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Mixed Supply Function and Quantity Bidding in Two-Stage Settlement Markets

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Motivated by electricity markets, we study the incentives of heterogeneous participants (firms and consumers) in a two-stage settlement market with a mixed bidding mechanism, in which firms participate using supply function bids and consumers use quantity bids. We carry out an equilibrium analysis of the market outcome and obtain closed-form solutions. The characterization of the equilibria allows us to gain insights into the market-power implications of mixed bidding and uncover the importance of accounting for consumers' strategic behavior in a two-stage market, even when their demand is completely inelastic with respect to price. We show that strategic consumers are able to exploit firms' strategic behavior to maintain a systematic difference between the forward and spot prices, with the latter being higher. Notably, such a strategy does bring down consumer payment and undermines the supply-side market power. However, it is only effective when firms are behaving strategically. We also observe situations where firms lose profit by behaving strategically, a sign of overturn of the conventional supply-side market power. Our results further suggest that market competition has a heterogeneous impact across consumer sizes, particularly benefiting small consumers. Our analysis can accommodate other market policies, and we demonstrate this versatility by examining the impact of some example policies, including virtual bidding, on the market outcome.

Key words: two-stage settlement, forward market, spot market, supply function bidding, Cournot competition, electricity market, extensive-form game, equilibrium analysis

1. Introduction

Market operations under two-stage settlement, commonly composed of a forward market followed by a spot market, have increasingly become the norm since deregulation of the electricity sector in the United States by the Energy Policy Act of 1992. Indeed, the risk-hedging nature of forward contracting offers slow-response generation units, with ramp constraints and long startup time, the necessary protection against the fast-changing prices of the spot market. As a result, the majority of electricity is traded in the forward market, e.g., in North America and Europe (Imran and Kockar 2014). Deregulation introduces competition and aims to bring down unnecessary costs. However, the promise of deregulation to drive efficient investment and operation of the electricity sector has yet to be fully delivered, despite two decades' practice. For instance, Borenstein et al. (1995) and Evans (2014) present evidence of price manipulation (market power) on both the supply and demand side of the clearing process after deregulation. Moreover, the report by Zummo (2018) suggests that retail electricity prices are systematically higher in deregulated states in the US. Our study aims to understand and identify the intrinsic sources of incentive misalignment in two-stage settlement market designs with mixed bidding.

We consider a model where a finite number of firms, i.e., producers, bid to meet an infinitely divisible yet inelastic demand for a commodity from a finite number of consumers over a two-stage settlement market. We focus on a uniform pricing mechanism for market clearing of each stage that sets a single per-unit price for the commodity based on participants' bids. Such a mechanism is prevalent in a wide variety of marketplaces beyond electricity trading (Kahn et al. 2001), e.g., those for government bonds (Malvey and Archibald 1998), and initial public offerings (Bennouri and Falconieri 2008). By accounting for the incentives of both firms and consumers, expressed via respective profit and negative payment maximization problems, we investigate the distinct roles that each type of participants play in such a two-stage market competition, paying special attention to the opportunities for price manipulation by consumers with inelastic demand.

A large body of work has investigated key aspects of market design that have direct impacts on market outcomes, including forward contracting, participant incentives, and bidding mechanisms. Forward contracting is known for enabling market power mitigation. This fact was first uncovered in the seminal work by

Allaz and Vila (1993), where a market with elastic demand becomes more competitive because the incentive to sell forward encourages all firms to produce more at equilibrium. In the meantime, distinct incentives of participants lead to different market participation strategies. For instance, fixed costs tend to drive firms to make binary on/off decisions (Yang et al. 2014), while variable costs may yield smoother price-driven production for cost recovery (Guo et al. 2021). The impact of bidding mechanisms, which aim to elicit truthful participant's behavior, has also been considered. The classical Cournot (Allaz and Vila 1993) and Bertrand (Spulber 1995) approaches investigate competitions that are based on quantity and price bids, respectively. In addition, supply function bidding has been increasingly studied (Holmberg and Newbery 2010, Baldick et al. 2004, Anderson and Philpott 2002), since it can more accurately reflect variable production costs.

However, little attention has been paid to the two-stage structure of the clearing process enabling consumers, especially with inelastic demand, to behave strategically. Such flexibility that the market conveys to consumers leads to an inter-group competition that has not been considered. Our study of the two-stage market competition among strategic firms and consumers aims to bridge this gap in the literature. We demonstrate situations where consumers are able to exercise market power over firms – a generally unlikely observation in the presence of inelastic demand. Our suite of results show that even within this simple setting the interplay between these two groups of participants leads to drastically different market outcomes and highlights the importance of a holistic view for market analysis where all participants' incentives are considered simultaneously. This work serves as a stepping stone to untangle complex market interactions and gain insights into the market-power implications of two-stage settlement designs.

1.1. Contributions

We analyze the effect of a mixed bidding mechanism on the equilibrium outcome of a two-stage market. In particular, the market competition is modeled as an extensive-form game among *homogeneous* firms and *heterogeneous* consumers, with perfect foresight and complete information. Such an approach allows tractable analysis while preserving the inter-group interplay as well as the intra-group interplay among consumers. Firms are assumed to have quadratic costs and their actions are modeled through supply function bidding in each stage; i.e., they report a schedule for each stage. Each consumer is assumed to have a fixed demand that is inelastic with respect to price, though the decision on how to allocate it across stages is price sensitive. This assumption reflects the typical case of a utility company participating in US electricity markets. Our first contribution is to derive a closed-form characterization of the *unique* (partially symmetric) Nash equilibrium in the setting where firms are homogeneous and make identical bids but consumers are heterogeneous. This analytical result further enables the evaluation of market outcomes in terms of surplus allocation. The identification of such a closed-form equilibrium is predicated on the assumption of homogeneity among firms, which, though unrealistic, offers several insights into the inter-group market power that is present between supply and demand.

Our main results point to the importance of accounting for the strategic behavior of consumers, even when their demand is completely inelastic with respect to price. First and foremost, the equilibrium analysis suggests that consumers' ability to allocate demand across stages is only valuable in the presence of strategic firms. If all firms are price-takers, only an equilibrium with equal two-stage prices is attainable, leaving consumers with no room for price manipulation. On the contrary, when firms are behaving strategically, their bidding strategy can be exploited by consumers to create a systematic two-stage price difference by allocating less than the actual demand in the forward market to lower the clearing price. Note that this goes against the no-arbitrage condition commonly assumed in the literature, e.g., Allaz and Vila (1993), Murphy and Smeers (2010), Cai et al. (2020). Such bidding behavior enables consumers to undermine the conventional supply-side market power and enjoy a reduction in payment. Our closed-form equilibrium characterization further allows us to identify the specific circumstances under which the strategic consumer behavior can overturn the supply-side market power and reduce firms' profit below that achieved at the competitive equilibrium. We also present comparative statics regarding how the equilibrium outcomes change as the number of participants on each side grows, and as the cost parameters change, which offer qualitative insights into the shift of market power. Our results further show that the market rules have a heterogeneous impact across consumer sizes, particularly benefiting small consumers.

Our analysis provides a means to evaluate different market policies. As examples, we look at policies that respectively target the supply side (uniform supply function bidding in Appendix I), the demand side (virtual bidding in Section 5), or both (spot-market transaction charge in Appendix F). The results suggest that all three policies tend to constrain the flexibility of consumers in allocating demand across stages, which contributes to the restoration of supply-side market power. As a whole, our study uncovers a new way in which forward markets can mitigate supply-side market power if one accounts for consumers' strategic behavior, even under the extreme condition of inelastic demand.

1.2. Related Literature

We use equilibrium analysis to investigate the interplay of strategic firms and consumers under a mixed bidding mechanism over a two-stage market, which is arguably the simplest yet still informative form that features cross-stage market competition among participants with heterogeneous incentives and different bidding mechanisms. In this subsection we review the relevant literature on these aspects and explain how our work complements existing studies.

Forward Contracting: Forward contracting mainly targets issues of uncertainty risk and market power, as pointed out in Ausubel and Cramton (2010). The rationale of forward contracting to hedge against risk of uncertainty is straightforward, by allowing participants to lock in prices and quantities so as to limit exposure to the more volatile spot market. The impact of uncertainty in forecast on market equilibria is explicitly explored in Tang et al. (2016), Mather et al. (2017), leading to the intuition that improved forecast accuracy alleviates the loss of efficiency.

The role of forward contracting on market power mitigation, even with perfect foresight, has also been extensively studied. In their seminal work, Allaz and Vila (1993) identified the possibility of mitigating the market power of firms with forward positions in the presence of elastic demand. This discovery has inspired follow-up studies that have led to the reaffirmation or invalidation of this effect under different assumptions, e.g., Gans et al. (1998), Newbery (1998), Green (1999). These works along with others have led to a general consensus that forward contracting often mitigates market power, yet counter-example cases do exist, e.g., Green (1999), Murphy and Smeers (2010), Cai et al. (2020). This observation is further corroborated in the context of electricity markets where more practical factors need to be accounted for in market models. For example, Kamat and Oren (2004) and Yao et al. (2007, 2008) deal with the additional complexity of network congestion and price caps, but limit their analysis to numerical simulations. In the presence of binding production capacity, market power of firms can be either enhanced or mitigated, as analytically verified in Murphy and Smeers (2010) and Cai et al. (2020).

Compared to the above studies, our work is distinctive in that it captures how forward contracting can be taken advantage of by strategic supply and demand sides simultaneously. Such an interplay yields an interesting finding that forward contracting enables consumers with *inelastic* demand to mitigate the conventional supply-side market power, which has not been revealed in the literature to the best of our knowledge. Further, the strategy in which consumers exploit their flexibility of allocating demand across stages maintains a gap between the two-stage prices at equilibrium. This strikingly goes against the *no-arbitrage condition* that is commonly assumed in the extant literature.

Bidding Mechanisms and Participant Incentives: The study of the effect of the bidding mechanisms on market outcomes has a very long history. Here we review the most common bidding mechanisms specifically for market participation of supply and demand.

The characterization of supply-side competition has always been the center of attention. Quantity bidding in Cournot competition, e.g., Allaz and Vila (1993), Cai et al. (2020), and price bidding in Bertrand competition, e.g., Mahenc and Salanié (2004), Liski and Montero (2006), are two classical forms of bidding mechanisms that are favored in different settings and indeed have distinct impacts on market outcomes. For instance, Mahenc and Salanié (2004) shows that price bidding in the presence of forward contracting increases equilibrium clearing prices, as opposed to the common role of forward markets in mitigating market power when firms bid quantity. In addition, supply function bidding for firms is gaining increasing popularity since it allows better adaptation to changing market conditions and requires less communication to control private information revelation, as discussed in Klemperer and Meyer (1989). Game-theoretic supply function equilibrium has also been broadly studied due to its implications in wholesale auctions, e.g., Holmberg and Newbery (2010), Vives (2011), Johari and Tsitsiklis (2011), Ruddell et al. (2017).

Historically there has been less study of the demand side, though the literature is rapidly growing due to the prevailing intelligence in demand-side management, especially in modern smart grids. In a lot of works, consumers, specified with utility or cost functions, are assumed to participate in markets using parameterized function bids, e.g., Li et al. (2015), Xu et al. (2015). Allowing quantity and price based bids from consumers is less commonly observed, and in most situations serves as a symmetric counterpart for the supply side, e.g., Weber and Overbye (1999), Song et al. (2002).

Our work complements the existing studies by providing an analysis of the two-stage interplay between strategic firms and consumers with inelastic demand under a mixed bidding mechanism. Even under this simple inter-group market competition, our work highlights the importance of accounting for the strategic behavior of consumers, despite having completely inelastic demand. That is, unlike one-stage mechanisms, two-stage settlement allows inelastic consumers to compete against firms. Further, our analysis not only allows closed-form characterization of market equilibria, but also offers a means to evaluate the explicit impact of many potential market policies.

2. Market Model

In this section, we introduce our market model. We start with a detailed description of the market mechanism. We then discuss participants' incentive structures, and end the section with the game model for the two-stage competition.

2.1. Market Mechanism

Consider a two-stage settlement market consisting of a forward market and a subsequent spot market, where a set $\mathcal{G}$ of firms and a set $\mathcal{L}$ of consumers participate to trade a certain commodity. These participants make individual bids into the two-stage market, where both stages are cleared based on these bids with guaranteed balance between supply and demand.

Bidding: We consider a model where firms are price-sensitive and allowed to bid a supply function for each stage to reflect their varying marginal costs, while the total demand of consumers is inelastic with respect price. We focus on the linear form for a supply function, which provides a robust analytic tool and has been widely-used to gain insights into Nash equilibrium properties of electricity markets (Green 1996, Baldick et al. 2004, Rashedi and Kebriaei 2014, Delbono and Lambertini 2015). Despite fixed demand, consumers have the flexibility of distributing demand across stages in this two-stage setting. As a result, they are allowed to make bids of quantity, indicating the split demand amount required from each stage.

For each firm $j \in \mathcal{G}$, we denote its two-stage supply function bids as

$$q_j^f(\lambda^f) = \beta_j^f \lambda^f \ , \tag{1a}$$

$$q_j^s(\lambda^s) = \beta_j^s \lambda^s \ , \tag{1b}$$

where q_j^f and q_j^s represent its supply in the forward and spot markets, respectively, while λ^f and λ^s denote the corresponding market prices. The linear supply functions are parameterized by non-negative scalars β_j^f and β_j^s to indicate firm j's price-incentivized production in the two-stage market. The larger these parameters, the larger the quantity firm j is willing to produce at those prices. To concentrate on the effect of a firm's cost-driven supply function bidding and facilitate concise closed-form analysis, we ignore its capacity limit. We remark, however, that capacity limits do have an important impact on two-stage market outcomes as pointed out in Cai et al. (2020).

For each consumer $l \in \mathcal{L}$, we denote its inelastic demand as d_l, which needs to be fulfilled from the two-stage market in aggregate. Let its demand allocation between the forward and spot markets be d_l^f and d_l^s, respectively, which we refer to as quantity bids from consumer l, subject to

$$d_l^f + d_l^s = d_l \ . \tag{2}$$

Clearing: Based on these bids from firms and consumers, the forward and spot markets clear sequentially their corresponding supply and demand, i.e.,

$$\sum_{j \in \mathcal{G}} q_j^f = \sum_{l \in \mathcal{L}} d_l^f \ , \tag{3a}$$

$$\sum_{j \in \mathcal{G}} q_j^s = \sum_{l \in \mathcal{L}} d_l^s \ , \tag{3b}$$

and yield clearing prices

$$\lambda^f = \frac{\sum_{l \in \mathcal{L}} d_l^f}{\sum_{j \in \mathcal{G}} \beta_j^f} \ , \tag{4a}$$

$$\lambda^s = \frac{\sum_{l\in\mathcal{L}} d_l^s}{\sum_{j\in\mathcal{G}} \beta_j^s} \ , \tag{4b}$$

by respecting firms' supply functions and substituting (1) into (3), if the denominators are nonzero. We discuss the degenerate cases with either denominator being zero and specify the corresponding pricing rules in Appendix A.

Settlement: In the forward (resp. spot) market with the clearing price λ^f (resp. λ^s), firm j is dispatched to supply quantity q_j^f (resp. q_j^s) and collects $\lambda^f q_j^f$ (resp. $\lambda^s q_j^s$) in revenue, while consumer l is dispatched to consume quantity d_l^f (resp. d_l^s) and pays $\lambda^f d_l^f$ (resp. $\lambda^s d_l^s$). In the settlement, the total money paid by consumers equals the total money collected by firms, which is guaranteed by the clearing mechanism.

For notational convenience, let $\bar{\beta}^f := \sum_{j\in\mathcal{G}} \beta_j^f$ and $\bar{\beta}^s := \sum_{j\in\mathcal{G}} \beta_j^s$ be the respective sum of firms' bids in the forward and spot markets. We further define $\bar{\beta}_{-j}^f := \sum_{k\in\mathcal{G}\setminus\{j\}} \beta_k^f$ and $\bar{\beta}_{-j}^s := \sum_{k\in\mathcal{G}\setminus\{j\}} \beta_k^s$. Similarly, we also define on the consumer side $\bar{d}^f := \sum_{l\in\mathcal{L}} d_l^f$, $\bar{d}^s := \sum_{l\in\mathcal{L}} d_l^s$, $\bar{d}_{-l}^f := \sum_{k\in\mathcal{L}\setminus\{l\}} d_k^f$ and $\bar{d}_{-l}^s := \sum_{k\in\mathcal{L}\setminus\{l\}} d_k^s$. $|\mathcal{G}| =: G$ and $|\mathcal{L}| =: L$ are the numbers of firms and consumers, respectively.

2.2. Participant Incentives

We next explicitly model the incentives of individual market participants and characterize their bidding behavior. To insulate the fundamental market interactions from the plethora of other factors that appear in the presence of uncertainty, we assume perfect foresight in decision making for every participant.

A profit-maximizing firm $j \in \mathcal{G}$ is paid $\lambda^f q_j^f$ and $\lambda^s q_j^s$ for supplying quantities q_j^f and q_j^s in the forward and spot markets, respectively, and incurs a quadratic cost $\frac{1}{2}c_j\left(q_j^f + q_j^s\right)^2$ with respect to the total dispatched production. The profit of firm j, denoted as π_j, is given by

$$\pi_j := \lambda^f q_j^f + \lambda^s q_j^s - \frac{c_j}{2}\left(q_j^f + q_j^s\right)^2 \ , \tag{5}$$

where the first and second terms represent its revenue streams from the forward and spot markets, respectively.

A consumer $l \in \mathcal{L}$ seeks to minimize the amount paid subject to satisfying its demand. Its total payment in the two-stage market, denoted as ρ_l, is

$$\rho_l := \lambda^f d_l^f + \lambda^s d_l^s \ . \tag{6}$$

Before proceeding to characterize the bidding behavior of each participant, we define two types of participants that differ in their ability to strategize, or account for how their bids impact prices. Though these notions are standard, they are formally stated for completeness. The first type is price-takers defined below.

Definition 1 (Price-Taker) *A market participant is a price-taker if it treats two-stage prices as given when deciding its bids.*

In other words, a price-taker does not anticipate that its bidding decision will affect market prices. In the case of a firm $j \in \mathcal{G}$, we can formulate its bidding problem as follows.

Bidding problem for price-taking firm j

$$\max_{\beta_j^f \geq 0, \beta_j^s \geq 0} \ \pi_j(\beta_j^f, \beta_j^s; \lambda^f, \lambda^s) = {\lambda^f}^2 \beta_j^f + {\lambda^s}^2 \beta_j^s - \frac{c_j}{2}\left(\lambda^f \beta_j^f + \lambda^s \beta_j^s\right)^2 \tag{7}$$

where we have plugged in the supply function bids (1). More importantly, the special structure of the quadratic program (7) implies that its closed-form solution is given by

$$\begin{cases} \beta_j^f = \dfrac{1}{c_j}\ , \ \ \beta_j^s = 0\ , \quad \text{if } \lambda^f > \lambda^s\ ; \\ \beta_j^f + \beta_j^s = \dfrac{1}{c_j}\ , \ \ \beta_j^f \geq 0, \beta_j^s \geq 0\ , \quad \text{if } \lambda^f = \lambda^s\ ; \\ \beta_j^f = 0\ , \ \ \beta_j^s = \dfrac{1}{c_j}\ , \quad \text{if } \lambda^f < \lambda^s\ . \end{cases} \tag{8}$$

This solution illustrates how firms favor the stage with a higher price and are willing to produce any quantity of commodities with the marginal production cost below or equal to the price. Note that the profit π_j of firm j could take on a different form in degenerate cases. However, that form of profit is never optimal for price-taking firm j, as we will discuss in Appendix A, and is thus ignored here.

Similarly, we can formulate the bidding problem of a consumer $l \in \mathcal{L}$ as follows.

Bidding problem for price-taking consumer l

$$\min_{d_l^f, d_l^s} \ \rho_l(d_l^f, d_l^s; \lambda^f, \lambda^s) = \lambda^f d_l^f + \lambda^s d_l^s \tag{9a}$$

$$\text{s.t.} \ \ (2) \tag{9b}$$

(9) is a simple linear program with a straightforward solution

$$\begin{cases} d_l^f = -\epsilon\,, \;\; d_l^s = \epsilon + d_l\,, \quad \text{if } \lambda^f > \lambda^s\,; \\ d_l^f + d_l^s = d_l\,, \quad \text{if } \lambda^f = \lambda^s\,; \\ d_l^f = \epsilon + d_l\,, \;\; d_l^s = -\epsilon\,, \quad \text{if } \lambda^f < \lambda^s\,; \end{cases} \tag{10}$$

with $\epsilon \to \infty$. Intuitively, a consumer favors the stage with a lower price. In the absence of bid caps, consumers will have the incentive to infinitely arbitrage over any two-stage price difference.

The second type of participants are price anticipators, a.k.a. *strategic participants*. We explicitly define them below in the context of two-stage settlement.

Definition 2 (Strategic Participant) *A market participant is strategic if it treats other participants' bids within the same stage as given when deciding its bids. More specifically, a strategic participant anticipates the impact of its bidding decision on the clearing price in each market, and also anticipates the impact of its bidding decision in the forward market on the subsequent spot market outcome.*

Under Definition 2, a strategic participant bids in a way that exploits the market clearing and pricing laws and maximizes its own benefit. If a firm j is strategic, its two-stage bids are determined sequentially and this process is formulated below as a nested bidding problem.

Bidding problem for strategic firm j

$$\max_{\beta_j^f \geq 0:(4\text{a})} \quad \pi_j(\beta_j^f, \beta_j^s; \bar{\beta}_{-j}^f, \bar{d}^f, \bar{\beta}_{-j}^s, \bar{d}^s) \tag{11a}$$

$$\text{s.t.} \quad \beta_j^s = \arg \max_{\beta_j^s \geq 0:(4\text{b})} \pi_j(\beta_j^s; \bar{\beta}^f, \bar{d}^f, \bar{\beta}_{-j}^s, \bar{d}^s) \tag{11b}$$

where the right-hand side of (11b) outputs the (unique) optimal bid β_j^s in the subsequent spot market given a choice of β_j^f. (11a) and (11b) form a nested structure where the spot market bidding depends on the forward market bidding and is fully accounted for by the latter. Current market pricing is anticipated through (4).

Unlike a strategic firm, due to inelasticity, a consumer l only has one shot to determine its demand allocation across the two stages in the forward market, even if it is strategic. The resulting formulation is given as follows.

Bidding problem for strategic consumer l

$$\min_{d_l^f, d_l^s} \quad \rho_l(d_l^f, d_l^s; \bar{\beta}^f, \bar{d}_{-l}^f, \bar{\beta}^s, \bar{d}_{-l}^s) \tag{12a}$$

$$\text{s.t.} \quad (2), (4) \tag{12b}$$

where consumer l anticipates market pricing through (4), subject to its fixed demand requirement (2).

2.3. Extensive-Form Game

Given the above two types of participants (price-taking vs. strategic), we are ready to model their interaction over the two-stage market. The case where all firms and consumers are price-takers is taken as a benchmark. On top of this, we consider the cases with only strategic consumers, only strategic firms, and both to parse their separate impact on market outcomes. When participants are strategic, we observe the nested decision structure that renders the competition an extensive-form game (Ritzberger et al. 2016). More specifically, we account for the two-stage temporal sequence by modeling the spot market competition as a subgame of the forward market competition.

To shed light on the particular role consumers play in the market, we will assume *all the strategic firms are homogeneous* in the sense that they share the same cost function. It is reasonable to further infer that they should take *symmetric* positions in the market, i.e., make identical bids. Therefore, we will be particularly interested in the analysis of such symmetric cases. This assumption is summarized below.

Assumption 1 *Strategic firms are homogeneous and make identical bids at equilibrium.*

Similar assumptions have been frequently observed in the economics literature, e.g., Cai et al. (2020), Ehrenmann (2004), Sherali et al. (1983), Sherali (1984). As we will show later, this assumption of homogeneity on the supply side contributes towards capturing some key interactions in the two-stage market. In particular, it enables the closed-form characterization of market equilibria, which provides insights into the inter-group competition between firms and consumers.

3. Market Equilibria

Given the mixed bidding mechanism, we now define and characterize the equilibria among participants over the two-stage market. An equilibrium outcome of the market satisfies two properties: (a) every participant is content with its current bids in both stages and therefore has no incentive to make a change; (b) the two-stage market is cleared. More formally, the definition for a market equilibrium is summarized below.

Definition 3 *A market equilibrium over the two-stage market is a tuple* $\Xi := ((\beta_j^f, j \in \mathcal{G}), (\beta_j^s, j \in \mathcal{G}), (d_l^f, l \in \mathcal{L}), (d_l^s, l \in \mathcal{L}), \lambda^f, \lambda^s)$*, consisting of participants' bids and two-stage clearing prices, that satisfies the following:*

- *The supply function bids* (β_j^f, β_j^s) *achieve the maximum profit for each firm* $j \in \mathcal{G}$*;*
- *The quantity bids* (d_l^f, d_l^s) *achieve the minimum payment for each consumer* $l \in \mathcal{L}$*;*
- *Supply and demand are balanced in both stages with the clearing prices* λ^f *and* λ^s*, respectively.*

Based on the types of firms and consumers, being either price-taking or strategic, their bidding behavior tends to drive the market towards different equilibria, and the differences across equilibria reflect their marginal impact on market clearing. We now describe the four major equilibria that are sufficient to articulate the role firms and consumers play in the two-stage market.

3.1. Competitive Equilibrium

If the firms $j \in \mathcal{G}$ and the consumers $l \in \mathcal{L}$ are all price-takers, a market equilibrium that satisfies Definition 3 is the canonical competitive equilibrium. In this context, individual firms and consumers optimally respond to given two-stage prices by solving their own bidding problems (7) or (9). The closed-form optimal bidding behavior of all the participants, captured in (8), (10), immediately suggests the following characterization of a competitive equilibrium (to avoid heavy notations, we do not use extra marks to denote an equilibrium point).

Proposition 1 *A competitive equilibrium* Ξ *over the two-stage market exists and is given by*

$$\lambda^f = \lambda^s = \left(\sum_{j \in \mathcal{G}} \frac{1}{c_j} \right)^{-1} \sum_{l \in \mathcal{L}} d_l \, , \tag{13a}$$

$$\beta_j^f + \beta_j^s = \frac{1}{c_j}\ , \ \ \beta_j^f \geq 0\ , \ \ \beta_j^s \geq 0\ , \ \ \forall j \in \mathcal{G}\ , \tag{13b}$$

$$d_l^f + d_l^s = d_l\ , \ \ \forall l \in \mathcal{L}\ , \tag{13c}$$

$$\sum_{j\in\mathcal{G}} \beta_j^f \lambda^f = \sum_{l\in\mathcal{L}} d_l^f > 0\ , \tag{13d}$$

$$\sum_{j\in\mathcal{G}} \beta_j^s \lambda^s = \sum_{l\in\mathcal{L}} d_l^s\ . \tag{13e}$$

The proof of Proposition 1 is available in Appendix B. Note that (i) this solution does not assume homogeneous price-taking firms; (ii) the competitive equilibrium is not unique; (iii) there are in fact infinitely many solutions to (13).

Remark 1 *The competitive equilibrium* (13) *is intuitive: price-taking firms and consumers prefer the higher-price and lower-price stages, respectively, and supply and demand cannot be matched, if there is a price difference between stages. This enforces equal two-stage prices at the competitive equilibrium* (13)*, a.k.a. no arbitrage.*

One interesting consequence of Proposition 1 is that, even in the presence of perfect foresight, a two-stage clearing process does not necessarily provide incentives for consumers to allocate their entire demand in the forward market. This is evidenced by the fact that (13) has infinitely many solutions. However, it can be shown that the dispatched supply $((q_j^f = \beta_j^f \lambda^f, j \in \mathcal{G}), (q_j^s = \beta_j^s \lambda^s, j \in \mathcal{G}))$ of any such competitive equilibrium is an optimal solution to the following market planner's problem.

Market planner's problem

$$\min_{\substack{q_j^f \geq 0, j\in\mathcal{G} \\ q_j^s \geq 0, j\in\mathcal{G}}} \ \ \sum_{j\in\mathcal{G}} \frac{c_j}{2}\left(q_j^f + q_j^s\right)^2 \tag{14a}$$

$$\text{s.t.} \ \ (2), (3) \tag{14b}$$

The market planner's problem (14) solves for the minimum aggregate production cost to meet the total inelastic demand in the two-stage market. In particular, the two-stage equilibrium prices in (13a) equal the marginal cost of an optimal solution to the market planner's problem (14). This implies that the market is competitive whenever the aggregate bid $\beta_j^f + \beta_j^s$ of each firm j equals the reciprocal of its truthful cost coefficient c_j in (13b). In this sense, the mixed bidding mechanism renders a desirable efficient competitive equilibrium when all market participants are price-takers. Note that the case $\bar{\beta}^f = 0$ and $\bar{d}^f = 0$ is excluded from the competitive equilibrium set due to Rule 2 in Appendix A that is set in favor of forward transactions.

3.2. Demand-Side Nash Equilibrium

We now proceed to consider the impact of participants' strategic behavior on the market equilibrium. We start with strategic consumers. Let all the firms $j \in \mathcal{G}$ be price-takers and all the consumers $l \in \mathcal{L}$ be strategic. We refer to the resulting market equilibrium as a demand-side Nash equilibrium. In this context, each individual consumer solves the bidding problem (12), seeking to manipulate prices to its advantage. However, the following proposition indicates that such behavior leads to Nash equilibria that constitute a subset of the competitive equilibria.

Proposition 2 *Firms behave as price-takers, and consumers behave strategically. A demand-side Nash equilibrium Ξ over the two-stage market exists and is given by*

$$\lambda^f = \lambda^s = \left(\sum_{j\in\mathcal{G}} \frac{1}{c_j}\right)^{-1} \sum_{l\in\mathcal{L}} d_l \ , \tag{15a}$$

$$\beta_j^f + \beta_j^s = \frac{1}{c_j} \ , \ \ \beta_j^f \geq 0 \ , \ \ \beta_j^s \geq 0 \ , \ \ \forall j \in \mathcal{G} \ , \tag{15b}$$

$$d_l^f + d_l^s = d_l \ , \ \ \forall l \in \mathcal{L} \ , \tag{15c}$$

$$\sum_{j\in\mathcal{G}} \beta_j^f \lambda^f = \sum_{l\in\mathcal{L}} d_l^f > 0 \ , \tag{15d}$$

$$\sum_{j\in\mathcal{G}} \beta_j^s \lambda^s = \sum_{l\in\mathcal{L}} d_l^s > 0 \ . \tag{15e}$$

Refer to Appendix C for the proof of Proposition 2.

Remark 2 *Based on the pricing mechanism* (4)*, given any (nonzero) aggregate supply function bids in both stages, strategic consumers tend to maintain equal two-stage prices via demand allocation such that their individual payment is minimal. Otherwise, the payment could always be lowered by shifting demand towards the lower-price stage.*

Proposition 2 shows that when consumers with inelastic demand behave strategically in the presence of price-taking firms, they cannot take advantage of their anticipation to gain benefit in the market, despite the flexibility in allocating their demand in different stages. As a consequence, the equilibrium outcome is also a solution to the market planner's problem (14), achieving the minimum aggregate production cost, even with heterogeneous firms.

3.3. Supply-Side Nash Equilibrium

As a counterpart on the supply side, we consider a particular Nash equilibrium under Definition 3, where all the firms $j \in \mathcal{G}$ are strategic, homogeneous, and make identical bids, while all the consumers $l \in \mathcal{L}$ are price-takers. At such an equilibrium, individual firms solve the coupled two-stage bidding problem (11) in the market. Let $c_j = c$, $\forall j \in \mathcal{G}$, be their shared cost coefficient. The supply-side Nash equilibrium can be described as follows.

Proposition 3 *Let Assumption 1 hold. Consumers behave as price-takers, and firms behave strategically. If there are at least three firms, i.e.,* $G \geq 3$*, a supply-side Nash equilibrium* Ξ *over the two-stage market exists and is given by*

$$\lambda^f = \lambda^s = \frac{G-1}{G-2} \left(\sum_{j \in \mathcal{G}} \frac{1}{c} \right)^{-1} \sum_{l \in \mathcal{L}} d_l = \frac{G-1}{G-2} \cdot \frac{c \sum_{l \in \mathcal{L}} d_l}{G} \ , \tag{16a}$$

$$\beta_j^f = \frac{G-2}{G-1} \cdot \frac{1}{c} \ , \ \ \beta_j^s = 0 \ , \ \ \forall j \in \mathcal{G} \ , \tag{16b}$$

$$d_l^f + d_l^s = d_l \ , \ \ \forall l \in \mathcal{L} \ , \tag{16c}$$

$$\sum_{l \in \mathcal{L}} d_l^f = \sum_{l \in \mathcal{L}} d_l \ , \tag{16d}$$

$$\sum_{l \in \mathcal{L}} d_l^s = 0 \ . \tag{16e}$$

Refer to Appendix D for the proof of Proposition 3.

Remark 3 *Due to the two-stage sequential settlement, strategic firms maintain a fixed spot price above the marginal cost of an optimal solution to the market planner's problem* (14)*, independent of demand allocation, as illustrated in Figure 1. When an equilibrium among strategic firms exists, from the consumer* *perspective, the forward price increases as a function of the forward market demand and equals the spot price only when all the demand is allocated to the forward market. Since price-taking consumers prefer the lower-price stage, they will shift demand to the forward market until there is no price difference, i.e., the supply-side Nash equilibrium* (16).

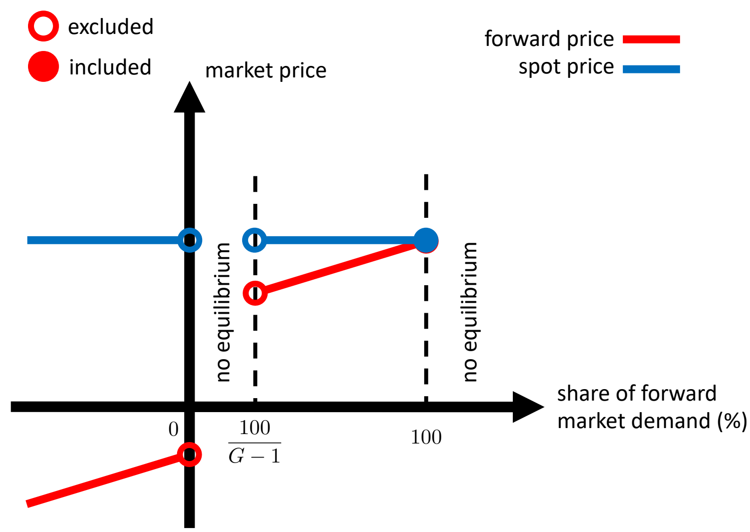

Figure 1 Equilibrium prices reached by strategic firms, as a function of the forward market demand (%). If the share of the forward market demand is in $(-\infty, 0) \cup (\frac{100}{G-1}\%, 100\%]$, there is a symmetric equilibrium among firms. Otherwise, there exists no equilibrium. Refer to the derivation in Appendix D.

Compared with the previous two equilibria (13), (15), the key distinction is that the two-stage clearing prices (16a) are above the marginal cost of an optimal solution to the market planner's problem (14), i.e., $c \sum_{l \in \mathcal{L}} d_l / G$, due to the strategically depressed supply function bids (16b). Therefore, the anticipation does benefit firms. Moreover, the sequential bidding breaks the tie between the two stages and renders an equilibrium where all the demand is cleared by the forward market.

Remark 4 *When there are less than three firms, no supply-side Nash equilibrium exists since the dominant firm(s) can benefit from making arbitrarily small bids. This condition is also seen in the literature, e.g., see Lemmas 1-3 in Li et al. (2015), which basically argue that each firm will always supply less than half of the total demand at the equilibrium in our setting.*

We also notice that the efficiency of the market dispatch that renders the minimum aggregate production cost is trivially maintained at this supply-side Nash equilibrium. This is a direct consequence of the homogeneous firm bidding in Assumption 1 and the demand inelasticity. However, even though this equilibrium is efficient, the firms still manipulate market prices to increase their profit. To better capture the change in the market equilibrium (16) due to the strategic bidding of firms, we adopt a more instructive metric to evaluate market outcomes in Section 4.

3.4. Nash Equilibrium

We now investigate the inter-group competition in the two-stage market between the firms $j \in \mathcal{G}$ and the consumers $l \in \mathcal{L}$ that are both strategic. In this case, we call an equilibrium that satisfies Definition 3 simply a Nash equilibrium (in contrast with a demand-side or supply-side one), where individual firms make identical bids in the market due to homogeneity. The strategic bidding behavior of the participants is defined in (11) for each firm and in (12) for each consumer. Then a Nash equilibrium is identified and summarized in the following proposition.

Proposition 4 *Let Assumption 1 hold. If there are at least three firms, i.e., $G \geq 3$, a Nash equilibrium Ξ over the two-stage market exists. Further, this equilibrium is unique and given by*

$$\lambda^f = \frac{L}{L+1} \cdot \frac{G-1}{G-2} \left(\sum_{j \in \mathcal{G}} \frac{1}{c} \right)^{-1} \sum_{l \in \mathcal{L}} d_l = \frac{L}{L+1} \cdot \frac{G-1}{G-2} \cdot \frac{c \sum_{l \in \mathcal{L}} d_l}{G} \ , \tag{17a}$$

$$\lambda^s = \frac{G-1}{G-2} \left(\sum_{j \in \mathcal{G}} \frac{1}{c} \right)^{-1} \sum_{l \in \mathcal{L}} d_l = \frac{G-1}{G-2} \cdot \frac{c \sum_{l \in \mathcal{L}} d_l}{G} \ , \tag{17b}$$

$$\beta_j^f = \frac{L(G-1)+1}{L(G-1)} \cdot \frac{G-2}{G-1} \cdot \frac{1}{c} \ , \ \ \forall j \in \mathcal{G} \ , \tag{17c}$$

$$\beta_j^s = \frac{1}{L+1} \cdot \left(\frac{G-2}{G-1} \right)^2 \cdot \frac{1}{c} \ , \ \ \forall j \in \mathcal{G} \ , \tag{17d}$$

$$d_l^f = \frac{L(G-1)+1}{L(L+1)(G-1)} \sum_{k \in \mathcal{L}} d_k \ , \ \ \forall l \in \mathcal{L} \ , \tag{17e}$$

$$d_l^s = d_l - d_l^f \ , \ \ \forall l \in \mathcal{L} \ . \tag{17f}$$

The proof of Proposition 4 is given in Appendix E. A direct consequence of Proposition 4 is the following characterization of the demand that is allocated to each stage of the market.

Corollary 1 *The respective demand allocated in the forward and spot markets at the Nash equilibrium* (17) *is*

$$\sum_{l\in\mathcal{L}} d_l^f = \frac{L(G-1)+1}{(L+1)(G-1)} \sum_{l\in\mathcal{L}} d_l \in \left(0.5\sum_{l\in\mathcal{L}} d_l \ , \ \sum_{l\in\mathcal{L}} d_l\right) \ , \tag{18a}$$

$$\sum_{l\in\mathcal{L}} d_l^s = \frac{G-2}{(L+1)(G-1)} \sum_{l\in\mathcal{L}} d_l \in \left(0 \ , \ 0.5\sum_{l\in\mathcal{L}} d_l\right) \ . \tag{18b}$$

Remark 5 *Strategic consumers take into account the response of strategic firms to their demand allocation, as in Figure 1, and allocate as in* (18) *to fully exploit the lower price in the forward market. This is in contrast with the price-taking behavior, where the demand fully shifts to the forward market, erasing the price difference.*

Some further discussion of these results is in order. Despite the fact that both supply-side and demand-side Nash equilibria render equal two-stage prices, the Nash equilibrium (17) maintains a systematic price difference across stages. Such a difference is typically a sign of incentive misalignment in the market (You et al. 2019). In particular, the spot price (17b), which is higher than the forward price (17a) due to the strategic bidding of the firms, matches the two-stage prices of the supply-side Nash equilibrium (16a). On this basis, the forward price (17a) is discounted, pushed by the consumers, but will still be above the marginal cost of an optimal solution to the market planner's problem (14), i.e., $c\sum_{l\in\mathcal{L}} d_l/G$, if $L > G-2$ holds (less consumers lead to a lower forward price). In this sense, the effect of strategic bidding on the clearing prices is decoupled between the firms and the consumers. As the number of firms G increases, both forward and spot prices drop. As the number of consumers L increases, the price difference tends to diminish.

Therefore, the price difference across stages can be attributed to consumers' strategic bidding, exploiting their flexibility in demand allocation. However, the equal two-stage prices of the demand-side Nash equilibrium (15) indicate that the strategic bidding of firms is a necessary enabler. Only by taking advantage of firms' intra-group competition over the two sequential stages can consumers maintain the two-stage prices at different values and enjoy a lower forward price. The strategic allocation (18) indicates that the majority (but not all) of the total demand is in the forward market. Such a phenomenon is often seen in real-world two-stage markets, e.g., the New York ISO electricity market where a small fraction of demand is left to be met in the spot market (You et al. 2019). Our analysis may be construed as a possible explanation of these observations. Another remarkable phenomenon, possibly associated with our homogeneous firm bidding assumption (Assumption 1), is that at the Nash equilibrium (17) all the consumers allocate the same amount of demand in the forward market, despite having possibly different inelastic demand d_l.

A comparison across all the four equilibria above in terms of the share of forward market demand and two-stage prices is illustrated in Figure 2.

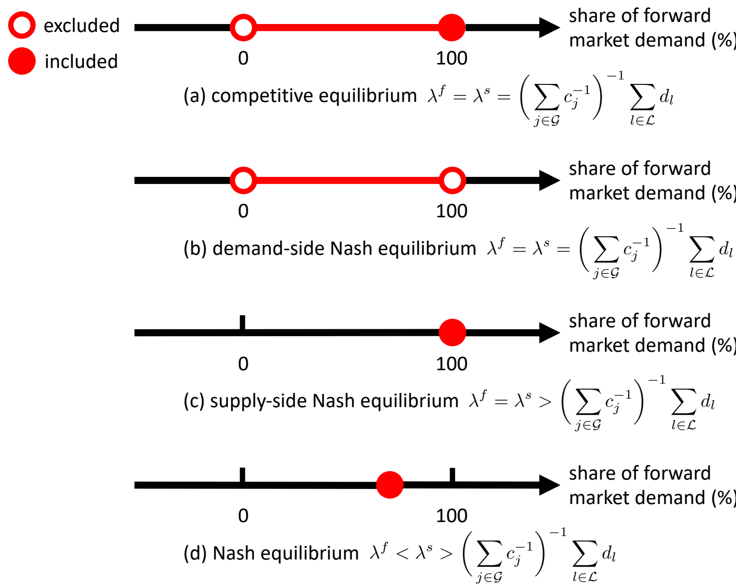

Figure 2 Illustration of demand allocation and two-stage prices for the four equilibria considered. (a) Competitive equilibrium: the two-stage prices equalize and match the marginal cost, with any share of forward market demand in $(0, 100\%]$. (b) Demand-side Nash equilibrium: the two-stage prices equalize and match the marginal cost, with any share of forward market demand in $(0, 100\%)$. (c) Supply-side Nash equilibrium: the two-stage prices equalize above the marginal cost, with a unique share of forward market demand at 100%. (d) Nash equilibrium: the two-stage prices diverge, with the spot price above the marginal cost and a unique share of forward market demand in $(50\%, 100\%)$.

4. Market Power

The ability of a strategic market participant to benefit from the exploitation of extra information in bidding is technically termed its *market power*; see Bose et al. (2014). In general, market power is not favored by market designers nor regulators, since it can have an adverse impact on the market outcome, e.g., by deteriorating market efficiency, or discouraging market participation due to biased allocation of social surplus. In this section we leverage the equilibrium analysis of the previous section to gain insights into the market power of strategic participants in the two-stage settlement market.

4.1. Metric

Recall that market efficiency is achieved at the minimum aggregate production cost to meet the total inelastic demand in the market, defined by the solution to the market planner's problem (14). It is realized not only by the desirable competitive equilibrium (13) and the demand-side Nash equilibrium (15), but also by the supply-side Nash equilibrium (16) and the Nash equilibrium (17), with our assumption of the homogeneous supply dispatch to meet inelastic demand. However, the latter two exhibit definite market changes that are not reflected by efficiency.

To capture the shift in the market outcome due to the strategic bidding of participants, we propose a metric, rooted in fairness of surplus allocation, to quantify their market power via net gain. In our context, the net gain of each firm $j \in \mathcal{G}$ is the profit π_j, while the net gain of each consumer $l \in \mathcal{L}$ is the negative payment $-\rho_l$. We then define the surplus in a market to be the total net gain of all participants through cleared transactions. The surplus amounts to the negative of the aggregate production cost necessary to meet the total inelastic demand in the market, i.e.,

$$\sum_{j \in \mathcal{G}} \pi_j - \sum_{l \in \mathcal{L}} \rho_l = -\sum_{j \in \mathcal{G}} \frac{c_j}{2} \left(q_j^f + q_j^s \right)^2 . \tag{19}$$

Note that the payment goes from the demand side to the supply side and is cancelled out from the global market perspective. When firms are homogeneous with $c_j = c$, $j \in \mathcal{G}$, the surplus (19) will be uniform across the four equilibria of interest. However, its allocation among individual participants, i.e., the contributing components π_j of each firm and $-\rho_l$ of each consumer, varies, depending on their respective financial settlements.

We propose to employ participants' net gains as an indicator for individual market power. In particular, an increase in a participant's net gain implies a reduction in the net gain of some others, thus it is a clear signal

Equilibrium	Individual firm profit π_j	Individual consumer payment ρ_l
Competitive equilibrium	$\frac{1}{2} \cdot \frac{c\left(\sum_{l \in \mathcal{L}} d_l\right)^2}{G^2}$	$\frac{c \sum_{k \in \mathcal{L}} d_k}{G} \cdot d_l$
Demand-side Nash equilibrium	$\frac{1}{2} \cdot \frac{c\left(\sum_{l \in \mathcal{L}} d_l\right)^2}{G^2}$	$\frac{c \sum_{k \in \mathcal{L}} d_k}{G} \cdot d_l$
Supply-side Nash equilibrium	$\left(\frac{1}{2} + \frac{1}{G-2}\right) \cdot \frac{c\left(\sum_{l \in \mathcal{L}} d_l\right)^2}{G^2}$	$\left(1 + \frac{1}{G-2}\right) \cdot \frac{c \sum_{k \in \mathcal{L}} d_k}{G} \cdot d_l$
Nash equilibrium	$\left(\frac{1}{2} + \frac{1}{G-2}\right) \cdot \frac{c\left(\sum_{l \in \mathcal{L}} d_l\right)^2}{G^2} - \frac{L(G-1)+1}{(L+1)^2(G-2)} \cdot \frac{c\left(\sum_{l \in \mathcal{L}} d_l\right)^2}{G^2}$	$\left(1 + \frac{1}{G-2}\right) \cdot \frac{c \sum_{k \in \mathcal{L}} d_k}{G} \cdot d_l - \frac{L(G-1)+1}{L(L+1)^2(G-2)} \cdot \frac{c\left(\sum_{k \in \mathcal{L}} d_k\right)^2}{G}$

Table 1 Surplus allocation compared across equilibria. We insert $c_j = c, \forall j \in \mathcal{G}$, into the competitive equilibrium and the demand-side Nash equilibrium for ease of comparison.

of one's market power being capitalized on. We specify the competitive equilibrium (13) as the benchmark of a fair allocation since it corresponds to both primal (efficiency) and dual (pricing) optimum of the market planner's problem (14). The two-stage prices thereof reflect the marginal cost. Our metric to quantify market power is then formally defined as follows.

Definition 4 *A participant is able to exercise market power if its net gain (at equilibrium) exceeds that of the competitive equilibrium* (13)*, i.e., $\pi_j > \pi_j^{\text{comp}}$ for a firm, and $\rho_l < \rho_l^{\text{comp}}$ for a consumer, where π_j^{comp} and ρ_l^{comp} are their profit and payment at the competitive equilibrium, respectively.*

Based on the metric, we can compare the other three equilibria against the competitive equilibrium to get a sense of individual participants' net gain change, as summarized in Table 1.

4.2. Discussion

Due to the symmetry among firms, the aggregate net gain of the supply side is a fixed multiple (G) of the profit of an individual firm. As a result, the column of individual firm profit π_j in Table 1 can suggest the inter-group market power shifts between firms and consumers, given the same market surplus across equilibria (rows). On the other hand, the column of individual consumer payment ρ_l shows the intra-group market power shifts among heterogeneous consumers.

4.2.1. Unilateral Market Power Analysis Both the individual firm profit π_j and the individual consumer payment ρ_l remain the same at the demand-side Nash equilibrium and the competitive equilibrium, suggesting that strategic consumers alone cannot exercise any market power when firms are all price-takers. However, if only firms are strategic, they benefit from anticipation to increase their profit at the supply-side Nash equilibrium. This extra net gain quantifies the supply-side market power, which enables them to lift the two-stage clearing prices by jointly exaggerating the cost. Note that this portion of surplus is shifted from the demand side; namely, consumers are paying more to their cross-group competitors. Table 1 shows that each consumer contributes to this stripped surplus proportionately based on its demand, in light of the equal two-stage prices. Comparative statics further indicate that the increment in both a firm's profit and a consumer's payment decreases with the number of strategic firms G. In particular, in the case of $G = 3$, the smallest number required for the existence of this equilibrium, these three firms can triple their profit to the maximum extent. On the contrary, when G grows large, this surplus shift tends to diminish, thus recovering the fair allocation. The reduced net gain, i.e., diluted market power, is attributed to the intensified intra-group competition introduced by more firms. We can observe that if either supply or demand side is price-taking, a market equilibrium entails equal two-stage prices. Under this circumstance, only strategic firms are able to exercise market power with their intrinsic flexibility, while consumers with inelastic demand entirely lose their allocation flexibility in the absence of a price difference.

4.2.2. Market Power of Inelastic Demand In the case where both firms and consumers are strategic, we also observe from Table 1 that at the Nash equilibrium each firm incurs an additional loss in profit, when compared with the supply-side Nash equilibrium. This loss is a consequence of strategic consumers taking advantage of strategic firms' bidding to create a price difference between stages and, in this way, counterbalancing the supply-side market power. This portion of surplus is returned to the demand side. However, strikingly, it is split evenly among all consumers, regardless of the heterogeneity in their individual demand. The intuition is that the strategic bidding of consumers lowers the forward price. At the same time, it yields a dilemma where allocating more demand in the forward market to capitalize on the lower price will diminish the price gap and reduce their own profitability. Therefore, at equilibrium each consumer enjoys the lower forward price by allocating the same amount of forward market demand. An implication here is that a small consumer is more likely to attain market power such that its payment drops below the threshold of the competitive equilibrium. More precisely, *a consumer pays less at the Nash equilibrium than at the competitive equilibrium if its demand satisfies*

$$d_l < \frac{L(G-1)+1}{L(L+1)^2} \sum_{k \in \mathcal{L}} d_k \ . \tag{20}$$

Moreover, with

$$G > \frac{L^3+2L^2+2L-1}{L} \ , \ \ i.e., \ \ \frac{L(G-1)+1}{L(L+1)^2} > 1 \ , \tag{21}$$

all consumers are better off and gain market power over firms. In fact, a consumer can make a strictly positive profit from the two-stage market as long as its demand is small enough and satisfies

$$d_l < \frac{L(G-1)+1}{L(L+1)^2(G-1)} \sum_{k \in \mathcal{L}} d_k \ . \tag{22}$$

Such ability is attributed to their smaller net demand to satisfy, which allows them to take advantage of the lower forward price for arbitrage by purchasing more than needed in the forward market. However, the above threshold suggests that it is not possible for all consumers to earn money simultaneously. This fact that a small consumer is favored may further create incentives for large consumers to split for market participation, instead of merging to consolidate.

4.2.3. Nullified Supply-Side Market Power Finally, recall that the strategic bidding of consumers is only valuable in the presence of strategic firms. However, as we have shown above, the market power of strategic firms may be overturned by the strategic bidding of consumers. In our analysis, *a general condition for firms to earn strictly less profit than that of the competitive equilibrium, a sign of the supply-side market power nullified, is*

$$G > L+3 \ . \tag{23}$$

The condition unveils the impact of the increasing supply-side intra-group competition on inter-group market power shifts. Therefore, strategic consumers, even if they are bound to completely inelastic demand, still have a chance to overturn the typical dominance of strategic firms in the market.

5. Extension: Virtual Bidding

In this section we extend our analysis to accommodate the policy of virtual bidding that is practiced in various two-stage settlement electricity markets (Hogan 2016). It is a form of speculation, similar to futures trading in other commodity markets. A virtual bidder is commonly a financial entity without physical generation units or electrical appliances, and therefore trades electricity without ever producing or consuming it. In other words, for any amount of electricity involved in trades by virtual bidders in the forward market, it will be reversed by trading in the spot market before actual delivery. The goal of virtual bidding is to exploit the potential price difference of the two-stage settlement for arbitrage. Using the Nash equilibrium (17) as a baseline, we analytically evaluate the impact of this type of virtual bidders on the two-stage competition among firms and consumers.

As pointed out in (17), there is a systematic price difference between the forward and spot markets at the Nash equilibrium among strategic firms and consumers, which is an appealing situation for virtual bidding. With a higher spot price, a virtual bidder in this case tends to buy from the forward market at a lower price and then sells back in the spot market. In our setting, it is equivalent to a consumer with zero inelastic demand. We denote $\mathcal{V}$ as a set of such virtual bidders with $V := |\mathcal{V}|$. In a sense, all the virtual bidders $v \in \mathcal{V}$ are homogeneous consumers with $d_v = 0$. Similarly, we define d_v^f and d_v^s as the quantity of commodities each virtual bidder v trades in the forward and spot markets, respectively, which are required to satisfy

$$d_v^f + d_v^s = 0 \; . \tag{24}$$

Its payment as a (virtual) consumer in both stages sums up to

$$\rho_v := \lambda^f d_v^f + \lambda^s d_v^s \; . \tag{25}$$

The smaller its payment (typically in negative values), the larger its gain through arbitrage. Then the payment minimization, or gain maximization, bidding problem for virtual bidder v can be formulated as

Bidding problem of virtual bidder v

$$\min_{d_v^f, d_v^s} \quad \rho_v(d_v^f, d_v^s; \bar{\beta}^f, \bar{d}_{-v}^f, \bar{\beta}^s, \bar{d}_{-v}^s) \tag{26a}$$

$$\text{s.t.} \quad (24) \tag{26b}$$

where we have particularly defined $\bar{d}_{-v}^f := \sum_{k\in\mathcal{L}\bigcup\mathcal{V}\setminus\{v\}} d_k^f$ and $\bar{d}_{-v}^s := \sum_{k\in\mathcal{L}\bigcup\mathcal{V}\setminus\{v\}} d_k^s$ to capture the same type of quantity bids of consumers and virtual bidders.

In this framework, a virtual bidder is a special form of a consumer and the addition of such a set $\mathcal{V}$ of virtual bidders into the market competition does not affect our previous equilibrium analysis. In particular, the total demand in the market remains $\sum_{k\in\mathcal{L}\bigcup\mathcal{V}} d_k = \sum_{l\in\mathcal{L}} d_l$, while the total number of consumers, including virtual consumers, now amounts to $L+V$. As a result, the Nash equilibrium with virtual bidding follows immediately from (17), and is summarized next.

Proposition 5 *Let Assumption 1 hold. If there are at least three firms, i.e., $G \geq 3$, a Nash equilibrium with virtual bidding $\Xi_v := ((\beta_j^f, j\in\mathcal{G}), (\beta_j^s, j\in\mathcal{G}), (d_l^f, l\in\mathcal{L}), (d_l^s, l\in\mathcal{L}), (d_v^f, v\in\mathcal{V}), (d_v^s, v\in\mathcal{V}), \lambda^f, \lambda^s)$ over the two-stage market exists. Further, this equilibrium is unique and given by*

$$\lambda^f = \frac{L+V}{L+V+1} \cdot \frac{G-1}{G-2} \cdot \frac{c\sum_{l\in\mathcal{L}} d_l}{G} \;, \tag{27a}$$

$$\lambda^s = \frac{G-1}{G-2} \cdot \frac{c\sum_{l\in\mathcal{L}} d_l}{G} \;, \tag{27b}$$

$$\beta_j^f = \frac{(L+V)(G-1)+1}{(L+V)(G-1)} \cdot \frac{G-2}{G-1} \cdot \frac{1}{c} \;, \;\; \forall j \in \mathcal{G} \;, \tag{27c}$$

$$\beta_j^s = \frac{1}{L+V+1} \cdot \left(\frac{G-2}{G-1}\right)^2 \cdot \frac{1}{c} \;, \;\; \forall j \in \mathcal{G} \;, \tag{27d}$$

$$d_l^f = \frac{(L+V)(G-1)+1}{(L+V)(L+V+1)(G-1)} \sum_{k\in\mathcal{L}} d_k \;, \;\; \forall l \in \mathcal{L} \;, \tag{27e}$$

$$d_l^s = d_l - d_l^f \;, \;\; \forall l \in \mathcal{L} \;, \tag{27f}$$

$$d_v^f = \frac{(L+V)(G-1)+1}{(L+V)(L+V+1)(G-1)} \sum_{l\in\mathcal{L}} d_l \;, \;\; \forall v \in \mathcal{V} \;, \tag{27g}$$

$$d_v^s = -\frac{(L+V)(G-1)+1}{(L+V)(L+V+1)(G-1)} \sum_{l\in\mathcal{L}} d_l \;, \;\; \forall v \in \mathcal{V} \;. \tag{27h}$$

The proof of Proposition 5 follows that of Proposition 4 in Appendix E, by treating virtual bidders as a special subset of consumers with zero demand.

One prominent impact of virtual bidding is the shrinking gap between the two-stage clearing prices in (27a), (27b). Indeed, since there still exists a price difference, it is always possible to involve more virtual bidders, which in turn drives the two-stage clearing prices closer. Meanwhile, we can infer from (18) that the demand allocation will be biased towards the forward market as V increases,

$$\sum_{k\in\mathcal{L}\bigcup\mathcal{V}} d_k^f = \frac{(L+V)(G-1)+1}{(L+V+1)(G-1)}\sum_{l\in\mathcal{L}} d_l = \left(1-\frac{G-2}{(L+V+1)(G-1)}\right)\sum_{l\in\mathcal{L}} d_l\ , \tag{28}$$

until there is ultimately no demand left for fulfillment in the spot market. In the context of electricity markets, it is favored to encourage forward-market transactions due to physical limits, e.g., startup/shutdown time and ramp capability, that inhibit slow-responsive electric machines from participating in the spot market. As a result, virtual bidders can be seen as a source of market power mitigation that further helps clear most demand in the forward market.

Notably, this shift in demand towards the forward market, resulting from virtual consumers that aim to solve (26), has the effect of only limiting consumer market power. This can be explicitly evidenced from the surplus allocation at the Nash equilibrium with virtual bidding:

$$\pi_j = \left(\frac{1}{2}+\frac{1}{G-2}\right)\cdot\frac{c\left(\sum_{l\in\mathcal{L}} d_l\right)^2}{G^2} - \underbrace{\frac{(L+V)(G-1)+1}{(L+V+1)^2(G-2)}\cdot\frac{c\left(\sum_{l\in\mathcal{L}} d_l\right)^2}{G^2}}_{\to 0 \text{ as } V\to\infty}\ ,\ \ j\in\mathcal{G}\ , \tag{29a}$$

$$\rho_l = \frac{G-1}{G-2}\cdot\frac{c\left(\sum_{k\in\mathcal{L}} d_k\right)}{G}\cdot d_l - \underbrace{\frac{(L+V)(G-1)+1}{(L+V)(L+V+1)^2(G-2)}\cdot\frac{c\left(\sum_{k\in\mathcal{L}} d_k\right)^2}{G}}_{\to 0 \text{ as } V\to\infty}\ ,\ \ l\in\mathcal{L}\ , \tag{29b}$$

$$\rho_v = -\frac{(L+V)(G-1)+1}{(L+V)(L+V+1)^2(G-2)}\cdot\frac{c\left(\sum_{l\in\mathcal{L}} d_l\right)^2}{G}\ ,\ \ v\in\mathcal{V}\ , \tag{29c}$$

where again π_j, ρ_l, and ρ_v are respectively a firm's profit, a consumer's payment, and a virtual bidder's payment. One can see that both π_j and ρ_l increase when V increases, and in the limit $V\to\infty$ they restore the surplus allocation of the supply-side Nash equilibrium (16). In this sense, while virtual bidding seems to be a policy that helps mitigate market power and leads to early clearing of consumers' demand, it achieves this by unilaterally increasing competition on the demand side, thus allowing firms to maximize the effect of their price manipulation.

In addition to virtual bidding, our analysis is amenable to other potential policies or mechanism changes aimed at mitigating market power. See Appendices F and I for another two examples, where our analysis allows explicit characterization of their impact and offers more insights into the role of such changes. The approach can thus be used to evaluate different settings and serve as guidelines for policymakers.

6. Conclusion

This paper carries out an analysis that models the cross-group competition of firms and consumers with inelastic demand over a two-stage market and provides a closed-form characterization of the market equilibria. Particular insights into the inter-group and intra-group market power are offered through a quantitative metric of surplus allocation among participants, which highlight the importance of accounting for the strategic bidding behavior of consumers. Even if their demand is completely inelastic, the flexibility of allocating demand across stages endows them with the ability to undermine or even overturn the conventional supply-side market power. Remarkably, such an ability is only available when firms are themselves behaving strategically. Consumers can take advantage of firms' intra-group cross-stage competition and maintain a systematic price gap for payment reduction, which is however impossible if firms are price-takers. Notably, we see that small consumers are especially favored and can even take advantage of the price difference between stages for arbitrage. As a whole, this paper uses a novel approach to reveal market outcomes that have been unnoticed by prior literature, and opens up the possibility of further untangling complex market interactions, just as we demonstrate through the analytical evaluation of several market policies.

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Supplementary Materials

Appendix A: Pricing Rules for Degenerate Cases

To account for the degenerate cases where the supply function bids sum up to zero in a market, we propose the following rules.

Rule 1 *If all firms make zero bids in a market while there is demand to fulfill, the clearing price is set to zero and demand is split evenly across firms.*

Rule 2 *When all firms make zero bids in a market while there is zero demand to fulfill, the clearing price is set to (a) zero, if it occurs in the forward market; (b) equal the forward price, if it occurs in the spot market.*

The two rules are made for ease of equilibrium analysis. Intuitively, Rule 1 discourages aggregate zero bids from firms in the presence of demand, while Rule 2 differentiates between the forward and spot markets to favor forward transactions out of operational and economic concerns.

Rule 1 targets the degenerate case where all firms make zero supply function bids in the presence of demand, and is designed in particular to discourage strategic firms that anticipate infinite revenue out of such a situation. Rules of similar purposes include market rejection, e.g., in Li et al. (2015), and zero revenue enforcement despite infinite prices. However, Rule 1 is especially useful in two-stage market analysis since it specifies the market clearing price and dispatch even in the degenerate case such that the two-stage bidding problems are always well defined for participants. As we will show later, it contributes to avoiding undesired equilibria, jointly with Rule 2.

Next we present two observations based on Rule 1 that serve as prerequisites for characterization of a market equilibrium. They apply to either stage of a two-stage market.

Observation 1. If the total supply function bids sum up to zero in a market, a strategic consumer will exploit Rule 1 to request as much demand as possible free of charge. Therefore, there does not exist an equilibrium where all firms make zero supply function bids in the presence of strategic consumers.

Observation 2. If there is positive demand in a market, there does not exist an equilibrium where all firms make zero supply function bids.

We now show Observation 2 is correct in both cases of strategic and price-taking firms. In the case of strategic firms, each of them has an incentive to make a positive bid that is however as small as possible, since the extra revenue could go unbounded while the extra cost is bounded due to the finite demand. In the case of price-taking firms, we put the discussion in the context of a two-stage market. Based on their optimal bidding behavior (8), they may all bid zero in one stage only when the price in the other stage is no lower. Suppose such a situation occurs in the forward market, i.e., $\bar{d}^f > 0$, $\bar{\beta}^f = 0$, and $\lambda^f = 0$ by Rule 1.

Meanwhile, the spot price $\lambda^s \geq 0$ is given and we have $\bar{\beta}^s = \sum_{j\in\mathcal{G}} \frac{1}{c_j}$ due to (8). We fix the spot-market dispatch $q_j^s = \beta_j^s \lambda^s \geq 0$ for firm j. In this situation the profit of firm j is

$$\pi_j = 0 \cdot \frac{\bar{d}^f}{G} + \lambda^s \cdot q_j^s - \frac{1}{2} c_j \left(\frac{\bar{d}^f}{G} + q_j^s \right)^2 \,, \tag{30}$$

if it bids $\beta_j^f = 0$. However, if it bids an arbitrary $\beta_j^f > 0$, its profit becomes

$$\pi_j' = 0 \cdot (0 \cdot \beta_j^f) + \lambda^s \cdot q_j^s - \frac{1}{2} c_j \left(0 \cdot \beta_j^f + q_j^s\right)^2 > \pi_j \,, \tag{31}$$

where it does not anticipate to affect prices via the change in its bid. Therefore, a price-taking firm always has an incentive to deviate from such a situation by making a positive bid. By symmetry, this situation will also never occur in the spot market. Combining the analysis above proves Observation 2.

Appendix B: Proof of Proposition 1

We first prove the characterization for the competitive equilibrium in Proposition 1. Given price-taking firms and consumers, their optimal bids to respectively maximize profit and minimize payment are explicitly given in (8), (10). Therefore, a competitive equilibrium to satisfy Definition 3 essentially requires (8), (10) and two-stage supply-demand balance to hold simultaneously. Note that the optimal bidding strategy (10) of a price-taking consumer will never lead to an equilibrium if the two-stage prices are not equal. The only possibility for the existence of a competitive equilibrium is to enforce equal two-stage prices. In light of the optimal bidding behavior (8) of a price-taking firm, we obtain $\lambda^f = \lambda^s = (\sum_{j\in\mathcal{G}} c_j^{-1})^{-1} \sum_{l\in\mathcal{L}} d_l$, which then leads to the set of competitive equilibria in (13), with a special case of $\bar{d}^f = 0$ and $\bar{\beta}^f = 0$ excluded ($\lambda^f = 0 < \lambda^s$ by Rule 2). This completes the proof of Proposition 1.

Appendix C: Proof of Proposition 2

We now prove the characterization for the demand-side Nash equilibrium in Proposition 2. Firms are still price-takers and bid optimally according to (8). However, each individual consumer is strategic that aims to solve (12) with anticipation of market clearing, given other participants' bids. If $\bar{\beta}^f = 0$ or $\bar{\beta}^s = 0$, Observation 1 suggests that no equilibrium exists. We therefore focus on the case with $\bar{\beta}^f, \bar{\beta}^s > 0$. In particular, (12) can be cast into an unconstrained single-variable quadratic program in d_l^f for each consumer l:

$$\begin{aligned} \min_{d_l^f} \quad & \lambda^f d_l^f + \lambda^s d_l^s \\ = \quad & \frac{\bar{d}_{-l}^f + d_l^f}{\bar{\beta}^f} \cdot d_l^f + \frac{\sum_{k\in\mathcal{L}} d_k - (\bar{d}_{-l}^f + d_l^f)}{\bar{\beta}^s} (d_l - d_l^f) \\ = \quad & \left(\frac{1}{\bar{\beta}^f} + \frac{1}{\bar{\beta}^s} \right) {d_l^f}^2 + \left(\frac{\bar{d}_{-l}^f}{\bar{\beta}^f} - \frac{d_l + \sum_{k\in\mathcal{L}} d_k - \bar{d}_{-l}^f}{\bar{\beta}^s} \right) d_l^f + \text{constant} \end{aligned} \tag{32}$$

Its unique optimal solution is given by

$$d_l^f = -\frac{1}{2}\bar{d}_{-l}^f + \frac{\bar{\beta}^f}{2(\bar{\beta}^f + \bar{\beta}^s)} \left(d_l + \sum_{k\in\mathcal{L}} d_k \right) . \tag{33}$$

Reorganizing the above expression yields

$$\sum_{l\in\mathcal{L}} d_l^f = \frac{\bar{\beta}^f}{\bar{\beta}^f + \bar{\beta}^s} \sum_{l\in\mathcal{L}} d_l > 0 \ , \ \text{ and } \ \sum_{l\in\mathcal{L}} d_l^s = \frac{\bar{\beta}^s}{\bar{\beta}^f + \bar{\beta}^s} \sum_{l\in\mathcal{L}} d_l > 0 \ , \tag{34}$$

which imply

$$\lambda^f = \frac{\sum_{l\in\mathcal{L}} d_l^f}{\bar{\beta}^f} = \frac{\sum_{l\in\mathcal{L}} d_l}{\bar{\beta}^f + \bar{\beta}^s} = \frac{\sum_{l\in\mathcal{L}} d_l^s}{\bar{\beta}^s} = \lambda^s \ , \tag{35}$$

i.e., equal two-stage prices. We note from the bidding behavior (8) of price-taking firms that in this situation

$$\bar{\beta}^f + \bar{\beta}^s = \sum_{j\in\mathcal{G}} \frac{1}{c_j} \tag{36}$$

holds with $\bar{\beta}^f > 0$, $\bar{\beta}^s > 0$, and both stages are cleared at the price that reflects the marginal cost

$$\lambda^f = \lambda^s = \left(\sum_{j\in\mathcal{G}} \frac{1}{c_j} \right)^{-1} \sum_{l\in\mathcal{L}} d_l \ . \tag{37}$$

As a result, the demand-side Nash equilibria constitute a subset of competitive equilibria, excluding the case with $\bar{d}^s = 0$ and $\bar{\beta}^s = 0$, as captured in (15). This completes the proof of Proposition 2.

Appendix D: Proof of Proposition 3

We then prove the characterization for the supply-side Nash equilibrium in Proposition 3. Here consumers are price-takers with optimal bidding captured in (10), while each individual firm is strategic that aims to solve (11a), (11b) sequentially. It needs to anticipate and account for the spot market profit in its forward market bidding. Therefore, we study backwards from the spot market competition and represent its equilibrium outcome as a function of forward market bids.

Again note that any difference between the two-stage prices means arbitrage opportunities for price-taking consumers and no equilibrium will exist. Therefore, it is necessary to maintain equal two-stage prices at any potential equilibrium. Then consumers are satisfied with arbitrary demand allocation subject to $\bar{d}^f + \bar{d}^s = \sum_{l\in\mathcal{L}} d_l$, based on their optimal bidding strategy (10).

Case i: $\bar{d}^s \neq 0$. At the time of the spot market, the dispatch and clearing price from the forward market has already been determined. Therefore, for each firm j, its spot market bidding problem (11b) boils down to

$$\begin{aligned} \max_{\beta_j^s \geq 0} \quad & \lambda^{s2} \beta_j^s - \frac{1}{2} c_j \left(q_j^f + \lambda^s \beta_j^s \right)^2 \\ = \quad & \left(\frac{\bar{d}^s}{\bar{\beta}_{-j}^s + \beta_j^s} \right)^2 \beta_j^s - \frac{1}{2} c_j \left(q_j^f + \frac{\bar{d}^s}{\bar{\beta}_{-j}^s + \beta_j^s} \cdot \beta_j^s \right)^2 \end{aligned} \tag{38}$$

where the constant forward market revenue is ignored and the forward market dispatch q_j^f is given. Although the objective function is not necessarily concave in the feasible region, we can analyze its optimal bidding behavior from the monotonicity. Taking the first-order derivative with respect to β_j^s, we have

$$\frac{\mathrm{d}\pi_j(\beta_j^s;\bar{\beta}^f,\bar{d}^f,\bar{\beta}_{-j}^s,\bar{d}^s)}{\mathrm{d}\beta_j^s}=\frac{\bar{d}^s}{(\bar{\beta}_{-j}^s+\beta_j^s)^3}\left[\bar{d}^s(\bar{\beta}_{-j}^s-\beta_j^s)-c_j\bar{\beta}_{-j}^s\left(q_j^f(\bar{\beta}_{-j}^s+\beta_j^s)+\bar{d}^s\beta_j^s\right)\right]\ . \tag{39}$$

Note that the term inside the square brackets of (39) is linear in β_j^s and admits an only turning point in $\mathbb{R}$ where the sign of (39) changes. If the turning point is non-positive, the individual optimal bid of firm j will be either $+\infty$ or 0 (positive and arbitrarily close to zero), depending on how the sign changes. Under this circumstance, no equilibrium exists (a zero bid is not possible for any symmetric equilibrium specified by Assumption 1 due to Observation 2). Even if the turning point is positive, any potential equilibrium requires it to be a maximal turning point that brings firm j the maximal profit.

Under Assumption 1 of symmetric firm bids, we plug in $\bar{\beta}_{-j}^s=(G-1)\beta_j^s$ and $c_j=c$ to attain the turning point as

$$\beta_j^s=\frac{G-2}{G-1}\cdot\frac{\bar{d}^s}{Gq_j^f+\bar{d}^s}\cdot\frac{1}{c}\ ,\ \ \forall j\in\mathcal{G}\ . \tag{40}$$

Note that any potential symmetric equilibrium over the two stage market leads to $Gq_j^f+\bar{d}^s=\sum_{l\in\mathcal{L}}d_l>0$. Therefore, (40) is a positive maximal turning point, i.e., the unique individual optimal bid, and also yields the subgame Nash equilibrium in the spot market, only if $G\geq 3$ and $\bar{d}^s>0$ hold.

Given the spot market equilibrium bid (40), the forward market bidding problem for firm j is now explicitly given by

$$\begin{aligned}\max_{\beta_j^f\geq 0}\quad & {\lambda^f}^2\beta_j^f+{\lambda^s}^2\beta_s^f-\frac{1}{2}c(\lambda^f\beta_j^f+\lambda^s\beta_j^s)^2\\ =\ & \left(\frac{\bar{d}^f}{\bar{\beta}_{-j}^f+\beta_j^f}\right)^2\beta_f^s-\frac{1}{2}c\left(\frac{\bar{d}^f}{\bar{\beta}_{-j}^f+\beta_j^f}\cdot\beta_j^f\right)^2+\frac{c\bar{d}^s}{G(G-2)}\cdot\frac{\bar{d}^f}{\bar{\beta}_{-j}^f+\beta_j^f}\cdot\beta_j^f+\text{constant}\end{aligned} \tag{41}$$

Sub-case i: $\bar{d}^f=0$. We can observe from (41) that the forward market bidding objective of each firm j is constant. It has no bias for any specific bid $\beta_j^f\geq 0$. Suppose $\bar{\beta}^f>0$, then by definition of the pricing (4a), we have $\lambda^f=0$. In the case of $\bar{\beta}^f=0$, by Rule 2, we still have $\lambda^f=0$. Recall the positive spot price (45), then the optimal bidding behavior (10) of consumers should drive $\bar{d}^f=\sum_{l\in\mathcal{L}}d_l$ and $\bar{d}^s=0$, which contradict the prerequisite $\bar{d}^f=0$. Therefore, no equilibrium exists in this sub-case.
Sub-case ii: $\bar{d}^f\neq 0$. Taking the first-order derivative of (41) with respect to β_j^f, we have

$$\frac{\mathrm{d}\pi_j(\beta_j^f,\beta_j^s(\beta_j^f),\lambda^s(\beta_j^f);\bar{\beta}_{-j}^f,\bar{d}^f,\bar{d}^s)}{\mathrm{d}\beta_j^f}=\frac{\bar{d}^f}{(\bar{\beta}_{-j}^f+\beta_j^f)^3}\left[\bar{d}^f(\bar{\beta}_{-j}^f-\beta_j^f-c\bar{\beta}_{-j}^f\beta_j^f)+\frac{\bar{d}^s c\bar{\beta}_{-j}^f(\bar{\beta}_{-j}^f+\beta_j^f)}{G(G-2)}\right]. \tag{42}$$

Similarly, the term inside the square brackets of (42) is linear in β_j^f and admits an only turning point in $\mathbb{R}$. For an equilibrium to exist, it has to be a positive maximal turning point. We exploit Assumption 1 again to plug in $\bar{\beta}_{-j}^f = (G-1)\beta_j^f$ and obtain the turning point as

$$\beta_j^f = \frac{(G-2)^2 \bar{d}^f}{(G-1)^2 \bar{d}^f - (G-1)\sum_{l\in\mathcal{L}} d_l} \cdot \frac{1}{c} \ , \ \ \forall j \in \mathcal{G} \ , \tag{43}$$

which meets our criteria to be individual optimum at any potential equilibrium in the forward market, only if $\bar{d}^f < 0$ or $\frac{\sum_{l\in\mathcal{L}} d_l}{G-1} < \bar{d}^f < \sum_{l\in\mathcal{L}} d_l$ holds together with $G \geq 3$.

Under this circumstance the resulting forward price is given by

$$\lambda^f = \frac{\bar{d}^f}{G\beta_j^f} = \frac{G-1}{G-2} \cdot \frac{c[(G-1)\bar{d}^f - \sum_{l\in\mathcal{L}} d_l]}{G(G-2)} < \frac{G-1}{G-2} \cdot \frac{c\sum_{l\in\mathcal{L}} d_l}{G} \ , \tag{44}$$

where the inequality follows from the prerequisite $\bar{d}^f < \sum_{l\in\mathcal{L}} d_l$. However, (40) implies that the current spot price is higher:

$$\lambda^s = \frac{\bar{d}^s}{G\beta_s^f} = \frac{G-1}{G-2} \cdot \frac{c\sum_{l\in\mathcal{L}} d_l}{G} > \lambda^f \ , \tag{45}$$

which contradicts the initial condition of equal two-stage prices. Therefore, no equilibrium exists in this sub-case either.

Case ii: $\bar{d}^s = 0$. All the transactions occur in the forward market, and the forward price at the equilibrium of (single-stage) forward market competition can be readily computed as

$$\lambda^f = \frac{G-1}{G-2} \cdot \frac{c\sum_{l\in\mathcal{L}} d_l}{G} \ , \tag{46}$$

with $G \geq 3$. Meanwhile, in the spot market, firm j has no bias for any specific bid $\beta_j^s \geq 0$. There are two possibilities. Suppose $\bar{\beta}^s > 0$, then by definition of the pricing (4b), we have $\lambda^s = 0 < \lambda^f$. The optimal bidding behavior (10) of consumers will drive $\bar{d}^f = 0$ and $\bar{d}^s = \sum_{l\in\mathcal{L}} d_l$, which contradict the prerequisite $\bar{d}^s = 0$. In the situation of $\bar{\beta}^s = 0$, we learn from Rule 2 that the spot price is set to equal the forward price, which satisfies Definition 3 of an equilibrium and indeed is the unique supply-side Nash equilibrium, as characterized in (16).

This completes the proof of Proposition 3.

Appendix E: Proof of Proposition 4

We now prove the characterization for the holistic Nash equilibrium in Proposition 4. We still analyze backwards from the spot market but account for the interaction among strategic firms and consumers. Likewise, the spot-market bid d_l^s of each consumer l is fixed, so is the total spot market demand $\bar{d}^s$, given the day-ahead dispatch. Only firms have flexibility to make adjustments in the spot market.

Case i: $\bar{d}^s \neq 0$. The spot market equilibrium analysis follows that in Appendix D and leads to the symmetric firms' bids in (40), requiring at least three firms and positive spot-market demand. Then in the forward market, we have also discussed there the potential symmetric equilibrium bids of strategic firms, with respect to the total demand allocation between the two stages. We now describe the strategic bidding behavior of each strategic consumer and combine both for a holistic equilibrium analysis.

We just need to consider $\bar{\beta}^f > 0$ due to Observation 1 and modify the individual consumer bidding problem (32) in Appendix C to reflect the anticipated equilibrium bids (40) of strategic firms in the spot market:

$$\begin{aligned}
\min_{d_l^f} \quad & \lambda^f d_l^f + \lambda^s d_l^s \\
= \; & \frac{\bar{d}_{-l}^f + d_l^f}{\bar{\beta}^f} \cdot d_l^f + \frac{G-1}{G-2} \cdot \left(\frac{\beta_j^f}{\bar{\beta}^f} (\bar{d}_{-l}^f + d_l^f) + \frac{\sum_{k \in \mathcal{L}} d_k - \bar{d}_{-l}^f - d_l^f}{G} \right) c \left(d_l - d_l^f \right) \\
= \; & \left[\frac{\bar{d}_{-l}^f}{\bar{\beta}^f} + \frac{G-1}{G-2} \cdot \left(\left(\frac{\beta_j^f}{\bar{\beta}^f} - \frac{1}{G} \right) d_l - \left(\frac{\beta_j^f}{\bar{\beta}^f} - \frac{1}{G} \right) \bar{d}_{-l}^f - \frac{\sum_{k \in \mathcal{L}} d_k}{G} \right) c \right] d_l^f \\
& + \left[\frac{1}{\bar{\beta}^f} - \frac{G-1}{G-2} \cdot \left(\frac{\beta_j^f}{\bar{\beta}^f} - \frac{1}{G} \right) c \right] {d_l^f}^2 + \text{constant}
\end{aligned} \tag{47}$$

This unconstrained quadratic program has a unique minimizer

$$d_l^f = -\frac{1}{2} \bar{d}_{-l}^f + \frac{G-1}{G-2} \cdot \frac{c \beta_j^f \sum_{k \in \mathcal{L}} d_k}{2} \;, \tag{48}$$

where we have applied Assumption 1 of symmetric firm bids. Combining all such individual optimizers (necessary for an equilibrium) and reorganizing terms lead to positive total demand in the forward market, given by

$$\bar{d}^f = \sum_{l \in \mathcal{L}} d_l^f = \frac{L}{L+1} \cdot \frac{G-1}{G-2} \cdot c \beta_j^f \sum_{l \in \mathcal{L}} d_l > 0 \;. \tag{49}$$

We can further combine the potential equilibrium demand allocation (49) and the potential equilibrium bids (43) of firms, conditioning on $\frac{\sum_{l \in \mathcal{L}} d_l}{(G-1)} < \bar{d}^f < \sum_{l \in \mathcal{L}} d_l$, to derive a holistic equilibrium in the forward market. It essentially solves for β_j^f and $\bar{d}^f$ from (43), (49), and leads to the unique solution as

$$\begin{aligned}
\beta_j^f &= \frac{L(G-1)+1}{L(G-1)} \cdot \frac{G-1}{G-1} \cdot \frac{1}{c} \;, \\
\bar{d}^f &= \frac{L(G-1)+1}{(L+1)(G-1)} \sum_{l \in \mathcal{L}} d_l \;.
\end{aligned} \tag{50}$$

It can be verified that here $\frac{\sum_{l \in \mathcal{L}} d_l}{(G-1)} < \bar{d}^f < \sum_{l \in \mathcal{L}} d_l$ indeed holds. Therefore, this yields a Nash equilibrium over the two-stage market, as explicitly captured in (17).

Case ii: $\bar{d}^s = 0$. As also discussed in Appendix D, at the forward market equilibrium, the forward price is positive and given by (46) with

$$\beta_j^f = \frac{G-2}{G-1} \cdot \frac{1}{c} \;, \;\; \forall j \in \mathcal{G} \;. \tag{51}$$

In the spot market, any decision of $\beta_j^s \geq 0$ makes no difference for firm j under this circumstance. Suppose $\bar{\beta}^s > 0$, then by the standard definition (4b) of pricing, we have $\lambda^s = 0$. This is not an equilibrium since each consumer l has an incentive to increase its d_l^s to exploit a lower spot price. In the other situation of $\bar{\beta}^s = 0$, despite that the corresponding spot price equals the forward price by Rule 2, Observation 1 suggests that this cannot be an equilibrium in the presence of strategic consumers. Therefore, there exists no equilibrium in this case.

As a whole, there is only one unique Nash equilibrium over the two-stage market, given by (17). This completes the proof of Proposition 4.

Appendix F: Spot-Market Transaction Charge

In many two-stage markets, it is generally desired – to the extent possible – to clear the demand in the forward market. However, when both firms and consumers behave strategically, such an outcome is not expected as described in (17). While virtual bidding may be considered a solution to such a problem, it requires sufficient large numbers of virtual bidders. An alternative regulatory policy to discourage trading in the spot market is to impose an extra spot transaction charge on any participant that trades in this market. Such a policy is expected to break a tie between the forward and spot markets and drive transactions to the former. However, to figure out its precise impact on the market outcome, we accommodate this policy in our analysis.

To reflect the spot-market transaction charge, we modify the profit of a firm $j \in \mathcal{G}$ as

$$\pi_j := \lambda^f q_j^f + \lambda^s q_j^s - \frac{c_j}{2}\left(q_j^f + q_j^s\right)^2 - \gamma q_j^s \ , \tag{52}$$

and the payment of a consumer $l \in \mathcal{L}$ as

$$\rho_l := \lambda^f d_l^f + \lambda^s d_l^s + \gamma d_l^s \ , \tag{53}$$

where we define a unit price of this spot-market transaction charge to be linear in the quantity of commodities traded in the spot market:

$$\gamma := \frac{\alpha}{2} \sum_{k \in \mathcal{G}} q_k^s = \frac{\alpha}{2} \sum_{k \in \mathcal{L}} d_k^s \ . \tag{54}$$

Here α is a constant linear coefficient, and (54) holds due to the need to enforce balance between supply and demand at market clearing (3). In general, there may exist many other forms for such spot-market transaction charges. For the moment we stick to the linearly increasing charge with γ in (54) that strengthens the penalty as the quantity of goods traded in the spot market grows.

With this modification, we now re-evaluate specifically the resulting competitive equilibrium and Nash equilibrium over the two-stage market. A price-taker should treat the price γ as given, along with the forward and spot prices. The competition among such firms and consumers leads to a unique competitive equilibrium in the set of original competitive equilibria (13).

Proposition 6 *In the presence of the spot-market transaction charge* (54)*, a competitive equilibrium* Ξ *over the two-stage market exists and is given by*

$$\lambda^f = \lambda^s = \left(\sum_{j \in \mathcal{G}} \frac{1}{c_j} \right)^{-1} \sum_{l \in \mathcal{L}} d_l \ , \tag{55a}$$

$$\beta_j^f = \frac{1}{c_j} \ , \ \ \beta_j^s = 0 \ , \ \ \forall j \in \mathcal{G} \ , \tag{55b}$$

$$d_l^f + d_l^s = d_l \ , \ \ \forall l \in \mathcal{L} \ , \tag{55c}$$

$$\sum_{l \in \mathcal{L}} d_l^f = \sum_{l \in \mathcal{L}} d_l \ , \tag{55d}$$

$$\sum_{l \in \mathcal{L}} d_l^s = 0 \ . \tag{55e}$$

Refer to Appendix G for the proof of Proposition 6. The competitive equilibrium (55) still achieves market efficiency of the minimum aggregate production cost to meet the total inelastic demand. Further, all trades are shifted to the forward market, and no one incurs the spot-market transaction charge at the equilibrium. For price-takers, the spot-market transaction charge serves as as a tie-breaker between the two stages. Instead, if participants are aware of this spot-market transaction charge (54) and behave strategically, they will arrive at a different but also unique Nash equilibrium, as summarized below.

Proposition 7 *Let Assumption 1 hold. In the presence of the spot-market transaction charge* (54)*, if there are at least three firms, i.e.,* $G \geq 3$*, a Nash equilibrium* Ξ *over the two-stage market exists. Further, this equilibrium is unique and given by*

$$\lambda^f = \frac{L}{L+1} \cdot \frac{G-1}{G-2} \cdot \frac{c \sum_{l \in \mathcal{L}} d_l}{G} + \left(1 - \frac{Lz}{L+1}\right) \cdot \frac{G-1}{G-2} \cdot \frac{(2G-3)\alpha}{(2G-3)Gz\alpha + 2(G-1)c} \cdot c \sum_{l \in \mathcal{L}} d_l \ , \tag{56a}$$

$$\lambda^s = \frac{G-1}{G-2} \cdot \frac{c \sum_{l \in \mathcal{L}} d_l}{G} + \left(1 - \frac{Lz}{L+1}\right) \cdot \frac{G-1}{G-2} \cdot \frac{(G-1)\alpha}{(2G-3)Gz\alpha + 2(G-1)c} \cdot c \sum_{l \in \mathcal{L}} d_l \ , \tag{56b}$$

$$\beta_j^f = \frac{G-2}{G-1} \cdot z \cdot \frac{1}{c} \ , \ \ \forall j \in \mathcal{G} \ , \tag{56c}$$

$$\beta_j^s = \frac{G-2}{G-1} \cdot \frac{2(L+1-Lz)(G-1)c}{(L+1)[(2G-3)Gzc\alpha + 2(G-1)c^2] + (L+1-Lz)(G-1)Gc\alpha} \ , \ \ \forall j \in \mathcal{G} \ , \tag{56d}$$

$$d_l^f = \frac{z}{L+1} \cdot \frac{2(G-1)c}{(2G-3)Gz\alpha + 2(G-1)c} \cdot \sum_{k \in \mathcal{L}} d_k + \frac{(2G-3)Gz\alpha}{(2G-3)Gz\alpha + 2(G-1)c} \cdot d_l \ , \ \ \forall l \in \mathcal{L} \ , \tag{56e}$$

$$d_l^s = d_l - d_l^f \ , \ \ \forall l \in \mathcal{L} \ , \tag{56f}$$

where z *is a constant coefficient defined as*

$$z := 1 + \frac{2(G-1)c}{2L(G-1)^2 c + (L+1)(2G-3)(G-2)G\alpha} \in (1, 1 + \frac{1}{L(G-1)}) \ .$$

Refer to Appendix H for the proof of Proposition 7.

Corollary 2 *The respective demand allocated in the forward and spot markets at the Nash equilibrium* (56) *is*

$$\sum_{l\in\mathcal{L}} d_l^f = \left(\frac{Lz}{L+1}\cdot\frac{2(G-1)c}{(2G-3)Gz\alpha+2(G-1)c}+\frac{(2G-3)Gz\alpha}{(2G-3)Gz\alpha+2(G-1)c}\right)\sum_{l\in\mathcal{L}} d_l\ , \tag{57a}$$

$$\sum_{l\in\mathcal{L}} d_l^s = \left(1-\frac{Lz}{L+1}\right)\frac{2(G-1)c}{(2G-3)Gz\alpha+2(G-1)c}\sum_{l\in\mathcal{L}} d_l \in \left(0\ ,\ \sum_{l\in\mathcal{L}} d_l\right)\ , \tag{57b}$$

where $\frac{Lz}{L+1}<1$ *holds for* $G\geq 3$ *and* $L\geq 1$.

Corollary 3 *The spot-market transaction charge at the Nash equilibrium* (56) *is*

$$\gamma = \left(1-\frac{Lz}{L+1}\right)\frac{(G-1)\alpha c}{(2G-3)Gz\alpha+2(G-1)c}\cdot\sum_{l\in\mathcal{L}} d_l \tag{58}$$

We next highlight several observations that illustrate the impact of the spot-market transaction charge through the coefficient α. We specifically vary α from zero, which boils down the original setting, to plus infinity, which represents an extreme case.

Fact 1: $\sum_{l\in\mathcal{L}} d_l^f$ increases in α and $\sum_{l\in\mathcal{L}} d_l^s$ decreases in α. In the limit of $\alpha\to\infty$, all the demand is fulfilled in the forward market, i.e., $\sum_{l\in\mathcal{L}} d_l^f \to \sum_{l\in\mathcal{L}} d_l$ and $\sum_{l\in\mathcal{L}} d_l^s \to 0$.

This is consistent with the intuition that with a spot-market transaction charge, the benefit in participating in the spot market will diminish so as to foster a demand shift towards the forward market. Notably, a sensitivity analysis on the initial impact of introducing a small α gives us

$$\frac{\partial \sum_{l\in\mathcal{L}} d_l^f}{\partial\alpha}\Big|_{\alpha=0} \sim \mathcal{O}\left(\frac{G}{L}\cdot\frac{1}{c}\right)\ . \tag{59}$$

Intuitively, a larger cost coefficient c implies higher market clearing prices (with $\alpha=0$), at which the demand sensitivity to introducing α will be lower. Similarly, more firms imply less price inflation; recall the uplift coefficient $\frac{G-1}{G-2}$ in (17a), (17b). Then the demand allocation tends to be more sensitive to the spot-market transaction charge, or α. On the contrary, more consumers already suggest a larger share of forward-market transactions, recall its allocation ratio $\frac{L(G-1)+1}{(L+1)(G-1)}$ in (18), thus the incremental effect of α is attenuated.

Fact 2: λ^f, λ^s and γ are all raised as α increases, yet with a positive invariant $\lambda^s+\gamma-\lambda^f$. In the limit of $\alpha\to\infty$, they all converge to finite values, where λ^f jumps most with a relative increase $\frac{L+1}{L}$ and equals the original spot price with $\alpha=0$.

The two-stage price markup is propelled by strategic firms as a means of cost recovery despite that the extra transaction charge is only imposed in the spot market. Notably, $\lambda^s>\lambda^f$ holds regardless of α. Moreover, the invariant $\lambda^s+\gamma-\lambda^f$ suggests a fixed gap between the unit costs of purchasing commodities in the two stages. Strikingly, despite such a price gap in the limiting case, all the demand will be shifted to the forward market. This suggest that in this setting the price gap should also be caused in part by strategic firms to hedge against high spot-market transaction penalty. A similar sensitivity analysis of the two-stage prices with respect to $\alpha = 0$ yields

$$\left.\frac{\partial \lambda^f}{\partial \alpha}\right|_{\alpha=0} \sim \mathcal{O}\left(\frac{\sum_{l\in\mathcal{L}} d_l}{L}\right) \ , \ \text{and} \ \left.\frac{\partial \lambda^s}{\partial \alpha}\right|_{\alpha=0} \sim \mathcal{O}\left(\frac{\sum_{l\in\mathcal{L}} d_l}{L}\right) \ , \tag{60}$$

both of which are strikingly on the order of magnitude of average demand per consumer. Given the fixed total demand, less consumers, or large consumers in general, imply a more significant hit by the spot-market transaction charge.

Fact 3: Individual firm profit π_j increases in α for $\forall j \in \mathcal{G}$. An arbitrary individual consumer payment ρ_l, $l \in \mathcal{L}$, is not necessarily monotonic in α, but definitely increases when α is sufficiently large. With $\alpha \to \infty$, they approach their corresponding values at the supply-side Nash equilibrium (16), i.e.,

$$\pi_j = \left(\frac{1}{2} + \frac{1}{G-2}\right) \cdot \frac{c\left(\sum_{l\in\mathcal{L}} d_l\right)^2}{G^2} \ , \tag{61a}$$

$$\rho_l = \frac{G-1}{G-2} \cdot \frac{c\sum_{k\in\mathcal{L}} d_k}{G} \cdot d_l \ . \tag{61b}$$

Note that this case occurs when all the demand is fulfilled in the forward market. Thus, there is no spot-market transaction charge for both firms and consumers. This is a similar limiting case to that of virtual bidding with an infinite number of virtual bidders. However, the rationale is slightly different here: the potential high spot-market transaction penalty drives all the consumers off on their own initiative without convergence of the two-stage clearing prices (while virtual bidding renders zero net demand yet still transactions in the spot market). The impact of α on individual consumer payment ρ_l is convoluted. However, in general when α grows large, all individual consumers would end up paying more. Further, a smaller consumer is more likely affected by the spot-market transaction charge. This matches with intuition since smaller consumers were the most benefited from shifting demand across stage. A penalty on demand shifts will thus hit them the most. In this sense, once again, the spot-market transaction charge contributes to enhancing market power of firms that tend to transfer the risk of extra cost to the demand side.

Appendix G: Proof of Proposition 6

In the presence of the spot market transaction charge, by an argument similar to Observation 2, it can be readily justified that *there does not exist an equilibrium where all firms make zero supply function bids to meet positive demand in the market*. Given this fact, we now prove the characterization for the competitive equilibrium in Proposition 6. We re-characterize the optimal bidding behavior of individual price-taking firms and consumers based on their current objectives (52), (53). Firm j earns the maximum profit with the following bid:

$$\begin{cases} \beta_j^f = \dfrac{1}{c_j} \ , \ \ \beta_j^s = 0 \ , \quad \text{if } \lambda^f > \lambda^s - \gamma \, ; \\ \beta_j^f + \beta_j^s = \dfrac{1}{c_j} \ , \ \ \beta_j^f \geq 0, \beta_j^s \geq 0 \ , \quad \text{if } \lambda^f = \lambda^s - \gamma \, ; \\ \beta_j^f = 0 \ , \ \ \beta_j^s = \dfrac{1}{c_j} \ , \quad \text{if } \lambda^f < \lambda^s - \gamma \, ; \end{cases} \tag{62}$$

while consumer l achieves the minimum payment by bidding according to

$$\begin{cases} d_l^f = -\epsilon \ , \ \ d_l^s = \epsilon + d_l \ , \quad \text{if } \lambda^f > \lambda^s + \gamma \, ; \\ d_l^f + d_l^s = d_l \ , \quad \text{if } \lambda^f = \lambda^s + \gamma \, ; \\ d_l^f = \epsilon + d_l \ , \ \ d_l^s = -\epsilon \ , \quad \text{if } \lambda^f < \lambda^s + \gamma \, ; \end{cases} \tag{63}$$

with $\epsilon \to \infty$.

We notice from (63) that whenever $\lambda^f \neq \lambda^s + \gamma$ holds, no equilibrium would exist since consumers have the incentive to unlimitedly arbitrage over the price difference. In the case of $\lambda^f = \lambda^s + \gamma$, we consider the three sub-cases in (62): (i) $\lambda^f > \lambda^s - \gamma$. All supply shifts to the forward market and no transaction occurs in the spot market, i.e., $\gamma = 0$. However, this leads to contradiction due to $\lambda^f > \lambda^s - \gamma = \lambda^s + \gamma = \lambda^f$. (ii) $\lambda^f < \lambda^s - \gamma$. All supply shifts to the spot market to serve the demand, i.e., $\gamma > 0$. However, this leads to contradiction due to $\lambda^f = \lambda^s + \gamma > \lambda^s - \gamma$. (iii) $\lambda^f = \lambda^s - \gamma$. This sub-case enforces $\gamma = 0$ and there exists such an equilibrium as long as

$$\beta_j^f = \frac{1}{c_j} \ , \ \ \beta_j^s = 0 \ , \ \ \forall j \in \mathcal{G} \ , \ \text{ and } \ \sum_{l \in \mathcal{L}} d_l^f = \sum_{l \in \mathcal{L}} d_l \ , \ \ \sum_{l \in \mathcal{L}} d_l^s = 0 \tag{64}$$

hold with equal two-stage prices

$$\lambda^f = \lambda^s = \left(\sum_{j \in \mathcal{G}} \frac{1}{c_j} \right)^{-1} \sum_{l \in \mathcal{L}} d_l \ . \tag{65}$$

Such an equilibrium is exactly captured by (55). This completes the proof of Proposition 6.

Appendix H: Proof of Proposition 7

The pipeline of showing Proposition 4 still applies here to the proof of Proposition 7. We highlight the backbone to arrive at the unique Nash equilibrium in the presence of the spot market transaction charge and skip discussing all trivial possibilities in detail. See Appendix E for an elaborate procedure.

We still start backwards from the spot market. Suppose $\bar{d}^s > 0$ is fixed, then the spot market bidding problem (38) for each firm j is modified as

$$\begin{aligned} \max_{\beta_j^s \geq 0} \quad & \lambda^{s2} \beta_j^s - \frac{1}{2} c_j \left(q_j^f + \lambda^s \beta_j^s \right)^2 - \gamma q_j^s \\ = \ & \left(\frac{\bar{d}^s}{\bar{\beta}_{-j}^s + \beta_j^s} \right)^2 \beta_j^s - \frac{1}{2} c_j \left(q_j^f + \frac{\bar{d}^s}{\bar{\beta}_{-j}^s + \beta_j^s} \cdot \beta_j^s \right)^2 - \frac{1}{2} \alpha \frac{\bar{d}^{s2}}{\bar{\beta}_{-j}^s + \beta_j^s} \cdot \beta_j^s \end{aligned} \tag{66}$$

To solve for the optimal bid, we evaluate its first-order derivative with respect to β_j^s:

$$\frac{\mathrm{d}\pi_j(\beta_j^s;\bar{\beta}^f,\bar{d}^f,\bar{\beta}_{-j}^s,\bar{d}^s)}{\mathrm{d}\beta_j^s} = \frac{\bar{d}^s}{(\bar{\beta}_{-j}^s+\beta_j^s)^3}\left[\bar{d}^s(\bar{\beta}_{-j}^s-\beta_j^s) - c_j\bar{\beta}_{-j}^s\left(q_j^f(\bar{\beta}_{-j}^s+\beta_j^s)+\bar{d}^s\beta_j^s\right) - \frac{\alpha\bar{d}^s}{2}\bar{\beta}_{-j}^s(\bar{\beta}_{-j}^s+\beta_j^s)\right] . \tag{67}$$

Under Assumption 1, a similar monotonicity analysis leads to the subgame Nash equilibrium in the spot market where all the firms make the symmetric bids of positive maximal turning points

$$\beta_j^s = \frac{G-2}{G-1}\cdot\frac{\bar{d}^s}{Gq_j^f c+\bar{d}^s(c+\frac{G}{2}\alpha)}\ ,\ \ \forall j\in\mathcal{G}\ , \tag{68}$$

with $G\geq 3$.

Given such a spot market equilibrium, the forward market competition is also affected. We first remark that the bidding problem of each firm j in the forward market remains the form of (41) except minor changes to the constant terms. Therefore, the potential equilibrium bids of firms are the same as (43), in the only case where an equilibrium may exist. On the demand side, the penalty coefficient α comes into play through the equilibrium spot price. Therefore, the bidding problem (47) of each consumer l is modified as

$$\begin{aligned}
\min_{d_l^f}\quad & \lambda^f d_l^f + \lambda^s d_l^s + \gamma d_l^s \\
= \ & \frac{\bar{d}_{-l}^f + d_l^f}{\bar{\beta}^f}\cdot d_l^f + \frac{G-1}{G-2}\cdot\left[\frac{\beta_j^f}{\bar{\beta}^f}(\bar{d}_{-l}^f+d_l^f)c + \frac{\sum_{k\in\mathcal{L}}d_k - \bar{d}_{-l}^f - d_l^f}{G}\left(c+\frac{G(2G-3)\alpha}{2(G-1)}\right)\right](d_l - d_l^f) \\
= \ & \left[\frac{\bar{d}_{-l}^f}{\bar{\beta}^f} + \frac{G-1}{G-2}\cdot c\left(\left(\frac{\beta_j^f}{\bar{\beta}^f}-\frac{1}{G}\right)d_l - \left(\frac{\beta_j^f}{\bar{\beta}^f}-\frac{1}{G}\right)\bar{d}_{-l}^f - \frac{\sum_{k\in\mathcal{L}}d_k}{G}\right) - \frac{(2G-3)\alpha}{2(G-2)}\left(\sum_{k\in\mathcal{L}}d_k + d_l - \bar{d}_{-l}^f\right)\right]d_l^f \\
& + \left[\frac{1}{\bar{\beta}^f} - \frac{G-1}{G-2}\cdot c\left(\frac{\beta_j^f}{\bar{\beta}^f}-\frac{1}{G}\right) + \frac{(2G-3)\alpha}{2(G-2)}\right]{d_l^f}^2 + \text{constant} \\
= \ & \left[\frac{\bar{d}_{-l}^f}{\bar{\beta}^f} - \frac{G-1}{G-2}\cdot\frac{c\sum_{k\in\mathcal{L}}d_k}{G} - \frac{(2G-3)\alpha}{2(G-2)}\left(\sum_{k\in\mathcal{L}}d_k + d_l - \bar{d}_{-l}^f\right)\right]d_l^f + \left(\frac{1}{\bar{\beta}^f} + \frac{(2G-3)\alpha}{2(G-2)}\right){d_l^f}^2 + \text{constant}
\end{aligned} \tag{69}$$

with the unique optimal bid

$$d_l^f = -\frac{1}{2}\bar{d}_{-l}^f + \frac{1}{2}\cdot\frac{[(2(G-1)c+G(2G-3)\alpha)]\,\beta_j^f\sum_{k\in\mathcal{L}}d_k + G(2G-3)\alpha\beta_j^f d_l}{G(2G-3)\alpha\beta_j^f + 2(G-2)}\ ,\ \ \forall l\in\mathcal{L}\ , \tag{70}$$

which implies the potential equilibrium total demand in the forward market:

$$\bar{d}^f = \sum_{l\in\mathcal{L}}d_l^f = \frac{\beta_j^f}{G(2G-3)\alpha\beta_j^f + 2(G-2)}\left[\frac{2L(G-1)c}{L+1} + G(2G-3)\alpha\right]\sum_{l\in\mathcal{L}}d_l > 0\ , \tag{71}$$

due to $\beta_j^f > 0$ and $G\geq 3$. Combining (71) and (43), we can solve for the equilibrium bids as

$$\beta_j^f = \frac{G-2}{G-1}\cdot z\cdot\frac{1}{c}\ ,\ \ \forall j\in\mathcal{G}\ , \tag{72a}$$

$$\beta_j^s = \frac{G-2}{G-1} \cdot \frac{2(L+1-Lz)(G-1)c}{(L+1)[(2G-3)Gzc\alpha + 2(G-1)c^2] + (L+1-Lz)(G-1)Gc\alpha} \ , \ \ \forall j \in \mathcal{G} \ , \tag{72b}$$

$$d_l^f = \frac{z}{L+1} \cdot \frac{2(G-1)c}{(2G-3)Gz\alpha + 2(G-1)c} \cdot \sum_{k \in \mathcal{L}} d_k + \frac{(2G-3)Gz\alpha}{(2G-3)Gz\alpha + 2(G-1)c} \cdot d_l \ , \ \ \forall l \in \mathcal{L} \ , \tag{72c}$$

$$d_l^s = d_l - d_l^f \ , \ \ \forall l \in \mathcal{L} \ , \tag{72d}$$

where z is a constant given by

$$z = 1 + \frac{2(G-1)c}{2L(G-1)^2 c + (L+1)(2G-3)(G-2)G\alpha} \ . \tag{73}$$

This is exactly the equilibrium captured in (56) and the resulting total demand allocation (57) satisfies $\frac{1}{G-1}\sum_{l\in\mathcal{L}} d_l < \bar{d}^f < \sum_{l\in\mathcal{L}} d_l$, suggesting that it is indeed the unique Nash equilibrium in the presence of the spot market transaction charge. This completes the proof of Proposition 7.

Appendix I: Uniform Supply Function Bidding for Firms

We have discussed policies that either add more competition on the consumer side (virtual bidding), or equally penalize firms' and consumers' participation in the spot market (spot-market transaction charge). The outcome, in both cases, has disproportionately affected the demand side. In an attempt to limit supply-side market power, we introduce and analyze here an alternative uniform supply function bidding mechanism that targets only firms.

In particular, the proposed mechanism requires each firm $j \in \mathcal{G}$ to bid one uniform supply function that will apply to both stages. Therefore, we redefine $\beta_j \geq 0$ to be this uniform bid and the supply dispatch in the forward and spot markets has to respect

$$q_j^f = \beta_j \lambda^f \ , \tag{74a}$$

$$q_j^s = \beta_j (\lambda^s - \lambda^f) \ , \tag{74b}$$

where the spot market accounts for any deviation from the forward-market dispatch based on the uniform bid β_j. In this setting, the spot price λ^s incentivizes the actual total dispatch:

$$q_j^f + q_j^s = \beta_j \lambda^s \ . \tag{75}$$

Under this mechanism, the market clearing law (3) sets the two-stage clearing prices at

$$\lambda^f = \frac{\sum_{l\in\mathcal{L}} d_l^f}{\sum_{j\in\mathcal{G}} \beta_j} \ , \tag{76a}$$

$$\lambda^s = \frac{\sum_{l\in\mathcal{L}} d_l^s}{\sum_{j\in\mathcal{G}} \beta_j} + \lambda^f \ , \tag{76b}$$

which follows from substituting (74) into (3). The spot pricing (76b) unveils the implicit coupling between the two-stage prices. In this case, we assume that simultaneous zero supply function bids from firms will be rejected by the market and are not considered. As before, we simplify the notation by defining $\bar{\beta} := \sum_{j\in\mathcal{G}} \beta_j$ and $\bar{\beta}_{-j} := \sum_{k\in\mathcal{G}\setminus\{j\}} \beta_k$.

To assess the impact of the uniform supply function bidding mechanism on the market competition, we concentrate our attention on a game featuring the interplay between strategic firms and consumers. In particular, their bidding problems are slightly modified to adapt to the mechanism:

Modified bidding problem of strategic firm j

$$\max_{\beta_j\geq 0} \quad \pi_j(\beta_j;\bar{\beta}_{-j},\bar{d}^f,\bar{d}^s) \tag{77a}$$

$$\text{s.t.} \quad (76) \tag{77b}$$

Modified bidding problem of strategic consumer l

$$\min_{d_l^f,d_l^s\geq 0} \quad \rho_l(d_l^f,d_l^s;\bar{\beta},\bar{d}_{-l}^f,\bar{d}_{-l}^s) \tag{78a}$$

$$\text{s.t.} \quad (2),(76) \tag{78b}$$

Predicated on the above characterization, a Nash equilibrium under Definition 3 is identified below.

Proposition 8 *Let Assumption 1 hold. Under the uniform supply function bidding mechanism for firms, if there are at least three firms, i.e.,* $G\geq 3$*, a Nash equilibrium* $\Xi_u := ((\beta_j, j\in\mathcal{G}),(d_l^f, l\in\mathcal{L}),(d_l^s, l\in\mathcal{L}),\lambda^f,\lambda^s)$ *over the two-stage market exists. Further, this equilibrium is unique and given by*

$$\lambda^f = \frac{L^2+L}{L^2+L+1}\cdot\frac{G-1}{G-2}\cdot\frac{c\sum_{l\in\mathcal{L}} d_l}{G}\ , \tag{79a}$$

$$\lambda^s = \frac{L^2+2L+1}{L^2+L+1}\cdot\frac{G-1}{G-2}\cdot\frac{c\sum_{l\in\mathcal{L}} d_l}{G}\ , \tag{79b}$$

$$\beta_j = \frac{L^2+L+1}{(L+1)^2}\cdot\frac{G-2}{G-1}\cdot\frac{1}{c}\ ,\ \ \forall j\in\mathcal{G}\ , \tag{79c}$$

$$d_l^f = \frac{1}{L+1}\sum_{k\in\mathcal{L}} d_k\ ,\ \ \forall l\in\mathcal{L}\ , \tag{79d}$$

$$d_l^s = d_l - d_l^f\ ,\ \ \forall l\in\mathcal{L}\ . \tag{79e}$$

The proof of Proposition 8 is given in Appendix J. Again, all consumers fulfill the same amount of demand d_l^f from the forward market, despite their heterogeneity in individual demand d_l. In the meantime, the aggregate demand allocation across stages turns out to be only dependent on the number of loads L:

Corollary 4 *The respective demand allocated in the forward and spot markets at the Nash equilibrium* (79) *is*

$$\sum_{l\in\mathcal{L}} d_l^f = \frac{L}{L+1}\cdot\sum_{l\in\mathcal{L}} d_l\ , \tag{80a}$$

$$\sum_{l\in\mathcal{L}} d_l^s = \frac{1}{L+1}\cdot\sum_{l\in\mathcal{L}} d_l\ . \tag{80b}$$

Similarly, due to the firm symmetry and the demand inelasticity, the market surplus (19), i.e., the negative of the aggregate production cost, at the Nash equilibrium (79) is also fixed at the optimum of the market planner's problem (14). Therefore, its allocation among all the participants can still be used to indicate the market power shifts based on Definition 4:

$$\pi_j = \left(\frac{1}{2} + \frac{1}{G-2}\right) \cdot \frac{c\left(\sum_{l\in\mathcal{L}} d_l\right)^2}{G^2}\ , \tag{81a}$$

$$\rho_l = \frac{G-1}{G-2} \cdot \frac{c\sum_{k\in\mathcal{L}} d_k}{G} \cdot \left(d_l + \frac{Ld_l - \sum_{k\in\mathcal{L}} d_k}{L^2+L+1}\right)\ . \tag{81b}$$

Unexpectedly, although the mechanism limits firms' flexibility by allowing them to submit only one bid (in the forward market), the profit of each firm in (81a) indicates that the supply side recovers the same-level market power as that of the supply-side Nash equilibrium (16), a sign of market dominance by firms. Arguably, the uniform supply function bidding mechanism restricts the flexibility of individual firms, but also hurts market competition in a way that instead enhances the supply-side market power.

The demand side is also affected by the mechanism. Compared with the supply-side Nash equilibrium, (81b) suggests that small consumers with demand below average, i.e., $d_l < \frac{1}{L} \cdot \sum_{k\in\mathcal{L}} d_k$, pay less while large consumers incur more payment. This mechanism again favors small consumers: in the case of $d_l < \frac{1}{L+1} \cdot \sum_{k\in\mathcal{L}} d_k$, consumer l performs arbitrage on the two-stage price difference; further, if d_l is sufficiently small, consumer l exploits intra-group market power such that its payment drops below that of the competitive equilibrium (13).

A counter-intuitive message that emerges here is that the additional constraints that uniform supply function bidding mechanism imposes on firms do not hinder the supply-side market power; rather it becomes even stronger. This analysis therefore illustrates the importance of achieving a qualitative characterization of inter-group competition that allows counter factual analysis of the impact of proposed policies.

Appendix J: Proof of Proposition 8

Note that Observations 1 and 2 still apply in this context. Therefore, we only need to focus on the case of $\bar{\beta} > 0$. We now prove the characterization for the Nash equilibrium in Proposition 8, under the policy of uniform supply function bidding for firms. In this setting, all the bidding decisions are made in the forward market based on (77), (78) for each individual firm and consumer, respectively.

The bidding problem (77) of firm j can be explicitly written as

$$\begin{aligned} \max_{\beta_j \geq 0} \quad & \lambda^f q_j^f + \lambda^s q_j^s - \frac{1}{2} c_j \left(q_j^f + q_j^f\right)^2 \\ = \quad & \left(\frac{\bar{d}^f}{\bar{\beta}_{-j} + \beta_j}\right)^2 \beta_j + \left(\frac{\bar{d}^f + \bar{d}^s}{\bar{\beta}_{-j} + \beta_j}\right) \frac{\bar{d}^s \beta_j}{\bar{\beta}_{-j} + \beta_j} - \frac{1}{2} c_j \left(\frac{\bar{d}^f \beta_j}{\bar{\beta}_{-j} + \beta_j} + \frac{\bar{d}^s \beta_j}{\bar{\beta}_{-j} + \beta_j}\right)^2 \end{aligned} \tag{82}$$

whose first-order derivative with respect to β_j is given by

$$\frac{\mathrm{d}\pi_j(\beta_j; \bar{\beta}_{-j}, \bar{d}^f, \bar{d}^s)}{\mathrm{d}\beta_j} = \frac{1}{(\bar{\beta}_{-j} + \beta_j)^3} \left[(\bar{d}^{f^2} + \bar{d}^f \bar{d}^s + \bar{d}^{s^2})(\bar{\beta}_{-j} - \beta_j) - c_j(\bar{d}^f + \bar{d}^s)^2 \bar{\beta}_{-j} \beta_j\right]\ . \tag{83}$$

The term inside the square brackets is linear in β_j and admits an only turning point in $\mathbb{R}$. Under Assumption 1 of symmetric bids for firms, we use $\bar{\beta}_{-j} = (G-1)\beta_j$ and $c_j = c$ to attain the positive maximal turning point

$$\beta_j = \frac{G-2}{G-1} \cdot \frac{{\bar{d}^f}^2 + \bar{d}^f \bar{d}^s + {\bar{d}^s}^2}{(\bar{d}^f + \bar{d}^s)^2} \cdot \frac{1}{c} > 0 \ , \ \ \forall j \in \mathcal{G} \ , \tag{84}$$

as long as $G \geq 3$ holds. (84) gives the optimal symmetric bids of firms at any potential equilibrium.

The bidding problem (78) of consumer l can be expanded as

$$\begin{aligned} \min_{d_l^f} \quad & \lambda^f d_l^f + \lambda^s d_l^s \\ = \ & \frac{\bar{d}_{-l}^f + d_l^f}{\bar{\beta}} \cdot d_l^f + \frac{\sum_{k \in \mathcal{L}} d_k}{\bar{\beta}} (d_l - d_l^f) \\ = \ & \frac{1}{\bar{\beta}} {d_l^f}^2 + \frac{\bar{d}_{-l}^f - \sum_{k \in \mathcal{L}} d_k}{\bar{\beta}} \cdot d_l^f + \text{constant} \end{aligned} \tag{85}$$

with the unique optimal bid as

$$d_l^f = -\frac{1}{2} \bar{d}_{-l}^f + \frac{1}{2} \sum_{k \in \mathcal{L}} d_k \ , \tag{86}$$

which implies the potential equilibrium demand allocation across the two stages:

$$\bar{d}^f = \sum_{l \in \mathcal{L}} d_l^f = \frac{L}{L+1} \sum_{l \in \mathcal{L}} d_l \ , \ \ \text{and} \ \ \bar{d}^s = \sum_{l \in \mathcal{L}} d_l^s = \frac{1}{L+1} \sum_{l \in \mathcal{L}} d_l \ . \tag{87}$$

Combining (84) and (87), we can solve for the explicit equilibrium bids as

$$\beta_j = \frac{L^2 + L + 1}{(L+1)^2} \cdot \frac{G-2}{G-1} \cdot \frac{1}{c} \ , \ \ \forall j \in \mathcal{G} \ , \tag{88a}$$

$$d_l^f = \frac{1}{L+1} \sum_{k \in \mathcal{L}} d_k \ , \ \ \forall l \in \mathcal{L} \ , \tag{88b}$$

$$d_l^s = d_l - d_l^f \ , \ \ \forall l \in \mathcal{L} \ , \tag{88c}$$

which is exactly the Nash equilibrium captured in (79). This completes the proof of Proposition 8.