

$$x_{s,t}^{d,bin} - x_{s,t-1}^{d,bin} = \frac{1}{E_s} u_{s,t}^{d,bin}, \; x_{s,t}^{d,adv} - x_{s,t-1}^{d,adv} = \frac{1}{E_s} u_{s,t}^{d,bin}. \quad (3)$$

Here $x_{s,0}^{d,bin}, x_{s,0}^{d,adv}$ denotes the initial SoC. The SoC evolution can be compactly expressed as

$$A x_s^{d,bin} = -\frac{1}{E_s} u_s^{d,bin}, \; A x_s^{d,adv} = -\frac{1}{E_s} u_s^{d,adv} \quad (4)$$

where

$$A = \begin{bmatrix} -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{T\times(T+1)}.$$

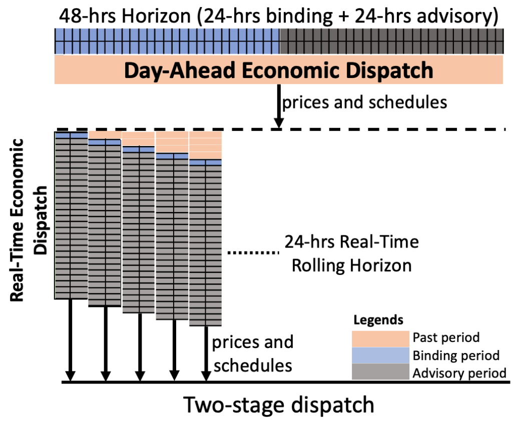

Fig. 1. Sequential decision making using rolling horizon framework.

Moreover, the normalized SoC of storage s is bounded as,

$$0 \le x_{s,t}^{d,bin} \le 1,\ 0 \le x_{s,t}^{d,adv} \le 1,\ t \in \{1,..,T\}. \tag{5}$$

Lastly, we assume periodicity constraints on storage to account for its cyclic nature in current markets, i.e.,

$$x_{s,0}^{d,bin} = x_{s,T}^{d,bin} = x_{s,0}^{d,adv} = x_{s,T}^{d,adv}. \tag{6}$$

Substituting (3) in (6), we get

$$\sum_{t\in\mathcal{T}} u_{s,t}^{d,bin} = 0,\ \sum_{t\in\mathcal{T}} u_{s,t}^{d,adv} = 0. \tag{7}$$

For simplicity, we define the combined demand forecast vector as $d^d := [d^{d,bin}; d^{d,adv}] \in \mathbb{R}^{2T}$. Similarly, we define the combined generator dispatch vector as $g_j^d := [g_j^{d,bin}; g_j^{d,adv}] \in \mathbb{R}^{2T}$, the combined storage dispatch vector as $u_s^d := [u_s^{d,bin}; u_s^{d,adv}] \in \mathbb{R}^{2T}$, and the combined SoC profile vector as $x_s^d := [x_s^{d,bin}; x_s^{d,adv}] \in \mathbb{R}^{2T+2}$

2) Real-Time market: The real-time market operates using a rolling T-period time horizon, with the real-time demand for the rolling window denoted as $d^r \in \mathbb{R}^T$. Specifically, the market clears sequentially based on the realized demand for the immediate period, denoted as $\hat{t}+1$, and an updated forecast for the remaining horizon, i.e., $\tau \in \{\hat{t}+2, ..., \hat{t}+T\}$, as illustrated in Figure 1. Without loss of generality, we assume that both markets operate on the same hourly timescale, although the analysis can be extended to other timescales. As the time window rolls forward, the operator receives a perfect forecast for the immediate period and updated forecasts for future advisory periods. The operator minimizes the dispatch cost, subject to operational constraints.

The dispatch of generator $j \in \mathcal{G}$ in rolling real-time window is denoted as $g_j^r \in \mathbb{R}^T$, subject to capacity constraints,

$$\underline{g}_{j,\tau}^r \le g_{j,\tau}^r \le \overline{g}_{j,\tau}^r,\ \tau \in \{\hat{t}+1, ..., \hat{t}+T\} \tag{8}$$

where $\underline{g}_{j,\tau}^r, \overline{g}_{j,\tau}^r$ denote the minimum and maximum generation limits, respectively. Similarly, the dispatch of storage $s \in \mathcal{S}$ in the real-time market is denoted as $u_s^r \in \mathbb{R}^T$, bounded as,

$$\underline{u}_{s,\tau}^r \le u_{s,\tau}^r \le \overline{u}_{s,\tau}^r,\ \tau \in \{\hat{t}+1, ..., \hat{t}+T\}. \tag{9}$$

Here $\underline{u}_{s,\tau}^r, \overline{u}_{s,\tau}^r$ denote the minimum and maximum storage rate limits, respectively.

We define the net two-stage demand $d \in \mathbb{R}^T$ over the rolling horizon $\tau \in \{\hat{t}+1, ..., \hat{t}+T\}$ as:

$$d_\tau := d_\tau^d + d_\tau^r. \tag{10}$$

Similarly, the net output g_j of generator j and u_s of storage s over the rolling horizon $\tau \in \{\hat{t}+1, ..., \hat{t}+T\}$, is:

$$g_{j,\tau} := g_{j,\tau}^d + g_{j,\tau}^r \tag{11a}$$

$$u_{s,\tau} := u_{s,\tau}^d + u_{s,\tau}^r \tag{11b}$$

and are subject to capacity constraints as,

$$\underline{g}_{j,\tau} \le g_{j,\tau} \le \overline{g}_{j,\tau},\ \tau \in \{\hat{t}+1, ..., \hat{t}+T\} \tag{12a}$$

$$\underline{u}_{s,\tau} \le u_{s,\tau} \le \overline{u}_{s,\tau},\ \tau \in \{\hat{t}+1, ..., \hat{t}+T\}. \tag{12b}$$

Here the pairs $(\overline{g}_{j,\tau}, \underline{g}_{j,\tau}), (\overline{u}_{s,\tau}, \underline{u}_{s,\tau})$ represents the maximum and minimum capacity limits for generation j and storage s, respectively. The net SoC profile over the rolling horizon $\tau \in \{\hat{t}+1, ..., \hat{t}+T\}$, as denoted by $x_s \in \mathbb{R}^{T+1}$ with initial SoC $x_{s,\tau-1}$ corresponding to the net storage dispatch u_s, follows:

$$Ax_s = -\frac{1}{E_s} u_s, \tag{13}$$

and is bounded as

$$0 \le x_{s,\tau} \le 1, \tau \in \{\hat{t}+1, ..., \hat{t}+T\}. \tag{14}$$

Lastly, the SoC profile is subject to periodicity constraints as,

$$\sum\nolimits_{\tau=\hat{t}+1}^{\hat{t}+T} u_{s,\tau} = 0, \tag{15}$$

Since the real-time market is dispatched sequentially to accommodate for last-minute adjustments owing to forecast errors, we relax the periodicity constraint (15) to allow storage for any such immediate adjustments.

B. Social Planner

The social planner's problem that aims to minimize the cost of net supply-demand balance, assuming perfect foresight for net demand $d_\tau, \tau \in \{1,..,T\}$, is given by

$$\min_{g_j, j\in\mathcal{G}, u_s, s\in\mathcal{S}, x_s, s\in\mathcal{S}} \sum_{j\in\mathcal{G}} C_j(g_j) + \sum_{s\in\mathcal{G}} C_s(u_s) \tag{16a}$$

$$\text{s.t.} \sum_{\tau=1}^{T}\sum_{j\in\mathcal{G}} g_{j,\tau} + \sum_{\tau=1}^{T}\sum_{s\in\mathcal{S}} u_{s,\tau} = d_\tau \tag{16b}$$

$$(12a), (12b), (13), (14), (15)$$

where (16b) denotes the power balance constraint over two stages. In this paper, we assume a quadratic cost function for the generators, given by

$$C_j(g_j) = \frac{c_j}{2} ||g_j||_2^2 + a_j \langle \mathbf{1}, g_j \rangle \tag{17}$$

where $\mathbf{1}$ represents a vector of all ones, i.e., each element as 1, and c_j, a_j are the cost coefficients[1]. For storage s, we adopt the convex cycle-based degradation cost as its operational cost. It combines the Rainflow cycle counting algorithm with a cycle stress function to identify and penalize the cost of

[1]For ease of analysis, we assume that $a_j = 0$. However, the analysis is generalizable for the case $a_j \neq 0$.

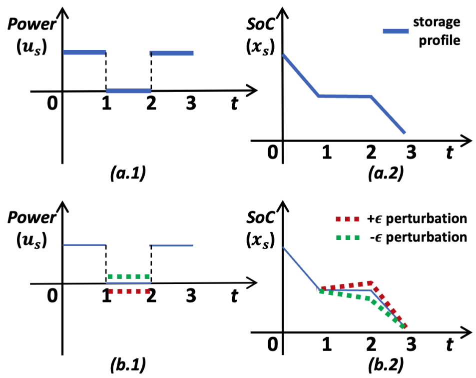

Fig. 2. An example storage (a.1) dispatch and associated (a.2) SoC profile. $\epsilon-$perturbed storage (b.1) dispatch and associated (b.2) SoC profile.

charge-discharge cycles [16]. In particular, the Rainflow cycle algorithm first identifies the vector of temporally coupled charge-discharge cycles $\nu_s \in \mathbb{R}^T$ associated with the net SoC profile u_s of storage s, i.e.,

$$\nu_s := \textit{Rainflow}(u_s) = N(u_s)u_s. \tag{18}$$

Here the matrix $N(u_s)$ is a function of the storage dispatch u_s and represents the non-smooth piece-wise linear map between the storage dispatch profile and half-cycle depths. More specifically, the matrix $N(u_s)$ is defined as [16]

$$N(u_s) = -\frac{1}{E}M(x_s)^T A^\dagger \tag{19}$$

where the matrix $M(x_s) \in \mathbb{R}^{T\times T}$ is the incidence matrix associated with a directed graph for the SoC profile x_s [20]. Here $A^\dagger$ represents the Moore–Penrose generalized inverse [22] of the matrix A, defined in the equation (4). We next illustrate this N-matrix and the underlying non-smooth map with a toy example SoC profile, see, e.g., [16] for more details.

Example 1. *The Rainflow counting algorithm iteratively compares the consecutive SoC points to extract the charge-discharge half-cycles [20]. For the example storage dispatch and SoC profile, as shown in the panels $(a.1)$ and $(a.2)$ in Figure 2, respectively, there is only one half-cycle of depth $(x_{s,0} - x_{s,3})$ and no full cycle exists. The associated matrix $N(u_s)$ is given by,*

$$N(u_s) = \frac{1}{E_s}\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \; s.t.\; \nu_s = N(u_s)u_s = \frac{\sum_{t=1}^{3} u_{s,t}}{E_s}.$$

The cycle stress function $\Phi(\cdot) : [0,1]^T \mapsto [0,1]$, often approximated as a quadratic penalty function [21], quantifies the normalized degradation associated with cycle depths ν_s:

$$\Phi(\nu_s) := \frac{\rho_s}{2}||\nu_s||_2^2 \tag{20}$$

where ρ_s is the quadratic cost coefficient. Therefore, the cost function is given by

$$C_s(u_s) = \frac{\rho_s B_s E_s}{2}||\nu_s||_2^2 = \frac{b_s}{2}||N(u_s)u_s||_2^2 \tag{21}$$

where $b_s := \rho_s B_s E_s$ is the cost coefficient. Here B_s denotes the storage capital cost such that $B_s E_s$ represents the replacement cost of storage. Furthermore, the piece-wise linear map results in a non-differentiable cost function, as illustrated for the example in Figure 2 below.

Remark 1. *The piecewise linear map between the storage dispatch and half-cycles results in a piecewise differentiable cost function, i.e., at points of non-differentiability, $\exists$ m possible matrices $N_k(u_s), k \in \{1, ..., m\}$, associated with a storage profile u_s, such that the following relation holds:*

$$\nu_s = N_k(u_s)u_s = N(u_s)u_s, \forall k \in \{1, ..., m\}.$$

We illustrate non-differentiability with a toy example below.

Example 2. *The example profile in Figure 2 is an interesting boundary case. If perturbed, i.e., $x_s \pm \epsilon$ for any $\epsilon \to 0^+$ shown in the panels $(b.1)$ and $(b.2)$ in Figure 2, respectively, leads to different associated N matrices. The storage profile with $+\epsilon$ perturbation is characteristically different from the profile in panel $(a.2)$ in Figure 2, i.e., the Rainflow algorithm extracts one full cycle. However, the profile with $-\epsilon$ perturbation is characteristically same with no full cycles. In particular, the N matrices are:*

$$N(u_s+\epsilon) = \frac{1}{E_s}\begin{bmatrix}1 & 1 & 1\\ 0 & -1 & 0\\ 0 & -1 & 0\end{bmatrix}, N(u_s-\epsilon) = \frac{1}{E_s}\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

but the depth vector is the same, i.e.,

$$\nu_s = N(u_s)u_s = N(u_s+\epsilon)u_s = N(u_s-\epsilon)u_s.$$

III. Two-Stage Market Mechanism

In this section, we describe the two-stage market clearing. We assume participants bid supply functions that map dispatch to marginal price, reflecting their willingness to participate. In practice, a resource submits a set of prices and generation quantities as a step function, which may result in a non-convex formulation. Therefore, smooth supply functions are often used in the literature for equilibrium analysis and solution tractability [23]–[25]. The resulting formulation, albeit optimistic, provides meaningful insights into the competition and behavior of the participants.

A. Day-Ahead Market

For ease of exposition, we focus on the binding interval of the day-ahead market. The advisory interval on Day 2 is cleared similarly in the day-ahead market. Each generator j submits a supply function parameterized by $\alpha_j^d \in \mathbb{R}$ in day-ahead market as,

$$g_j^d = \alpha_j^d \lambda^d \tag{22}$$

where $\lambda^d \in \mathbb{R}^T$ denotes the clearing prices in the day ahead. Analogously, each storage s submits an energy cycling function parameterized by $\beta_s^d \in \mathbb{R}$ that maps cycle depths $\nu_s^d \in \mathbb{R}^T$ to per cycle prices $\theta_s^d \in \mathbb{R}^T$, as

$$\nu_s^d = \beta_s^d \theta_s^d. \tag{23}$$

The market operator collects all the bids and associates a cost function with generator j as,

$$\sum_{t\in\mathcal{T}} \int_0^{g_{j,t}^d} \lambda_t^d \partial g_{j,t}^d = \sum_{t\in\mathcal{T}} \int_0^{g_{j,t}^d} \frac{1}{\alpha_j^d} g_{j,t}^d \partial g_{j,t}^d = \frac{\left(g_{j,t}^d\right)^2}{2\alpha_j^d} \tag{24}$$

and with storage s as,

$$\sum_{t\in\mathcal{T}}\int_0^{\nu^d_{s,t}}\theta^d_{s,t}\partial\nu^d_{s,t}=\sum_{t\in\mathcal{T}}\int_0^{\nu^d_{s,t}}\frac{1}{\beta^d_s}\nu^d_{s,t}\partial\nu^d_{s,t}=\frac{\left(\nu^d_{s,t}\right)^2}{2\beta^d_s}. \tag{25}$$

Given the bids $(\alpha^d_j, j\in\mathcal{G}, \beta^d_s, s\in\mathcal{S})$, the operator solves the day-ahead market clearing problem that minimizes the cost of supply-demand balance in the day-ahead market, given by:

$$\min_{g^d_j, j\in\mathcal{G}, (u^d_s,\nu^d_s), s\in\mathcal{S}} \sum_{j\in\mathcal{G}}\sum_{t\in\mathcal{T}}\frac{(g^d_{j,t})^2}{2\alpha^d_j}+\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\frac{(\nu^d_{s,t})^2}{2\beta^d_s} \tag{26a}$$

$$\text{s.t.} \sum_{j\in\mathcal{G}} g^d_j + \sum_{s\in\mathcal{S}} u^d_s = d^d \tag{26b}$$

$$\nu^d_s = N(u^d_s)u^d_s \tag{26c}$$

$$(1), (2), (5), (7)$$

where (26c) denotes the constraint associated with the Rainflow algorithm. The market clearing gives the optimal dispatch and prices, such that generator j dispatches g_j and gets paid $\langle\lambda^d, g^d_j\rangle$ while storage s produces a cycle depth schedule ν^d_s and gets paid $\langle\theta^d_s, \nu^d_s\rangle$. Here the prices λ^d, θ^d_s are given by the dual variables corresponding to the constraints (26b) and (26c), respectively.

In the day-ahead market, participants are competing against each other to maximize their profit. We assume participants as price-takers and the individual problems of generator j, is

$$\max_{g^d_j} \langle\lambda^d, g^d_j\rangle - C_j(g^d_j), \quad \text{s.t. } (1), (22). \tag{27}$$

Similarly, the individual problem of storage s, is

$$\max_{(u^d_s,\nu^d_s)} \langle\theta^d_s, \nu^d_s\rangle - C_s(u^d_s) \quad \text{s.t. } (2), (23). \tag{28}$$

Since the participants are price-takers, we assume that they do not anticipate their decisions in the real-time market.

B. Real-Time Market

In the real-time market, as the window rolls forward, we have the participants' decision $(g^d_{j,\tau}, u^d_{s,\tau}), \tau\in\{\hat{t}+1,...,\hat{t}+T\}$ from the day-ahead market. Analogous to the day-ahead stage, each generator j submits a supply function $f:\mathbb{R}\times\mathbb{R}^T\to\mathbb{R}^T$, parameterized with $\alpha^r_j\in\mathbb{R}$,

$$g^r_j = f(\lambda^r;\alpha^r_j) \tag{29}$$

where $\lambda^r\in\mathbb{R}^T$ denotes the clearing prices in the real-time. However, unlike storage in day-ahead markets, in the real-time market storage s bids a supply function $h:\mathbb{R}\times\mathbb{R}^T\to\mathbb{R}^T$, parameterized by $\beta^r_s\in\mathbb{R}$, as

$$u^r_s = h(\lambda^r;\beta^r_s). \tag{30}$$

Similarly, in the real-time, the market operator associates a cost function with generator j as,

$$C_f(g^r_j;\alpha^r_j):=\sum_{t\in\mathcal{T}}\int_0^{g^r_{j,t}}\lambda^r_t\partial g^r_{j,t}=\sum_{t\in\mathcal{T}}\int_0^{g^r_{j,t}}f^{-1}(g^r_j;\alpha^r_j)\partial g^r_{j,t} \tag{31}$$

and with storage s as,

$$C_h(u^r_s;\beta^r_j):=\sum_{t\in\mathcal{T}}\int_0^{u^r_{s,t}}\lambda^r_{s,t}\partial u^r_{s,t}=\sum_{t\in\mathcal{T}}\int_0^{u^r_{s,t}}h^{-1}(u^r_{s,t};\beta^r_s)\partial u^r_{s,t} \tag{32}$$

Here $f^{-1}(\cdot)$ and $h^{-1}(\cdot)$ represents the unique inverse functions, respectively. Given the bids $(\alpha^r_j, j\in\mathcal{G}, \beta^r_s, s\in\mathcal{S})$, the operator clears the real-time market and meets the supply-demand balance for each time period $t\in\mathcal{T}$, as given by:

$$\min_{g^r_j, j\in\mathcal{G}, u^r_s, s\in\mathcal{S}} \sum_{j\in\mathcal{G}} C_f(g^r_j;\alpha^r_j)+\sum_{s\in\mathcal{S}} C_h(u^r_s;\beta^r_s) \tag{33a}$$

$$\text{s.t.} \sum_{j\in\mathcal{G}} g^r_j + \sum_{s\in\mathcal{S}} u^r_s = d^r \tag{33b}$$

$$(8), (9), (12), (14), (15).$$

Given the prices in the real-time market and the participant's decision $g^d_{j,\tau},\ \tau\in\{\hat{t}+1,...,\hat{t}+T\}$ in the day-ahead market, the individual problem of generator j is given by,

$$\max_{g^r_j} \langle\lambda^d, g^d_j\rangle + \langle\lambda^r, g^r_j\rangle - C_j(g^d_j+g^r_j) \quad \text{s.t. } (12\text{a}), (29). \tag{34}$$

Similarly, given the prices in the real-time market and the participant's decision $u^d_{s,\tau},\ \tau\in\{\hat{t}+1,...,\hat{t}+T\}$ in the day-ahead market, the individual problem of storage s is

$$\max_{u^r_s} \langle\theta^d_s, \nu^d_s\rangle + \langle\lambda^r, u^r_s\rangle - C_s(u^d_s+u^r_s) \quad \text{s.t. } (12\text{b}), (30). \tag{35}$$

We refer to this market mechanism as a *mixed market mechanism* (MM), where storage submits cycle depth in the day-ahead market and is paid based on per-cycle prices. In the real-time market, storage submits conventional charge-discharge power bids and is paid according to energy prices.

C. Market Equilibrium

In this subsection, we characterize the properties of competitive equilibrium in a two-stage market, such that the market clears and participants do not deviate from their bid.

Definition 1. *Each stage of a two-stage market is at the competitive equilibrium if the participant bids and the clearing prices, i.e.,* $(\lambda^d, \theta^d_s, \alpha^d_j, \beta^d_s)$ *and* $(\lambda^r, \alpha^r_j, \beta^r_s)$*, in day-ahead and real-time markets, respectively, satisfy:*

1) *The bid* $\alpha^d_j(\alpha^r_j)$ *of generator* j *in the day-ahead (real-time) market maximizes its profit, given by equations* (28) *and* (35)*, respectively.*
2) *The bid* $\beta^d_s(\beta^r_s)$ *of storage* s *in the day-ahead (real-time) market maximizes its profit, given by equations* (27) *and* (34)*, respectively.*
3) *The inelastic demand* d^d, d^r *in the day-ahead and the real-time market is met, resulting in clearing prices* λ^d, θ^d_s *and* λ^r*, respectively.*

IV. Day-Ahead Decision Unaware Market Model

In this section, we investigate whether the mixed market mechanism, where storage submits a cycle-depth bid in the day ahead and a day-ahead-unaware bid in real-time, leads to a competitive equilibrium. For ease of exposition, we consider a simplified setting in this section where we only consider constraints (7), (15), (26b), (26c), and (33b) in the market clearing. However, our results generalize beyond this assumption, at the cost of a more involved analysis. We first provide a proposition that will enable us to define a time-varying uniform price market mechanism in the day-ahead market, which generates uniform prices for the participating heterogeneous storage units. It will also allow us to characterize the competitive equilibrium of the mixed market mechanism.

Proposition 1. *For any participants' bids* (α_j^d, β_s^d)*, where* $\sum\limits_{j\in\mathcal{G}}\alpha_j^d \neq 0$, $\sum\limits_{s\in\mathcal{S}}\beta_s^d \neq 0$*, there exists a unique set of proportionality coefficients*

$$\epsilon_s = \frac{\beta_s^d}{\sum_{i\in\mathcal{S}}\beta_i^d},\ s\in\mathcal{S}$$

such that day-ahead market clearing (26) *results in a unique set of uniform per-cycle prices, i.e.,* $\theta_s^{d*} := \theta^{d*}, \forall s\in\mathcal{S}$ *and the optimal storage dispatch is proportional to the energy surplus (defined as net demand minus total generation).*

Proof. We begin by deriving the KKT conditions for the economic dispatch problem (26). Next, we introduce a new set of primal-dual variables that satisfy these conditions. To demonstrate that such a primal-dual solution exists uniquely, we formulate an underlying convex optimization problem. Finally, we prove that the proposed mechanism results in uniform prices.

Step 1: For the dispatch problem (26), denote the the dual variables associated with constraints (7), (26b), and (26c) as $\delta_s^d, s\in\mathcal{S}$, λ^d, and $\theta_s^d, s\in\mathcal{S}$, respectively. The KKT conditions are given by,

$$d^d = \sum_{j\in\mathcal{G}} g_j^{d*} + \sum_{s\in\mathcal{S}} u_s^{d*},\ g_j^{d*} = \alpha_j^d\lambda^{d*} \tag{36a}$$

$$\nu_s^{d*} = N(u_s^{d*})u_s^{d*},\ \nu_s^{d*} = \beta_s^d\theta_s^{d*} \tag{36b}$$

$$\lambda^{d*} = \sum_k \gamma_k N_k(u_s^{d*})^T N_k(u_s^{d*})u_s^{d*} + \delta_s^{d*}\mathbf{1} \tag{36c}$$

$$\mathbf{1}^T u_s^{d*} = 0. \tag{36d}$$

where γ_k are convex coefficients associated with $k\in\mathcal{K}$ possible subgradients of the piecewise linear convex cost of storage cycling (21), see, e.g., Remark 1, for more details. However, the per-cycle prices θ_s^{d*} are not uniform.

Step 2: Now, we assume a proportional energy storage dispatch of the form

$$\hat{u}_s^d = \epsilon_s(d^d - \textstyle\sum_{j\in\mathcal{G}}\hat{g}_j^d),\ \text{where}\ \textstyle\sum_{s\in\mathcal{S}}\epsilon_s = 1. \tag{37}$$

Let's denote $\hat{u}^d := (d^d - \sum_{j\in\mathcal{G}}\hat{g}_j^d)$ such that $\hat{u}_s^d = \epsilon_s\hat{u}^d$. Further, the dual variables can then be defined in terms of primal variables, as

$$\hat{\lambda}^d = {\alpha_j^d}^{-1}\hat{g}_j^d \tag{38a}$$

$$\hat{\nu}_s^d = N(\hat{u}_s^d)\hat{u}_s^d \implies \hat{\theta}_s^d = {\beta_s^d}^{-1}\hat{\nu}_s^d. \tag{38b}$$

Then, for any participants' bid (α_j^d, β_s^d), $\exists$ solution $(\hat{u}^d, \hat{g}_j^d, j\in\mathcal{G}, \hat{\lambda}^d, \hat{\theta}_s^d, s\in\mathcal{S})$ that satisfies the KKT conditions (36) where $\hat{\delta}_s^d\in\mathbb{R}$. Therefore, it is locally optimal solution of the dispatch problem (26). Also, using the [16, Lemma 1] we can write

$$N(\hat{u}_s^d) = N(\epsilon_s\hat{u}^d) = N(\hat{u}^d).$$

Step 3: We next show that such a solution is globally optimal. Rewriting the KKT conditions (36), we get

$$\frac{d - \hat{u}^{d*}}{\sum_{j\in\mathcal{G}}\alpha_j^d} = \sum_k \gamma_k \frac{\epsilon_s}{\beta_s^{d*}} N_k(u^{d*})^T N_k(u^{d*})u^{d*} + \delta^{d*}\mathbf{1} \tag{39a}$$

$$\mathbf{1}^T u^{d*} = 0 \tag{39b}$$

where $\delta_s^{d*} := \delta^{d*}\ \forall s\in\mathcal{S}$. We next claim that the necessary conditions (39) is basically the KKT conditions of the convex optimization below, given by

$$\min_{u^d} \frac{||u^d||_2^2}{2\sum_{j\in\mathcal{G}}\alpha_j^d} - \frac{\langle d^d, u^d\rangle}{\sum_{j\in\mathcal{G}}\alpha_j^d} + \frac{||N(u^d)u^d||_2^2}{\sum_{s\in\mathcal{S}}\beta_s^d} \tag{40a}$$

$$\text{s.t. } \mathbf{1}^T u^d = 0 \tag{40b}$$

where u^{d*} is the optimal primal variable and δ^{d*} is the optimal dual variable associated with the constraint (40b). Using the fact that the storage cost function (21) is convex, we can observe that the piece-wise quadratic objective (40a) is convex. Further, the constraint (40b) is affine and linear constraint qualifications are satisfied.

Step 4: Therefore, the market clearing generates the uniform per-cycle prices, given by:

$$\hat{\theta}_s^{d*} = \frac{1}{\beta_s^d}N(\hat{u}_s^{d*})\hat{u}_s^{d*} = \frac{\epsilon_s}{\beta_s^{d*}}N(\hat{u}^{d*})\hat{u}^{d*} = \frac{1}{\sum\limits_{s\in\mathcal{S}}\beta_s^{d*}}N(\hat{u}^{d*})\hat{u}^{d*}$$

Hence, for the bids (α_j^d, β_s^d), $\exists$ a unique primal-dual solution $(u^d)^*, \delta^*$ to the convex optimization problem (40) which satisfies the KKT conditions (39). This completes the proof. □

Similar to generators in the existing market, the proposition provides a mechanism for market operators to assign a uniform price to storage units, accounting for their operational costs, such as degradation costs. Additionally, in the uniform price market mechanism, participants reveal their true cost functions, enabling the day-ahead market to clear efficiently. The proof relies on the fact that, under the price-taking assumption, storage units treat market prices as given and are incentivized to disclose their true costs at the competitive equilibrium. For a similar result in a non-uniform price market mechanism, please refer to the work in [16].

A. Day-Ahead Decision Unaware Real-Time Model

In this subsection, we discuss the day-ahead decision unaware participation strategy in the real-time market and characterize the competitive equilibrium. Each generator j bids

$$g_j^r = \alpha_j^r\lambda^r \tag{41}$$

and storage s bids

$$u_s^r = \beta_s^r\lambda^r \tag{42}$$

in the real-time market. Substituting (41) in (34), we get

$$\max_{\alpha_j^r} \langle\lambda^d, g_j^d\rangle + \alpha_j^r||\lambda^r||_2^2 - \frac{c_j}{2}||g_j^d + \alpha_j^r\lambda^r||_2^2. \tag{43}$$

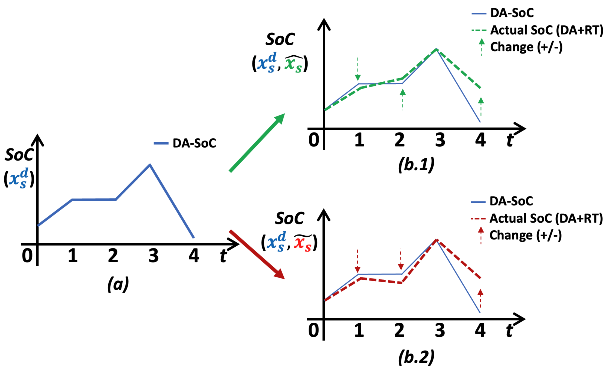

Fig. 3. An example storage (a) day-ahead SoC (DA-SoC) profile and actual two-stage SoC (day-ahead (DA) + real-time (RT)) profile in (b.1) scenarios 1 - $(x_s^d, \hat{x}_s)$ and (b.2) scenario 2 - $(x_s^d, \tilde{x}_s)$.

Similarly, substituting (42) in (35), we get

$$\max_{\beta_s^r} \langle \theta_s^d, \nu_s^d \rangle + \beta_s^r ||\lambda^r||_2^2 - \frac{b_s}{2} ||N(u_s^d + \beta_s^r \lambda^r)(u_s^d + \beta_s^r \lambda^r)||_2^2. \quad (44)$$

To account for the temporally coupled cost of storage degradation while making decisions sequentially in the real-time market, we assume that storage deviations are constrained in the neighborhood of its day-ahead decision such that

$$N(u_s^d + u_s^r) = N(u_s^d). \quad (45)$$

We next illustrate the constraint in equation (45) with a toy example SoC profile.

Example 3. *For the example day-ahead SoC profile, as shown in panel (a) in Figure 3, the associated N-matrix is given by,*

$$N(u_s^d) = \frac{1}{E_s} \begin{bmatrix} -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ s.t.\ \nu_s^d = \begin{bmatrix} x_{s,3}^d - x_{s,0}^d \\ x_{s,3}^d - x_{s,4}^d \\ 0 \\ 0 \end{bmatrix}.$$

Now, the net SoC profile $\hat{x}_s := (x_s^d + \hat{x}_s^r)$, with adjustments in the day-ahead commitments, in panel $(b.1)$ is characteristically similar to the day-ahead profile x_s^d, i.e., the N-matrix remains the same. However, the SoC profile $\tilde{x}_s := (x_s^d + \tilde{x}_s^r)$ in panel $(b.2)$ leads to a different N-matrix, as

$$N(\tilde{u}_s) = \frac{1}{E_s} \begin{bmatrix} -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \neq N(u_s^d)$$

where $\hat{u}_s, \tilde{u}_s$ denotes the storage dispatch profile associated with the SoC profile $\hat{x}_s, \tilde{x}_s$, respectively.

The assumption in equation (45) allows storage to participate in the market within a simplified setting, e.g., existing supply function, with the flexibility to change the optimal bid to account for the degradation cost. However, real-time market dispatch must constrain the storage to maintain the piece-wise linear map between cycle depths and SoC profile. Without such an assumption, the closed-form solution of the competitive equilibrium cannot be obtained analytically. Under a uniform price market mechanism, we characterize the competitive equilibrium in the following Theorem.

Theorem 1. *Suppose* (45) *holds. The competitive equilibrium in the real-time market, given the participants' decision in the day-ahead market, exists uniquely. Precisely, there exists a unique coefficient ω such that the equilibrium is given by:*

$$\lambda_r = \omega d^r \quad (46a)$$

$$\alpha_j^r = c_j^{-1} - \frac{< g_j^d, \lambda^r >}{||\lambda^r||_2^2} \quad (46b)$$

$$\beta_s^r = b_s^{-1} \frac{||\lambda^r||_2^2 - \langle \theta_s^d, N(u_s^d)\lambda^r \rangle}{||N(u_s^d)\lambda^r||_2^2} \quad (46c)$$

$$\omega = \frac{<\lambda^d, d^r>}{||d^r||_2^2} + \left(\sum_{j\in\mathcal{G}} c_j^{-1} + \sum_{s\in\mathcal{S}} b_s^{-1} \frac{||d^r||_2^2}{||\sum_k \gamma_k N_k(u_s^d) d^r||_2^2} \right)^{-1} \quad (46d)$$

where $\gamma_k \geq 0, \sum_k \gamma_k = 1$ are the convex coefficients associated with the subgradients of the piecewise linear convex cost function (21)*, see e.g., Remark 1 for more details.*

Proof. Given the day-ahead decisions and prices in the real-time market, we can solve for the optimal bid by taking the derivative of the convex optimization problem (43), as

$$||\lambda^r||_2^2 - c_j(g_j^d + \alpha_j^r \lambda^r)^T \lambda^r = 0 \implies \alpha_j^r = c_j^{-1} - \frac{\langle g_j^d, \lambda^r \rangle}{||\lambda^r||_2^2}. \quad (47a)$$

Similarly, taking the derivative of (44), we get:

$$||\lambda^r||_2^2 - b_s \left(\sum_k \gamma_k N_k(u_s^d)(u_s^d + \beta_s^r \lambda^r) \right)^T \left(\sum_k \gamma_k N_k(u_s^d) \right) \lambda^r = 0 \quad (48)$$

where we use $N(u_s^d + u_s^r) = N(u_s^d)$. Here γ_k are the convex coefficients associated with the piecewise linear cost of storage dispatch, see e.g., Remark 1. Solving (27) and (28) for optimal bids in the day-ahead market, we have

$$\alpha_s^{d*} = c_j^{-1}, \beta_s^{d*} = b_s^{-1}.$$

Substituting (18) and (23) in (48), we get

$$\implies ||\lambda^r||_2^2 - b_s \left(\nu_s^d + \beta_s^r \tilde{N}(u_s^d)\lambda^r \right)^T \left(\tilde{N}(u_s^d) \right) \lambda^r = 0 \quad (49a)$$

$$\implies ||\lambda^r||_2^2 - \left(\theta_s^d + b_s \beta_s^r \tilde{N}(u_s^d)\lambda^r \right)^T \left(\tilde{N}(u_s^d) \right) \lambda^r = 0 \quad (49b)$$

$$\implies \beta_s^r = b_s^{-1} \frac{||\lambda^r||_2^2 - \langle \theta_s^d, \tilde{N}(u_s^d)\lambda^r \rangle}{||\tilde{N}(u_s^d)\lambda^r||_2^2} \quad (49c)$$

where $\tilde{N}(u_s^d) := \sum_k \gamma_k N_k(u_s^d)$. At the equilibrium, (33b), (47a), and (49c) must hold simultaneously. Since λ^r is proportional to d^r, let's assume that $\exists\, \omega \in \mathbb{R}$ such that $\lambda^r = \omega d^r$. Substituting (22) in (47a), as

$$\alpha_j^r = c_j^{-1} \left(1 - \frac{< \lambda^d, \lambda^r >}{||\lambda^r||_2^2} \right) = c_j^{-1} \left(1 - \frac{1}{\omega} \frac{< \lambda^d, d^r >}{||d^r||_2^2} \right) \quad (50a)$$

$$\implies \sum_{j\in\mathcal{G}} \alpha_j^r = \sum_{j\in\mathcal{G}} c_j^{-1} \left(1 - \frac{1}{\omega} \frac{< \lambda^d, d^r >}{||d^r||_2^2} \right). \quad (50b)$$

Similarly, substituting (36c) in (49c) and simplifying as

$$\beta_s^r = b_s^{-1} \frac{||\lambda^r||_2^2 - \langle \lambda^d, \lambda^r \rangle}{||\sum_k \gamma_k N_k(u_s^d)\lambda^r||_2^2} = b_s^{-1} \frac{||d^r||_2^2 - \omega^{-1} \langle \lambda^d, d^r \rangle}{||\sum_k \gamma_k N_k(u_s^d) d^r||_2^2}. \quad (51)$$

Substituting (50b) in (51), as

$$\implies \beta_s^r = b_s^{-1} \frac{\sum_{j\in\mathcal{G}} \alpha_j}{\sum_{j\in\mathcal{G}} c_j^{-1}} \frac{||d^r||_2^2}{||\sum_k \gamma_k N_k(u_s^d) d^r||_2^2} \tag{52a}$$

$$\implies \sum_{s\in\mathcal{S}} \beta_s^r = \sum_{s\in\mathcal{S}} b_s^{-1} \frac{\sum\limits_{j\in\mathcal{G}} \alpha_j}{\sum\limits_{j\in\mathcal{G}} c_j^{-1}} \frac{||d^r||_2^2}{||\sum_k \gamma_k N_k(u_s^d) d^r||_2^2} \tag{52b}$$

where we use the fact that energy dispatch is proportional and $N(u_s^d) = N(u^d)$ in the uniform price market mechanism in equation (52b). Substituting (50b) and (52b) in (33), we get

$$\omega = \frac{<\lambda^d, d^r>}{||d^r||_2^2} + \left(\sum_{j\in\mathcal{G}} c_j^{-1} + \sum_{s\in\mathcal{S}} b_s^{-1} \frac{||d^r||_2^2}{||\sum_k \gamma_k N_k(u_s^d) d^r||_2^2} \right)^{-1}.$$

Hence, under the price-taking assumption the optimal bid and clearing prices $(\alpha_j^r, j \in \mathcal{G}, \beta_s^r, s \in \mathcal{S}, \lambda^r)$ exist uniquely. □

Although a unique competitive equilibrium exists, solving it requires a customized iterative algorithm, where bids and clearing prices are repeatedly updated until convergence.Additionally, the algorithm involves a subroutine to update the constraint set to ensure that the assumption in equation (45) holds, see an example SoC profile in Figure 3. To address these challenges, we propose a modified bidding approach in the following subsection that accounts for the day-ahead decisions into the bidding function.

V. Day-Ahead Decision Aware Market Model

In this section, we propose an alternative mixed market mechanism, characterize the competitive equilibrium, and conduct a numerical study.

A. Day-Ahead Decision Aware Real-Time Model

In the day-ahead-aware mechanism, participants account for their day-ahead decisions or commitments in the bidding function itself. In particular, given the day-ahead decisions, each generator j bids

$$g_j^r = \alpha_j^r \lambda^r - g_j^d \tag{53}$$

and storage s bids

$$u_s^r = \beta_s^r \lambda^r - u_s^d \tag{54}$$

in the real-time market. Substituting (53) in (34), the individual profit maximization problem of generator j, is:

$$\max_{\alpha_j^r} \; \alpha_j^r ||\lambda^r||_2^2 + \langle \lambda^r, g_j^d \rangle - \frac{c_j}{2} ||\alpha_j^r \lambda^r||_2^2 \tag{55}$$

and substituting (54) in (35), we get the individual profit maximization problem of storage s, as:

$$\max_{\beta_s^r} \; \beta_s^r ||\lambda^r||_2^2 + \langle \lambda^r, u_s^d \rangle - \frac{b_s}{2} ||N(\beta_s^r \lambda^r)(\beta_s^r \lambda^r)||_2^2. \tag{56}$$

Note that we drop the terms associated with the day-ahead revenue in (55) and (56), as it does not affect the objective in the individual problem. Once all the bids (α_j^r, β_s^r) are collected, the market operator clears the real-time market to achieve supply-demand balance, given by

$$d^r = \sum_{j\in\mathcal{G}} \left(\alpha_j^r \lambda^r - g_j^d \right) + \sum_{s\in\mathcal{S}} \left(\beta_s^r \lambda^r - u_s^d \right). \tag{57}$$

Theorem 2. *The competitive equilibrium of the mixed market mechanism with day-ahead decision-aware bids in the real-time market exists uniquely, as*

$$\lambda^r = \phi d, \tag{58a}$$

$$\alpha_j^r = c_j^{-1}, \tag{58b}$$

$$\beta_s^r = b_s^{-1} \frac{||\lambda^r||_2^2}{||N(\lambda^r)\lambda||_2^2}, \tag{58c}$$

$$\phi^{-1} = \left(\sum_{s\in\mathcal{S}} b_s^{-1} \frac{||d||_2^2}{||N(d)d||_2^2} + \sum_{j\in\mathcal{G}} c_j^{-1} \right). \tag{58d}$$

Proof. Writing the derivative of (55) for the optimal bid of each generator j, we get

$$||\lambda^r||_2^2 (1 - \alpha_j^r c_j) = 0 \implies \alpha_j^r = c_j^{-1}, \; \forall j \in \mathcal{G}. \tag{59}$$

Similarly, we take the derivative of (56) for storage, as

$$||\lambda^r||_2^2 - b_s \beta_s^r ||N(\lambda^r)\lambda^r||_2^2 = 0 \implies \beta_s^r = b_s^{-1} \frac{||\lambda^r||_2^2}{||N(\lambda^r)\lambda^r||_2^2} \tag{60}$$

where we use [16, Lemma 1] in (60). At the equilibrium (57), (59), and (60) must hold simultaneously. Since λ^r is proportional to d, let's assume $\exists \; \phi \in \mathbb{R}$ such that $\lambda^r = \phi d$. Substituting (26b), (59), (60) in (57), we can solve for ϕ as

$$\phi^{-1} = \left(\sum_{s\in\mathcal{S}} b_s^{-1} \frac{||d||_2^2}{||N(d)d||_2^2} + \sum_{j\in\mathcal{G}} c_j{}^{-1} \right). \tag{61}$$

Hence the competitive equilibrium exists uniquely. □

We note that the competitive equilibrium in Theorem 2 always exists, unlike the mixed market mechanism in subsection IV-A. Additionally, the resulting market equilibrium with constant optimal participant bids can be solved fast enough for the needs of the real-time market using convex optimization.

B. Case Study

In this subsection, we provide a numerical case study that analyses the mixed market mechanism based on a day-ahead-aware bidding model with one generator and one storage unit. For the day-ahead market, we formulate a two-day optimization horizon consisting of Day 1 with 24 time periods as the *binding* period and Day 2 with 24 periods as the *advisory* period. In the real-time market, we use a rolling time horizon window of 24 h that has the 1st hour as the *binding* period and the rest as *advisory* periods. As the window rolls forward, the operator realizes the perfect forecast for the immediate period and an updated forecast for the advisory periods, as illustrated in Figure 1. We use forecast and real aggregate demand data for Aug 25-26, 2023, from the Millwood Zone in the New York ISO [26]. Furthermore, we assume one generator with aggregate cost coefficients $c = 0.28\$/(MW)^2$ [27] and capacity limits $\underline{g} = 0, \; \overline{g} = \max_t\{d_t\}$. Also, we assume one energy storage asset with fixed capacity cost B and energy capacity E. The empirical cost coefficient given by $b_s := \rho BE$ where $\rho = 5.24 \times 10^{-4}$ [20], [21]. We assume a 4-hour Li-ion battery such that the dispatch is bounded by $\underline{u} = -\frac{E}{4}$ and $\overline{u} = \frac{E}{4}$.

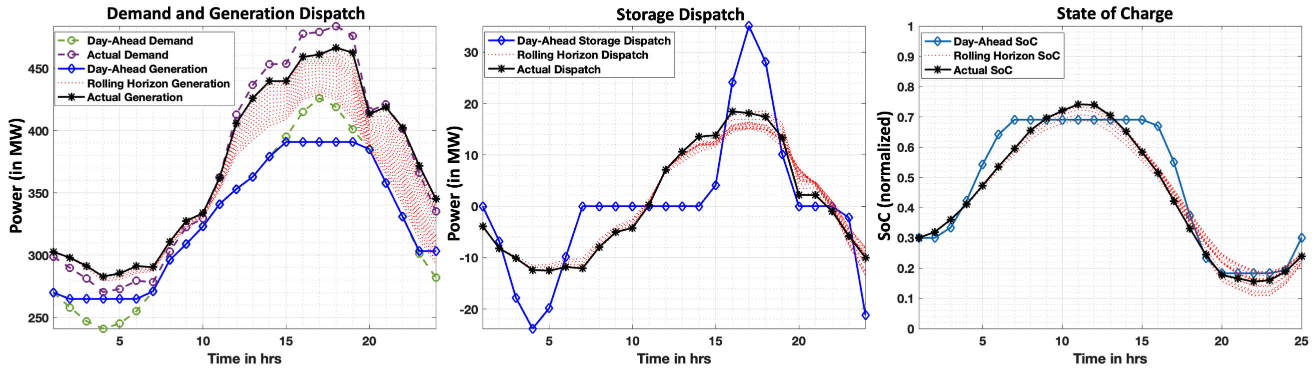

Fig. 4. Day-Ahead, Rolling Window, and Two-Stage: (a) demand and generator dispatch, (b) storage dispatch, and (c) state of charge with respect to storage capacity (E) of 200 MWh and capital cost (B) of 150 \$/kWh.

It is important to note that in real-time markets, the absence of a periodicity constraint may lead to periodicity constraint violations in cases of total two-stage dispatch. We use SoC targets from the day-ahead market, i.e., $x_{s,\tau} \geq x^d_{s,\tau}$ for the advisory intervals to mimic the periodicity constraint. The operator only considers the optimal real-time dispatch for the first interval of the horizon as binding as the horizon window moves forward. Therefore, we use the underlying social planner problem (16) with and without periodicity constraints as a benchmark to compare the performance of the proposed mixed market mechanism from different perspectives.

1) Market Dispatch and Degradation Cost of Storage: We first consider the individual resource perspective. Figure 4 illustrates the day-ahead, rolling window, and two-stage dispatch for the fixed storage capacity of $E = 200MWh$ and storage capital cost of $B = 150\$/kWh$. We plot the demand and generator dispatch, storage dispatch, and state of charge in panels (a)-(c), respectively. In this case study, the generator accounts for the majority of positive error in the demand forecast while storage participates in the intra-day arbitrage, as shown in the panel (a) in Figure 4. Although the sequential decision making framework leads to varying optimal bids for each time period, a quadratic cost as a function of dispatch power in real-time resulting in short-term strategy to meet the positive error in demand forecast, as shown in the panels (b) and (c) in Figure 4, respectively.

2) Comparison with Underlying Social Planner: In this subsection, we benchmark the performance of the proposed mixed market mechanism w.r.t the underlying social planner problem. Since the proposed mechanism may not satisfy the periodicity constraints due to deviation in the real-time markets, we relax the periodicity constraint in the underlying social planner problem, i.e., $\sum_{t\in\mathcal{T}} u_{s,t} = \pm\epsilon$, where ϵ is obtained from the simulation of the mixed market mechanism to ensure that the comparison is calibrated. Figure 5 illustrates the actual social cost (top panels) and storage profit (bottom panels) for the proposed mixed market mechanism and the underlying social planner problem, as we vary the storage capital cost and storage capacity. As expected, the social cost for both mechanisms increases with the storage capital cost, as shown in panel (a) for a fixed storage capacity of $E = 200MWh$, due to the expensive storage units. However, the social cost decreases for both mechanisms with an increase in storage capacity, as shown in panel (b) for fixed storage capital cost of $B = 150\$/kWh$. This is because storage can dispatch the required power with shallower cycle depths, leading to reduced capacity degradation. A similar trend can be observed for the storage profit, as shown in panels (c) and (d), respectively. The rolling horizon framework and a quadratic cost of operation in real-time that emphasizes on immediate revenue potential results in a reduction of net storage profit in the proposed market mechanism as compared to the underlying social planner problem with perfect foresight. Interestingly, the performance of the proposed mechanism remains within a narrow gap of 0.1%, as illustrated by the gap between the black and blue solid lines in Figure 5. However, this gap tends to widen as storage capacity increases, as shown in panel (b) of Figure 5.

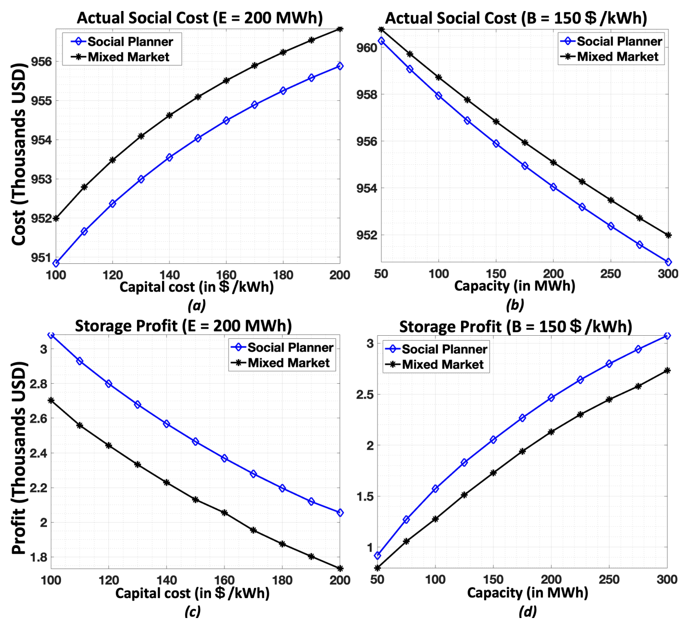

Fig. 5. Comparison of social planner and proposed mixed market mechanism - social cost for (a) fixed capacity (top-left panel) and (b) fixed capital cost (top-right panel), storage profit for (c) fixed capacity (bottom-left panel), and (b) storage profit for fixed capital cost (bottom-right panel)

3) Comparison with Existing Market Approach: We next compare the proposed mixed market mechanism with traditional generation-centric dispatch (GCD) strategies, which only consider generation dispatch costs and ignore any costs related to storage. This approach leads to frequent and deeper charge and discharge cycles, resulting in significant degrada-

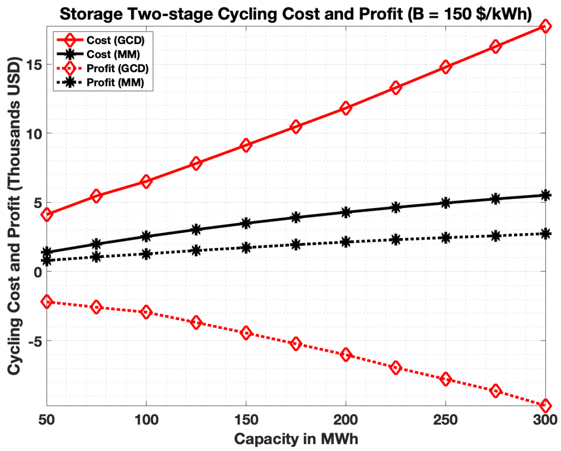

Fig. 6. Net two-stage (day-ahead + real-time) storage cycling cost and storage profit in the proposed mixed market mechanism (MM) and existing generation centric dispatch (GCD) w.r.t storage capacity.

tion losses for resource owners. We then assess these losses, i.e., we compute the resulting cycling cost for the optimal dispatch for the storage capital cost of $B = 150\$/kWh$, and adjust the storage profits to account for losses not included in the market. Figure 6 illustrates the net two-stage storage cycling costs and profits for the proposed market mechanism compared to the existing generation-centric strategy. As expected, cycling costs are much higher for the generation-centric strategy and increase significantly with increase in its capacity. The mixed market mechanism results in approximately 68% savings in cycling costs relative to the generation-centric strategy. Furthermore, the net two-stage profit for the proposed mechanism is significantly higher than the adjusted profits in the existing approach. The existing approach leads to substantial and increasing losses as storage capacity grows. Such losses result in out-of-market settlements that, in turn, distort market signals and may cause further efficiency losses.

VI. Conclusions

We propose a mixed market mechanism that accounts for storage degradation in the participation of energy storage. Specifically, each storage unit submits charge-discharge cycle bids in the day-ahead market, followed by charge-discharge power bids in the real-time market. Under the first approach, participants submit independent bids in both stages, while the second approach involves reflecting on their day-ahead decisions in the bidding functions themselves. Although both market mechanisms result in a unique competitive equilibrium, the first approach requires an iterative best response algorithm, which may not be desirable in real-time markets due to frequent market clearing. On the other hand, the second approach can be implemented using convex programming. Numerical simulations with real-world NYISO data show up to 68% cycling cost savings versus existing market designs that ignore storage operation costs.

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