Equilibrium Analysis of Electricity Markets with Day-Ahead Market Power Mitigation and Real-Time Intercept Bidding

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ABSTRACT

Electricity markets are cleared by a two-stage, sequential process consisting of a forward (day-ahead) market and a spot (real-time) market. While their design goal is to achieve efficiency, the lack of sufficient competition introduces many opportunities for price manipulation. To discourage this phenomenon, some Independent System Operators (ISOs) are planning to sequentially implement system-level market power mitigation policies that replace non-competitive bids, based on an estimation of generator costs, a.k.a. default bids. However, without fully accounting for all participants' incentives (generators and loads), the application of such a policy may lead to unintended consequences. In this paper, we model and study the interactions of generators and inelastic loads in a two-stage settlement where the system operator imposes default bids, based on an estimation of generators' cost function in the day-ahead market. We show that such policy, when accounting for generator and load incentives, leads to a generalized Stackelberg-Nash game where load decisions (leaders) are performed in day-ahead market and generator decisions (followers) are relegated to the real-time market. Furthermore, the use of conventional supply function bidding for generators in real-time, does not guarantee the existence of a Nash equilibrium. This motivates the use of intercept bidding, as an alternative bidding mechanism for generators in the real-time market. An equilibrium analysis in this setting, leads to a closed-form solution that unveils several insights. Particularly, it shows that, unlike standard two-stage markets, loads are the winners of the competition in the sense that their aggregate payments are less than that of the competitive equilibrium. Moreover, heterogeneity in generators cost has the unintended effect of mitigating loads' market power. Finally, an analysis on the effect of overestimation of generation cost and real-time demand uncertainty shows a tendency for generators to benefit from uncertainty in both cases. Numerical studies validate and further illustrate these insights.

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CCS CONCEPTS

• **Hardware → Power and energy**; • **Mathematics of computing** → *Mathematical analysis.*

KEYWORDS

electricity market, two-stage settlement, supply function bidding, Stackelberg game, equilibrium analysis

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1 INTRODUCTION

Many Regional Transmission Organizations (RTOs) and Independent System Operators (ISOs) conduct auctions to settle electricity transactions in a wholesale energy market. Typically, suppliers, e.g., generator owners, offer to sell electricity as a function of price, while consumers, e.g., utilities, offer to purchase electricity to meet their energy demand. After all the bids are collected, the market is cleared achieving supply-demand balance. Such an electricity market often constitutes of a two-stage settlement, namely day-ahead and real-time markets [2]. The first stage is the day-ahead market settlement - which is cleared based on the load forecasts for the next day on an hourly basis and accounts for the majority of energy trading in the market. The second stage, the real-time market settlement, occurs at a faster timescale, typically every five minutes, and it is used for participants to adjust their commitment to account for changes in forecasts [19, 22].

Though such a coupled two-stage market was designed, in spirit, to mitigate any form of speculation and arbitrage, the common price difference between the two stages in practice signals efficiency losses [8, 13]. The discrepancies occur, not only due to uncertainty in load forecast, unscheduled maintenance, or shutdowns, but also due to market manipulation by strategic participants who influence the market to their benefit [4, 21]. To discourage this price manipulation, some operators, like California Independent System Operator (CAISO), are considering imposing system-level policies aimed at substituting in, e.g., day-ahead [1], non-competitive bids with *default bids* that are based on an estimate of generators costs that leverages operator's knowledge of technology, fuel prices, and operational constraints [6].[1] Such market rules are straightforward. However, without accounting for the conflicting interest of individual participants, the effect of this policy in market outcome remains unknown. The goal of this paper is to study such possible policy and mitigate the possible unintended effects.

To this end, we model and study the competition of generators and loads in a two-stage settlement mechanism where generators' bids are substituted by default bids in day-ahead. Though in principle, it seems reasonable to impose such policy in both, day-ahead and real-time markets, such modifications are being considered separately. We thus focus here on analyzing the effect of imposing such a mitigation strategy solely in the day-ahead market, and leave the complementary case of analyzing the constraint in the real-time market as future work. We show that the introduction of such mandate, when combined with the loads' incentive to reduce payments across stages, leads to a form of Stackelberg-Nash game, where loads and generators compete with alike participants, yet as a group loads lead generator in their decision making. Our analysis further shows that in the case when generators bid in real-time using a linear supply function [7, 16, 20], strategic behavior may lead to the non-existence of a Nash equilibrium. In particular, leveraging recent analysis [20], we show that even in the setting of generators with homogeneous costs, a sufficiently large number of loads and the presence of negative demand in real-time can lead to unstable behavior signaled by the lack of a Nash equilibrium. This inability of supply function bidding to guarantee stable market outcome motivates the use of a different bidding mechanism that is better suited for participants that in a given market can generate or consume energy, i.e., prosumers [10, 15].

More precisely, following [10], we propose the use of the intercept of the supply function as the main parameter that generators are allowed to bid. To better differentiate these two different forms of supply function bids, we use (from now on) the term intercept function bidding when generators bid the intercept parameter; for consistency we also refer to the standard supply function mechanism as slope function bidding. A detailed Nash equilibrium analysis of this new market mechanism illustrates several analytic and practical advantages of the proposed solution.

Contributions: The main contributions in this paper are summarized below:

(1) We study a sequential game formulation of market competition among generators/prosumers, that bid the intercept of the supply function and seek to maximize their profit, and loads, that bid demand quantities in the two-stages. The proposed policy leads to a generalized Stackelberg-Nash game, where demand acts as a leader in the day-ahead market, and generators act as followers in the real-time market.
(2) We characterize the competitive equilibrium of such a game and show that it is optimal w.r.t the social planner's problem. We further show that the Nash equilibrium of this game always exists, and can be characterized in closed-form.
(3) To understand the impact of strategic behavior in the market outcome (Nash equilibrium), we further characterize the market equilibrium for the cases of unilateral strategic behavior, i.e., either loads or generators behave strategically. Our analysis broadly suggest that the combination of default bid substitution in day-ahead and real-time intercept bidding successfully limits generators' market power while simultaneously guaranteeing stable market outcomes.
(4) We further provide a detailed numerical study to illustrate several additional insights of the proposed solution, such as the fact that the (fixed) slope parameter of the intercept bidding can be tuned to obtain a Nash equilibrium arbitrarily close to the competitive equilibrium, as well as the odd fact that heterogeneity in generator cost can limit the market power of the game leaders, i.e., loads.
(5) Finally, we discuss the impact of uncertainty in cost estimation and demand randomness on the market equilibria. Our analysis shows that at the Nash equilibrium overestimation of generation cost, and randomness in real-time demand tend to penalize demand by increasing their payments and benefit generators by increasing their revenue.

[1] Such policies are inspired by existing Local Market Power Mitigation strategies that CAISO uses today to mitigate local market power in the presence of congestion [3].

Related work: Several works have analyzed the competition between cross-group participants under different market settings. In particular [10, 14, 16] focus on the strategic behaviors in a single stage, for example, in the day-ahead market, where participants maximize their profit or minimize their payment while participating either as prosumers [10] or as traditional generators and loads [14]. Another related line of work looks at a two-stage market setting where generators participate via the widely adopted linear supply function [7, 16]. In particular, references [11, 18] look at a perfect competition where participants cannot manipulate market prices and accept prevailing prices in the market. Reference [12] analyzes the market competition between cross-group participants for general strategic behavior but lacks theoretical guarantees on the existence of equilibrium in such a market. While reference [20] investigates the impact of strategic participants under the assumption of homogeneous cost functions and symmetric participation. However, despite the extensive studies, to the best of our knowledge, this work is the first one to formally analyze the effect of default bid substitution on the market outcome.

Paper organization: The rest of the paper is organized as follows. In Section 2 we introduce the market model, participants' behavior and generalized Stackelberg-Nash game. In Section 3 we characterize the competitive and Nash equilibrium achieved by intercept function bidding and compare it with the social planner optimal solution. The market power, comparison of different equilibria, and numerical study on the role of market parameters are discussed in Section 4. To streamline the presentation, we relegate the analysis of the slope bidding function to Section 5 and the impact of uncertain market conditions, like the discrepancy in the cost function of generators and demand uncertainty, to Section 6. We provide the conclusions in Section 7.

2 MARKET MODEL

In this section, we first summarize the general market design goal in a social planer problem. We then lay out the two-stage settlement market setup with our proposed intercept bidding function in the real-time market. In the end we formally define different participants' behavior as either competitive or strategic participant.

2.1 Social Planner Problem

Consider a single-interval market in which a set $\mathcal{G}$ of generators interact with a set $\mathcal{L}$ of inelastic loads to meet aggregate demand $d \in \mathbb{R}$. The power output for each generator $j \in \mathcal{G}$ is denoted by $g_j \in \mathbb{R}$ and the inelastic demand of each load $l \in \mathcal{L}$ is denoted by $d_l \in \mathbb{R}$ respectively, where $\sum_{l \in \mathcal{L}} d_l = d$. We assume standard quadratic cost functions for the generators, parameterized by quadratic coefficients c_j, $j \in \mathcal{G}$ respectively. Then the social planner problem that minimizes the cost of dispatching generators to meet the aggregate demand is given by

SOCIAL PLANNER

$$\min_{g_j, j \in \mathcal{G}} \quad \sum_{j \in \mathcal{G}} \frac{c_j}{2} \left(g_j\right)^2 \tag{1a}$$

$$\text{s.t.} \quad \sum_{l \in \mathcal{L}} d_l = \sum_{j \in \mathcal{G}} g_j \tag{1b}$$

where (1b) enforces the power balance over the two-stage settlement operation.

2.2 Two-stage Market Mechanism

In this subsection, we define the market clearing process for the two stages to be studied in the paper, together with the proposed intercept function bids.

2.2.1 Day-Ahead Market: In the day-ahead market, the power output of generator j is denoted by g_j^{DA} and each generator j submit linear supply function as adopted widely in electricity market designs [16]

$$g_j^{DA} = \hat{\beta}_j \lambda^{DA} \tag{2}$$

where λ^{DA} denote the prices in the day-ahead market. The linear supply function is parameterized by $\hat{\beta}_j > 0$ indicating the willingness of generator j to produce g_j^{DA} at the price λ^{DA}. We assume that the market substitute generators' bids with the default bids in the day-ahead market i.e.,

$$\hat{\beta}_j = \frac{1}{c_j} \tag{3}$$

such that the linear supply function is equivalent to truthful quadratic cost in the market. Each load $l \in \mathcal{L}$ bids demand d_l^{DA} in the day-ahead market. Given the bids $(\hat{\beta}_j, d_l^{DA})$ the market operator solves the dispatch problem in the day-ahead market:

Day-Ahead Dispatch

$$\min_{g_j^{DA}, j \in \mathcal{G}} \sum_{j \in \mathcal{G}} \frac{c_j}{2} g_j^{DA^2} \tag{4a}$$

$$\text{s.t.} \sum_{j \in \mathcal{G}} g_j^{DA} = \sum_{l \in \mathcal{L}} d_l^{DA} = d^{DA} \tag{4b}$$

where (4b) constraint enforces the power balance constraint. The optimal solution to the dispatch problem (4) gives the optimal dispatch for the participants and the clearing prices λ^{DA} as function of dual variables associated with the power balance constraint (4b). Each generator $j \in \mathcal{G}$ and load $l \in \mathcal{L}$ produces or consumes g_j^{DA} and d_l^{DA} and are paid $\lambda^{DA} g_j^{DA}$ and $\lambda^{DA} d_l^{DA}$ as part of the market settlement.

2.2.2 Real-Time Market: In the real-time market, the power output of generator j is denoted by g_j^{RT} and the bid for generator j is specified as

$$g_j^{RT} = b\lambda^{RT} - \beta_j \,, \tag{5}$$

where λ^{RT} denote the prices in the real-time market and $b > 0$ is the constant slope parameter indicating positive correlation with prices, i.e., increase in price leads to increase in power supplied. The intercept bidding function bids are parameterized by $\beta_j \in \mathbb{R}$ indicating the willingness of generator j to produce g_j at the price λ^{RT}. In comparison to the slope bidding function where slope of the linear supply function is the parameter, intercept bidding function has intercept as the parameter taking both positive/negative values. This implies that the intercept bidding function allows generator to act as both supplier and consumer in the market. Each load $l \in \mathcal{L}$ partially allocates its inelastic demand d_l in real-time by submitting the quantity bids d_l^{RT}. Given the bids $(\beta_j, j \in \mathcal{G}, d_l^{RT}, l \in \mathcal{L})$ the operator associates a cost function with generator j and solves the real-time economic dispatch

Real-Time Dispatch

$$\min_{g_j^{RT}, j \in \mathcal{G}} \sum_{j \in \mathcal{G}} \left(\frac{1}{2b} g_j^{RT^2} + \frac{\beta_j}{b} g_j^{RT} \right) \tag{6a}$$

$$\text{s.t.} \sum_{j \in \mathcal{G}} g_j^{RT} = \sum_{l \in \mathcal{L}} d_l^{RT} = d^{RT} \tag{6b}$$

where (6b) constraint enforces the power balance constraint and load balance constraint respectively. Similar to the day-ahead market, the optimal solution to the dispatch problem (6) gives the optimal dispatch for the participants and the market clearing prices λ^{RT} as function of dual variables associated with the operational constraints. Each generator $j \in \mathcal{G}$ and load $l \in \mathcal{L}$ produces or consumes g_j^{RT} and d_l^{RT} and are paid $\lambda^{RT} g_j^{RT}$ and $\lambda^{RT} d_l^{RT}$ respectively as part of the real-time market settlement.

2.2.3 Market Goal: In this paper, we are interested in market conditions that lead to market dispatch that also solves the social planner problem, where

$$g_j^{DA} + g_j^{RT} = g_j, \ j \in \mathcal{G} \tag{7a}$$

$$d_l^{DA} + d_l^{RT} = d_l, \ l \in \mathcal{L} \tag{7b}$$

$$\textstyle\sum_{j \in \mathcal{G}} g_j = \sum_{l \in \mathcal{L}} d_l = d \tag{7c}$$

However this is not always possible in which case, we analyse the deviation between the market optimal dispatch and social planner problem.

2.3 Participation Behavior

In this section, for the purpose of our study, we introduce two different types of participant behavior, price-taking and strategic. In either case, the participants are assumed to be rational. That is, generators seek to maximize their profit π_j, and loads aim to minimize their payments ρ_l across the two stages. The generator profit π_j is given by:

Generator Profit

$$\pi_j(g_j^{DA}, g_j^{RT}, \lambda^{DA}, \lambda^{RT}) := \lambda^{RT} g_j^{RT} + \lambda^{DA} g_j^{DA} - \frac{c_j}{2}\left(g_j^{DA} + g_j^{RT}\right)^2 \quad (8)$$

The individual payment ρ_l for load l in the two-stage market is given by:
Load Payment

$$\rho_l(d_l^{DA}, d_l^{RT}, \lambda^{DA}, \lambda^{RT}) := \lambda^{DA} d_l^{DA} + \lambda^{RT} d_l^{RT} \quad (9)$$

2.3.1 Price-Taking Participants: A price taker is defined below:

Definition 2.1. A market participant is price-taking if it accepts the given market prices and cannot anticipate how its own bid affects market prices.

Given the prices in the day-ahead market λ^{DA} and real-time market λ^{RT}, the generator individual problem is given by:
Price-taking Generator Bidding problem

$$\max_{g_j^{RT}} \ \pi_j(g_j^{DA}, g_j^{RT}, \lambda^{DA}, \lambda^{RT}) \quad (10)$$

Substituting the intercept bidding function (5) in (8), we get

$$\begin{aligned}&\pi_j(g_j^{DA}, \beta_j, \lambda^{DA}, \lambda^{RT})\\ &= \lambda^{RT}(b\lambda^{RT} - \beta_j) + \lambda^{DA} g_j^{DA} - \frac{c_j}{2}\left(g_j^{DA} + b\lambda^{RT} - \beta_j\right)^2 \quad (11)\end{aligned}$$

Further substituting the linear supply function and the default bid in the day-ahead market (2),(3) in (11), we can simplify the generator profit π_j as

$$\begin{aligned}&\pi_j(\beta_j, \lambda^{DA}, \lambda^{RT})\\ &= \lambda^{RT}(b\lambda^{RT} - \beta_j) + \frac{1}{c_j}\lambda^{DA^2} - \frac{c_j}{2}\left(\frac{1}{c_j}\lambda^{DA} + b\lambda^{RT} - \beta_j\right)^2 \quad (12)\end{aligned}$$

Therefore the individual problem for generator j is given by:

$$\max_{\beta_j} \ \pi_j(\beta_j, \lambda^{DA}, \lambda^{RT}) \quad (13)$$

Similarly given the prices $\lambda^{DA}, \lambda^{RT}$, the individual bidding problem for load is given by:
Price-taking Load Bidding problem

$$\min_{d_l^{DA}, d_l^{RT}} \ \rho_l(d_l^{DA}, d_l^{RT}, \lambda^{DA}, \lambda^{RT}) \quad (14)$$

Substituting the coupling constraint for the load allocation across two stages (7b) in (9) we get,

$$\rho_l(d_l^{DA}, \lambda^{DA}, \lambda^{RT}) := \lambda^{DA} d_l^{DA} + \lambda^{RT}(d_l - d_l^{DA}) \quad (15)$$

Therefore the individual problem for load l is given by:

$$\min_{d_l^{DA}} \rho_l(d_l^{DA}, \lambda^{DA}, \lambda^{RT}) \quad (16)$$

For each load, $l \in \mathcal{L}$ the allocation in day-ahead market d_l^{DA} determines its allocation in the real-time market d_l^{RT} due to the demand inelasticity in the market. We next define the price-anticipating (or strategic) participants.

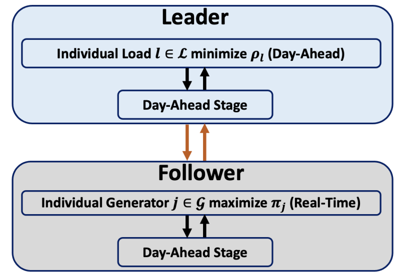

Figure 1: Stackelberg-Nash game between the participants

2.3.2 Price-Anticipating Participants. A strategic participant in the two-stage settlement market is defined below:

Definition 2.2. A market participant is price anticipating, or strategic, if it has complete knowledge of other participants' bids and the effect of its own bids on the prices in the two stages.

Since the market substitutes the generators' bids with default bids in the day-ahead, they behave strategically only in the real-time market, thus the individual problem is given by:
Strategic Generator Bidding problem

$$\max_{\beta_j} \ \pi_j\left(\beta_j, \lambda^{DA}\left(d^{DA}\right), \lambda^{RT}\left(\beta_j, \overline{\beta}_{-j}, d^{RT}\right)\right) \quad (17a)$$

$$\text{s.t. } (4), (6) \quad (17b)$$

where $\overline{\beta}_{-j} := \sum_{k \in \mathcal{G}, k \neq j} \beta_k$. The generator j maximizes its profit while anticipating the market clearing prices in the day-ahead market and real-time market (4),(6), and with complete knowledge of load bids $d_l^{DA}, d_l^{RT}, l \in \mathcal{L}$ in the market. Similarly, the individual problem for strategic load l with complete knowledge of prices in the day-ahead and real-time clearing (4),(6) is given by:
Strategic Load Bidding problem

$$\min_{d_l^{DA}} \ \rho_l\left(d_l^{DA}, \lambda^{DA}\left(d^{DA}\right), \lambda^{RT}\left(\beta_j, \overline{\beta}_{-j}, d^{RT}\right)\right) \quad (18a)$$

$$\text{s.t. } (4), (6) \quad (18b)$$

where the load l minimizes its payment in the two-stage market.

2.4 Stackelberg-Nash Interpretation of Sequential Game

We now provide an alternative formulation of the sequential game between price-anticipating participants, where the day-ahead market clears before the real-time market. We analyze the game backward from the real-time market where generators make decisions while participating as prosumers in the market and compute the equilibrium path. Since the inelastic loads only make decisions in the day-ahead market, this two-stage sequential game can be viewed as a leader-follower Stackelberg-Nash game with generators as a follower in the real-time market and loads as a leader in the day-ahead market, while each group of participants competes in the Nash game amongst themselves. Several works under different names have analyzed similar formulations in various markets [9, 17]. Though the terminology varies in the literature, here we follow the terminology used in [17]. The interaction structure is further illustrated in Figure 1. There, each generator $j \in \mathcal{G}$

maximize its individual profit (17a) using only the real-time bid parameter β_j which is used by the real-time market after the loads decisions d_l^{DA} has already been made. The load anticipates the behaviour of the generator in the real-time stage and allocates its demand d_l^{DA} by accounting for its effect on the real-time prices via $\lambda^{RT}(\beta_j, \beta_{-j}, d - d^{DA})$ in (18).

3 MARKET EQUILIBRIUM

In this section, we study the two-stage market equilibrium where no participant has an incentive to deviate from its current bid and both stages are cleared, as defined below.

Definition 3.1. We say the participant bids and market clearing prices $(g_j^{DA}, \beta_j, j \in \mathcal{G}, d_l^{DA}, d_l^{RT}, l \in \mathcal{L}, \lambda^{DA}, \lambda^{RT})$ in the day-ahead and real-time respectively form a two-stage market equilibrium if the following conditions are satisfied:

(1) For each generator $j \in \mathcal{G}$, the bid β_j maximizes their individual profit.
(2) For each load $l \in \mathcal{L}$, the allocation d_l^{DA}, d_l^{RT} minimizes their individual payment.
(3) The inelastic demand $d \in \mathbb{R}$ is satisfied with the market-clearing prices λ^{DA} given by (4) and λ^{RT} given by (6) over the two-stages of the market.

We will analyze particularly cases where all participants are either price-taking or strategic. These cases respectively lead to a competitive equilibrium and a Nash equilibrium. We will use the term strategic equilibrium in this paper to refer to Nash equilibrium.

3.1 Competitive Equilibrium

We first look at the competitive equilibrium when all the participants are price-takers. Given the market clearing prices λ^{RT} each generator $j \in \mathcal{G}$ maximize its profit (13). Similarly, given both clearing prices $\lambda^{DA}, \lambda^{RT}$, each load $l \in \mathcal{L}$ minimizes its individual payment (16). This setting leads to a set of competitive equilibrium, as characterized below.

Theorem 3.2. *The competitive equilibrium in the two-stage market mechanism exists as given by:*

$$g_j^{DA} = \frac{1}{c_j} \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ g_j^{RT} = 0,\ \forall j \in \mathcal{G} \tag{19a}$$

$$d_l^{DA} + d_l^{RT} = d_l\ \forall l \in \mathcal{L},\ d^{DA} = d,\ d^{RT} = 0 \tag{19b}$$

$$\beta_j = b\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \forall j \in \mathcal{G},\ \lambda^{DA} = \lambda^{RT} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{19c}$$

We provide the proof of the theorem in Appendix A. At the competitive equilibrium the load allocates all the demand in the day-ahead market even though the market clearing prices are equal across the two-stages of the market. This is desired as the majority of energy trading occurs in the day-ahead market, however it is not generally satisfied by other mechanisms [20].

Moreover, the equilibrium attains the desirable social planner objective in the market, as summarized below.

Corollary 3.3. *The competitive equilibrium* (19) *corresponds to an optimal solution to the social planner problem* (1).

The corollary uses the fact that the competitive equilibrium satisfies the KKT conditions associated with the convex social planner problem (1). We also consider the competitive equilibrium as a benchmark and compare it with the outcome of other forms of participation.

3.2 Nash Equilibrium

We next analyze the interplay between strategic generators $j \in \mathcal{G}$ and the strategic loads $l \in \mathcal{L}$ in the two-stage market setting. We first propose a theorem that will enable us to characterize the Nash equilibrium in the real-time market. In this generalized Stackelberg-Nash game, each strategic generator $j \in \mathcal{G}$ with dispatch g_j^{RT} also solves the underlying augmented convex social planner problem as defined:

Theorem 3.4. *Assume that there are at least two generators participating in the market, i.e.,* $|\mathcal{G}| \geq 2$. *Given the day-ahead dispatch* g_j^{DA}, *the real-time subgame equilibrium* (g_j^{RT}, λ^{RT}) *also corresponds to the optimal primal-dual solution of an augmented convex social planner problem and associated power balance constraint are given by:*

$$\min_{g_j^{RT}} \sum_{j\in\mathcal{G}} \left(\frac{1}{2b(G-1)} {g_j^{RT}}^2 + \frac{c_j}{2}\left(g_j^{DA} + g_j^{RT}\right)^2 \right) \tag{20a}$$

$$s.t.\ (6b) \tag{20b}$$

We provide the proof of the theorem in Appendix B. The real-time subgame shifts the dispatch of the generators compared to the competitive equilibrium (19) due to the strategic participation of generators. Hence the resulting Nash equilibrium is not socially optimal. Notice that the optimization problem is strongly convex since (20a) is a quadratic function of g_j^{RT} which implies that the real-time dispatch g_j^{RT} is unique. We use the closed-form dual solution for λ^{RT} in characterizing the closed-form Nash equilibrium.

The load in the day-ahead market anticipates the prices in the real-time subgame played by the follower and minimizes its payment while acting as leader in the Stackelberg-Nash game as shown in the Figure 1. The following theorem characterize the two-stage Nash equilibrium that satisfies the Definition (3.1).

Theorem 3.5. *If there are at least two generators participating in the market, i.e.,* $|\mathcal{G}| \geq 2$, *then the two-stage Nash equilibrium exists and is uniquely given by:*

$$g_j^{DA} = \frac{1}{c_j}\left(1 - \frac{1}{L+1}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, \forall j \in \mathcal{G} \tag{21a}$$

$$g_j^{RT} = \frac{1}{C_j}\frac{1}{L+1}\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, \tag{21b}$$

$$d_l^{DA} = d_l + \left(\frac{1}{L+1}d - d_l\right)\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \forall l \in \mathcal{L} \tag{21c}$$

$$d_l^{RT} = \left(d_l - \frac{1}{L+1}d\right)\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \forall l \in \mathcal{L} \tag{21d}$$

$$\beta_j = \left(b - \frac{b}{L+1}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}} - \frac{1}{C_j}\frac{1}{L+1}\right)\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \forall j \in \mathcal{G} \tag{21e}$$

$$\lambda^{DA} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} - \frac{1}{L+1}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, \tag{21f}$$

$$\lambda^{RT} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} + \frac{1}{L+1}\left(1 - \frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{21g}$$

where $C_j = \frac{1}{b(G-1)} + c_j$.

We provide the proof the theorem in Appendix C. Here $C_j = \frac{1}{b(G-1)} + c_j > c_j$ can be understood as the augmented cost coefficient of dispatching generators in the real-time market. The strategic generators manipulate the market by increasing the clearing prices in the real-time market. However, this ability to manipulate the market eventually diminishes as the number of generators increases in the market as discussed in the next section. Moreover, when there is only one firm, no Nash equilibrium exists and such a case reflects monopoly of generators in the market. In a monopoly the unique generator can bid arbitrarily large bids and earn arbitrarily large profit from the inflated prices it generates in the market.

Corollary 3.6. *The load allocation across the two stages at the Nash equilibrium* (21) *is given by:*

$$\textstyle\sum_{l\in\mathcal{L}} d_l^{DA} = \left(1 - \frac{1}{L+1}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right) d \in \left(\frac{d}{2}, d\right) \tag{22a}$$

$$\textstyle\sum_{l\in\mathcal{L}} d_l^{RT} = \frac{1}{L+1}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}} d \in \left(0, \frac{d}{2}\right) \tag{22b}$$

The proof uses the individual demand allocation at the Nash equilibrium (21) and the fact that $C_j > c_j$.

4 MARKET ANALYSIS

In this section, we study the impact of participants' strategic behavior as well as market parameters. We start by benchmarking market equilibria against the competitive equilibrium (19) as we toggle a particular group of participants, generators or loads, between price-taking and strategic. Although some equilibria are of no practical relevance, they allows us to separately understand the potential capability of different participants in market manipulation. We then analyze the impact of the market supply elasticity and the generator heterogeneity on market outcomes, and illustrate our analysis via numerical case studies.

4.1 Equilibrium Comparison For Different Participant Behaviours

We first propose two theorems that characterize the market equilibria when only one group of generators or loads is strategic while the other remains price-taking.

Theorem 4.1. (Generator Side Nash Equilibrium) *The market equilibrium with strategic generators and price-taking loads in the two-stage market exists and is uniquely given by:*

$$g_j^{DA} = \frac{1}{c_j}\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ g_j^{RT} = 0,\ \forall j \in \mathcal{G} \tag{23a}$$

$$d_l^{DA} + d_l^{RT} = d_l,\ \forall l \in \mathcal{L} \tag{23b}$$

$$\beta_j = b\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \forall j \in \mathcal{G},\ \lambda^{DA} = \lambda^{RT} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{23c}$$

Theorem 4.2. (Load Side Nash Equilibrium) *The market equilibrium with strategic loads and price-taking generators in the two-stage market exists and is uniquely given by:*

$$g_j^{DA} = \frac{L}{L+1}\frac{1}{c_j}\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, g_j^{RT} = \frac{1}{L+1}\frac{1}{c_j}\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, \forall j \in \mathcal{G} \tag{24a}$$

$$d_l^{DA} = \frac{1}{L+1}d,\ d_l^{RT} = d_l - \frac{1}{L+1}d,\ \forall l \in \mathcal{L} \tag{24b}$$

$$\beta_j = \left(b - \frac{1}{L+1}\frac{1}{c_j}\right)\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \forall j \in \mathcal{G} \tag{24c}$$

$$\lambda^{DA} = \frac{L}{L+1}\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, \lambda^{RT} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{24d}$$

We provide the proof of both theorems in Appendix D and Appendix E, respectively.

We now analyze the shift in the market outcome and efficiency loss due to the strategic bidding of participants as compared to the competitive equilibrium. For this we use the widely accepted metric like the social cost, aggregate profit of participants, and aggregate payment of participants as shown in Table 1.

4.1.1 Insights:

Generator Side Nash Equilibrium: In the case of only strategic generators, the market equilibrium found in Theorem 4.1 shows that generators cannot take advantage of their ability to manipulate prices (23c), since price-taking loads tend to allocate all the demand in the day-ahead market (23b). Though generators are the only participants with bidding flexibility in the real-time market, their strategic decisions cannot earn them any extra profit. This is partly attributed to their required truthfulness in the day-ahead market. Bidding in only one stage grant them no market opportunity, in the presence of price-taking loads.

Load Side Nash Equilibrium: On the contrary, with only strategic loads, i.e., the load side Nash equilibrium, the allocation (24) deviates from the competitive equilibrium (19) and favors loads. This is because in both stages the market clearing prices reflect marginal generations costs, with generators default bid in the day-ahead market and price-taking in the real-time market. The implication that the real-time price always equals the final system marginal cost $\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}$ is taken advantage of by loads to strategically participate in the day-ahead market with a lower price. However, more demand will raise the day-ahead price and the dilemma drives loads to reach the equilibrium where each of them allocates the same amount of day-ahead demand d_l^{DA} in (24b), regardless of sizes. As a whole, this change of market outcome is mainly reflected as a shift in surplus allocation, as shown in the third row of Table 1. The social cost remains the same while part of generators' profit is shifted to reduce loads' payment. However, we notice that as the number of loads increases, the increasing competition diminishes such a shift and the load side market equilibrium (24) converges to the competitive equilibrium (19). Remarkably, the number of generators does not contribute to the competition.

Table 1: Comparison between Competitive Equilibrium (CE), Generator Side Nash Equilibrium (GNE), Load Side Nash Equilibrium (LNE) and Nash Equilibrium (NE)

Instance	Social Cost	Generators Aggregate Profit	Loads Aggregate Payment
CE	$\frac{1}{2}\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}$	$\frac{1}{2}\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}$	$\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}$
GNE	$\frac{1}{2}\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}$	$\frac{1}{2}\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}$	$\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}$
LNE	$\frac{1}{2}\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}$	$\frac{1}{2}\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}\left(1-\frac{2L}{(L+1)^2}\right)$	$\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}\left(1-\frac{L}{(L+1)^2}\right)$
NE	$\frac{1}{2}\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}\left(1+\frac{1}{\sum_{j\in\mathcal{G}} c_j^{-1}}\frac{\Delta}{(L+1)^2}\right)$	$\frac{1}{2}\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}\left(1-\frac{2L}{(L+1)^2}\right)+\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}\left(\left(1-\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)\frac{L}{(L+1)^2}-\frac{1}{2}\frac{1}{\sum_{j\in\mathcal{G}} c_j^{-1}}\frac{\Delta}{(L+1)^2}\right)$	$\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}\left(1-\frac{L}{(L+1)^2}\right)+\frac{d^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}\left(1-\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)\frac{L}{(L+1)^2}$

where $\Delta := \left(\sum_{j\in\mathcal{G}}\frac{c_j}{C_j^2}-\frac{\left(\sum_{j\in\mathcal{G}} C_j^{-1}\right)^2}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right)$, $C_j = \frac{1}{b(G-1)}+c_j$

Nash Equilibrium: Our main result points to the Nash equilibrium (21) that features the strategic interplay of generators and loads. Compared with the load side market equilibrium (24), generators do strive for more profit by bidding strategically in the real-time market as in (21e), which leads to an inflated real-time price beyond the system marginal cost $\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}$. As a result, demand is driven towards the day-ahead market that also raises the day-ahead price in (21f). Market-wise, the Nash equilibrium incurs a higher social cost (the fourth row of Table 1) due to generators' strategic bidding.

Corollary 4.3. *The aggregate profit of generators at Nash equilibrium* (21) *is always less than that at competitive equilibrium* (19).

We finally remark on the impact of generator heterogeneity.

Corollary 4.4. *With homogeneous generators, i.e.* $c_j = c, \forall j \in \mathcal{G}$*, the market achieves the minimum social cost compared to the competitive equilibrium.*

The corollary follows from the fact of $\Delta = 0$ in Table 1 when generators are homogeneous.

4.2 Numerical Study: Impact of Market parameters

We now analyze the impact of market parameters on the Nash equilibrium in Theorem 3.5 using a numerical case study. For ease of analysis, we consider the test case of 2 strategic generators and 4 strategic loads in two-stage market setting. The individual aggregate inelastic demand bids for a mix of smaller and larger loads are given by $d_l = [0.2, 25.6, 106.6, 199.6]^T MW$ from the Pennsylvania, New Jersey, and Maryland (PJM) data miner day-ahead demand bids [5] with total aggregate inelastic demand $d = 332\ MW$.

First we look at the impact of the homogeneous slope constant b of the intercept bidding function (5) on the Nash equilibrium (21). Figure 2 illustrates the payment of loads and profit of generators as we increase the parameter b in the top panel and bottom panel respectively. In this case we fix the heterogeneous cost coefficients of the generators to be $c = [0.1, 0.11]^T \$/(MW)^2$ corresponding to the cost coefficients from the IEEE 300-bus system [21, 23]. As b increases, $C_j \to c_j$ and loads payment decreases, see bottom row in Table 1. This decrease in payment is attributed to the lower profit of generators in the market as shown in top and bottom panel in Figure 2. Though the larger value of b impacts the individual interests, its impact on social cost diminishes as $C_j \to c_j$ and

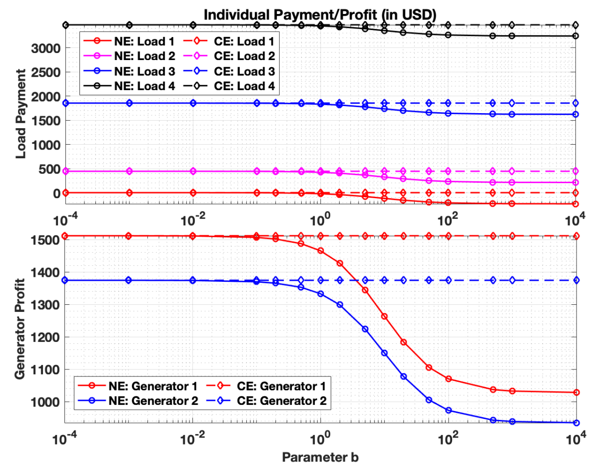

Figure 2: Individual load payment and generator profit w.r.t market parameter b for Nash Equilibrium (NE) and Competitive Equilibrium (CE).

$\Delta \to 0$ as shown in Table 1. Interestingly, the social cost at Nash equilibrium further aligns with the competitive equilibrium for smaller values of b as $C_j^{-1} \to 0$ and $\Delta \to 0$. This alignment suggests that market operator can possibly optimize for the parameter b to reduce efficiency loss in the case of Nash equilibrium and restore market efficiency.

In Figure 3 we show the demand allocation and generators dispatch in the two-stage as we change the parameter b in the left and right panel respectively. For this case also, we keep the same cost coefficients of generators $c = [0.1, 0.11]^T \$/(MW)^2$. The allocation at Nash equilibrium deviates from the competitive equilibrium due to price manipulation by participants. As b increase $C_j \to c_j$ and prices in the real-time market decreases (21g) leading to higher allocation of loads (21d) and higher generator dispatch (21b) in the real-time market. Amongst all the loads, smaller loads increase their allocation in the day-ahead market leading to negative demand allocation in the real-time stage at the expense of lower generator profit, and therefore as b increases, it earns money instead of making payments owing to higher prices in the real-time stage as shown in the Figure. 3. This also implies that loads have an incentive to break their demand into multiple smaller units, we skip such analysis in this paper for future work.

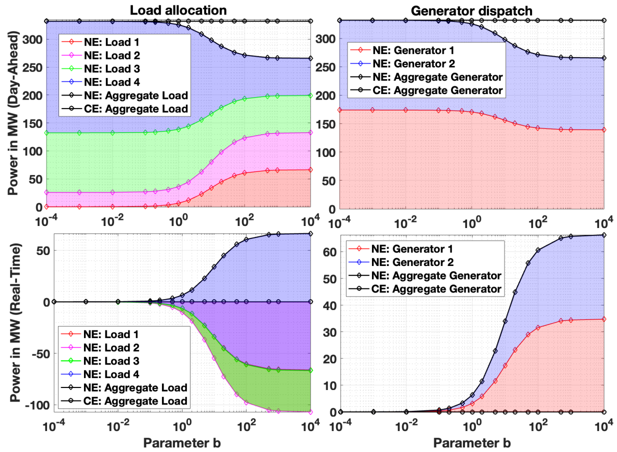

Figure 3: Day-Ahead and Real-Time load allocation and generator dispatch w.r.t parameter b at Nash Equilibrium (NE) and Competitive Equilibrium (CE).

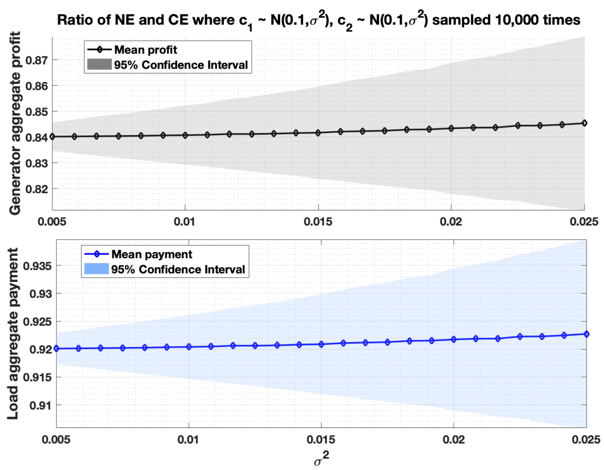

Figure 4: Ratio of Nash Equilibrium (NE) and Competitive Equilibrium (CE) for aggregate individual profit or payment w.r.t σ^2; $c_1 \sim \mathcal{N}(0.1, \sigma^2)$, $c_2 \sim \mathcal{N}(0.1, \sigma^2)$ sampled 10,000 times.

Now we analyze the impact of heterogeneity of generators in terms of different cost coefficients on Nash equilibrium (21). The coefficients are sampled 10, 000 times from a normal distribution with mean 0.1 as we increase σ^2, i.e. $c_1 \sim \mathcal{N}(0.1, \sigma^2)$, $c_2 \sim \mathcal{N}(0.1, \sigma^2)$ for fixed value of parameter $b = 10$. The top panel and bottom panel in Figure 4 plot mean value and 95% confidence interval for aggregate payment of loads and aggregate profit of generators at Nash equilibrium normalized with aggregate value of the competitive equilibrium, respectively. As σ^2 increases, we see two phenomena overlapping. First, it is more likely to obtain instances where generators are highly heterogeneous. Second, it also more likely to obtain different absolute values for $\sum_{j\in\mathcal{G}} c_j^{-1}$. This interplay leads to opportunities for generators (resp. loads) to increase (resp. decrease) their profit (resp. payments) and vice versa. The exact mechanism of this phenomena is further discussed below.

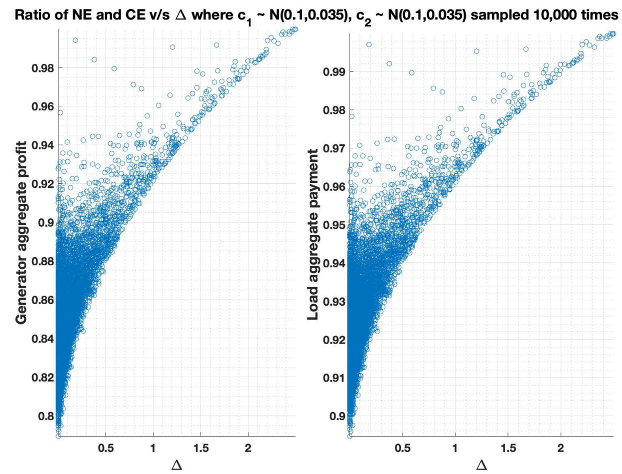

Figure 5: Ratio of Nash Equilibrium (NE) and Competitive Equilibrium (CE) for aggregate profit or payment w.r.t Δ; $c_1 \sim \mathcal{N}(0.1, 0.035)$, $c_2 \sim \mathcal{N}(0.1, 0.035)$ sampled 10,000 times.

We next analyze the relation between the aggregate profit or aggregate payment of participants w.r.t Δ, which serves as measure of the heterogeneity of the generators. The cost coefficients are again sampled 10, 000 times from a normal distribution with mean 0.1 and fixed variance 0.035, i.e. $c_1 \sim \mathcal{N}(0.1, 0.035)$, $c_2 \sim \mathcal{N}(0.1, 0.035)$ for fixed value of parameter $b = 10$. The parameter Δ associated with the generators depends non-linearly on the cost coefficients as shown in Table 1 and approaches to 0 as the difference between the cost coefficients $|c_1 - c_2| \longrightarrow 0$. In such a case, the ratio of aggregate profit of generators at the Nash equilibrium and competitive equilibrium is given by:

$$1 - \left(1 - \frac{1}{1 + bc(G-1)}\right) \frac{2L}{(L+1)^2}$$

Observe that as $c \longrightarrow 0$, the aggregate profit of generators aligns with the competitive equilibrium and misalignment increase with increase in cost coefficient c. Similar observations can be made in the case of heterogeneous generators also. In particular, if $|c_1 - c_2| \approx c_1$, or equivalently one of the generator is extremely cheap ($c_2 << c_1$), then $\left(1/\sum_{j\in\mathcal{G}} c_j^{-1}\right) \longrightarrow 0$ and the ratio of profit as mentioned in Table 1 and given by:

$$\left(1 - \frac{2L}{(L+1)^2}\right) + \left(\left(1 - \frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}}\right) \frac{2L}{(L+1)^2} - \frac{1}{\sum_{j\in\mathcal{G}} c_j^{-1}} \frac{\Delta}{(L+1)^2}\right)$$

aligns with competitive equilibrium. The left panel of the Figure 5 illustrates the ratio as Δ increases signaling higher levels of heterogeneity in generator cost. This implies that market with even one cheap generator can counter the market power of all the strategic participants.

5 EQUILIBRIUM COMPARISON WITH SLOPE BIDDING FUNCTION

In this section, we compare the intercept function bidding with the conventional linear supply function or as we call it, slope function bidding for the real-time market [7, 16]. More formally, in the case of linear supply function or the slope function bidding, the bid for generator j in real-time is specified as

$$g_j^{RT} = \hat{\beta}_j \lambda^{RT} \tag{25}$$

where λ^{RT} denote the prices in the real-time market. The supply function bids are parameterized by $\hat{\beta}_j \in \mathbb{R}$ indicating the willingness of generator j to produce g_j at the marginal price. We first characterize the competitive equilibrium of the two-stage market.

Theorem 5.1 (Proposition 1 [20]). *The competitive equilibrium in the two-stage market mechanism exists and is given by:*

$$g_j^{DA} = \frac{1}{c_j} \frac{d}{\sum_{j \in \mathcal{G}} c_j^{-1}},\ g_j^{RT} = 0,\ \forall j \in \mathcal{G} \tag{26a}$$

$$d_l^{DA} + d_l^{RT} = d_l,\ \forall l \in \mathcal{L},\ d^{DA} = d,\ d^{RT} = 0 \tag{26b}$$

$$\hat{\beta}_j = 0,\ \lambda^{DA} = \lambda^{RT} = \frac{d}{\sum_{j \in \mathcal{G}} c_j^{-1}} \tag{26c}$$

The competitive equilibrium characterized in Theorem 5.1 is ill-defined. In particular, the individual bids $\hat{\beta}_j$ and demand in the real-time market d^{RT} are both zero. Therefore prices in the real-time market given by,

$$\lambda^{RT} = \frac{d^{RT}}{\sum_{j \in \mathcal{G}} \hat{\beta}_j}$$

are not well defined and assumed to be same as prices in the day-ahead market [20].

For the case of Nash equilibrium, we assume that participants are homogeneous in the sense that they share the same cost function, i.e. $c_j := c, \forall j \in \mathcal{G}$ and bid symmetrically or take identical positions in the market i.e. $\beta_j := \beta, \forall j \in \mathcal{G}$ to compare the two mechanisms, since the closed-form analysis is otherwise hard for slope bidding function without these assumptions. The following theorem characterize the two-stage Nash equilibrium or strategic equilibrium in such a market that satisfies the Definition (3.1).

Theorem 5.2. *Assume that generators are homogeneous and bid symmetrically in the market. If there are at least three generators participating in the market i.e. $|\mathcal{G}| \geq 3$ and number of individual load satisfies $L < G - 2$, then the symmetric strategic market equilibrium of the market with homogeneous participants in such a two-stage setting exists and is uniquely given by:*

$$g_j^{DA} = \frac{L}{L+1} \frac{G-1}{G-2} \frac{d}{G},\ g_j^{RT} = \left(1 - \frac{L}{L+1} \frac{G-1}{G-2}\right) \frac{d}{G},\ \forall j \in \mathcal{G} \tag{27a}$$

$$d_l^{DA} = \frac{1}{L+1} \frac{G-1}{G-2} d,\ d_l^{RT} = \left(d_l - \frac{1}{L+1} \frac{G-1}{G-2} d\right),\ \forall l \in \mathcal{L} \tag{27b}$$

$$\hat{\beta}_j = \frac{1}{c} \left(\frac{G-2}{G-1} - \frac{L}{L+1}\right),\ \forall j \in \mathcal{G} \tag{27c}$$

$$\lambda^{DA} = \frac{L}{L+1} \frac{G-1}{G-2} \frac{cd}{G},\ \lambda^{RT} = \frac{G-1}{G-2} \frac{cd}{G}. \tag{27d}$$

Theorem 5 is an immediate result of Proposition 4 in [20] with truthful bids of generators in the day-ahead market. However, if the number of generators and loads satisfies

$$L \geq G - 2$$

then no symmetric Nash equilibrium exists and demand is non-positive in the real-time market. In such a case, the first order condition implies that the optimal bid $\hat{\beta}_j < 0$ and the clearing price in real-time $\lambda^{RT} > 0$ which means generators act as load and each generator j pays $\lambda^{RT} g_j^{RT}$ as part of the market settlement. However assuming all the generators bid $\hat{\beta}_j > 0$, then each generator j dispatch $g_j^{RT} < 0$ at the clearing prices $\lambda^{RT} < 0$ which leads to positive revenue as compare to earlier case. The symmetric bid $\hat{\beta}_j > 0$ fails to satisfy the first order condition and symmetric equilibrium does not exist. Since the individual bid $\hat{\beta}_j$ also depends on the given bids from other participants, the closed-form analysis is not easy to deal with and equilibrium existence cannot be guaranteed.

However in the case of intercept bidding with strategic participants, the equilibrium exists uniquely and no participant has an incentive to deviate from it. It also limits generators' market power, a property usually desired by system operators, and the motivating factor for substituting generators' bids with default bids in day-ahead. Moreover as a byproduct, our analysis shows that the intercept bidding mechanism also gives the equilibrium in the case of a duopoly among generators ($G = 2$), which is not the case for slope bidding function [20].

6 IMPACT OF MARKET UNCERTAINTY

In this section, we summarize the impact of uncertainties on market equilibrium. We consider respectively two sources — discrepancy in the generator dispatch cost (across stages) and demand uncertainty.

6.1 Discrepancy in Cost of Generator Dispatch

The discrepancy in the cost of generator dispatch can be understood as either due to inaccurate estimation of the truthful cost function in the day-ahead by the operator or high dispatch cost in real-time due to unplanned ramping of generator resources. We capture this discrepancy with a parameter $\epsilon_j \in \mathbb{R}$ that denotes the error in estimation or the difference in the cost function in the two stages such that given the bids d_l^{DA}, the market operator solves the dispatch problem in the day-ahead market as:

Day-Ahead Dispatch

$$\min_{g_j^{DA}, j \in \mathcal{G}} \sum_{j \in \mathcal{G}} \frac{c_j + \epsilon_j}{2} g_j^{DA^2} \tag{28a}$$

$$\text{s.t.}\ \ (4b) \tag{28b}$$

The generators submit the intercept function bidding (5) in the real-time market such that the real-time dispatch problem is still (6). Moreover, each generator j (load l) maximizes its profit (minimizes its payment) as introduced in Section 2.3.

For brevity, we assume $\epsilon_j = \epsilon c_j,\ \forall j \in \mathcal{G}$, for a constant parameter $\epsilon \in \mathbb{R}$. However, the results generalize for any arbitrary ϵ_j. We denote by $(\overline{g}_j^{DA}, \overline{g}_j^{RT}, \overline{\beta}_j\ \forall j \in \mathcal{G}, \overline{d}_l^{DA}, \overline{d}_l^{RT}\ \forall l \in \mathcal{L}, \overline{\lambda}^{DA}, \overline{\lambda}^{RT})$ the market equilibrium in Theorem 3.2 and Theorem 3.5 for the case of $\epsilon = 0$. We first characterize the competitive equilibrium

Theorem 6.1. *The competitive equilibrium in the two-stage market mechanism exists, and is given by:*

$$g_j^{DA} = \frac{1}{1+\epsilon} \overline{g}_j^{DA},\ g_j^{RT} = \frac{\epsilon}{1+\epsilon} \frac{1}{c_j} \frac{d}{\sum_{j=1}^{G} c_j^{-1}},\ \forall j \in \mathcal{G} \tag{29a}$$

$$d_l^{DA} + d_l^{RT} = d_l;\ d^{DA} = \frac{1}{1+\epsilon}\bar{d}^{DA},\ d^{RT} = \frac{\epsilon}{1+\epsilon}d \tag{29b}$$

$$\beta_j = \bar{\beta}_j - \frac{1}{c_j}\frac{\epsilon}{1+\epsilon}\frac{d}{\sum_{j=1}^{G} c_j^{-1}},\ \forall j \in \mathcal{G} \tag{29c}$$

$$\lambda^{DA} = \lambda^{RT} = \bar{\lambda}^{DA} = \bar{\lambda}^{RT} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{29d}$$

We provide the proof of the theorem in Appendix F. The competitive equilibrium in Theorem 6.1 still aligns with the social optimum (1), but the parameter ϵ_j reallocates load partially into the real-time market, as shown in (29b). At the equilibrium, the load enforces equal prices in the two stages. However, due to the existence of ϵ in the day-ahead, the generator's marginal cost is cheaper (expensive) when $\epsilon < 0$ (> 0), which leads to a higher (lower) day-ahead load allocation to guarantee equal prices at the equilibrium. The following theorem characterizes the two-stage Nash equilibrium.

Theorem 6.2. *If there are at least two generators participating in the market, i.e., $|\mathcal{G}| \geq 2$, then the two-stage Nash equilibrium exists and is uniquely given by*

$$g_j^{DA} = \left(1 + \epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}}\right)^{-1} \bar{g}_j^{DA}, \forall j \in \mathcal{G} \tag{30a}$$

$$g_j^{RT} = ((1+\epsilon(L+1))\left(1 + \epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}}\right)^{-1} \bar{g}_j^{RT}, \forall j \in \mathcal{G} \tag{30b}$$

$$d_l^{DA} = \left(1 + \epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}}\right)^{-1} \bar{d}_l^{DA},\ \forall l \in \mathcal{L} \tag{30c}$$

$$d_l^{RT} = \left(1 + \epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}}\right)^{-1} \left(\epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j\in\mathcal{G}} c_j^{-1}} + \bar{d}_l^{RT}\right),\ \forall l \in \mathcal{L} \tag{30d}$$

$$\lambda^{DA} = ((1+\epsilon)\left(1 + \epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}}\right)^{-1} \bar{\lambda}^{DA}, \tag{30e}$$

$$\lambda^{RT} = \epsilon\left(1 + \epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}}\right)^{-1} d + \left(1 + \epsilon\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}}\right)^{-1} \bar{\lambda}^{RT} \tag{30f}$$

where $C_j = \frac{1}{b(G-1)} + c_j$.

We provide the proof the theorem in Appendix G. Similarly, the parameter ϵ_j partially shifts the demand to the real-time market, yet with a price increase (for $\epsilon > 0$) in the day-ahead market due to

$$\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}} < 1$$

We use an example with 2 generators and 4 loads to numerically illustrate the individual profit and payment for each generator j and load l that depends non-linearly on the parameter ϵ, respectively, in Figure 6 . In this case, we fix the cost coefficients of generators as $c = [0.1, 0.11]^T \$/MW^2$ and slope parameter $b = 0.5$. As ϵ changes, the generators in the day-ahead market become relatively cheaper (expensive) — indicated by the left (right) region of the x-axis in Figure 6. The overestimation of generation cost penalizes demand in the market by increasing demand payment shown in the top panel and increasing generators' revenue shown in the bottom panel in Figure 6.

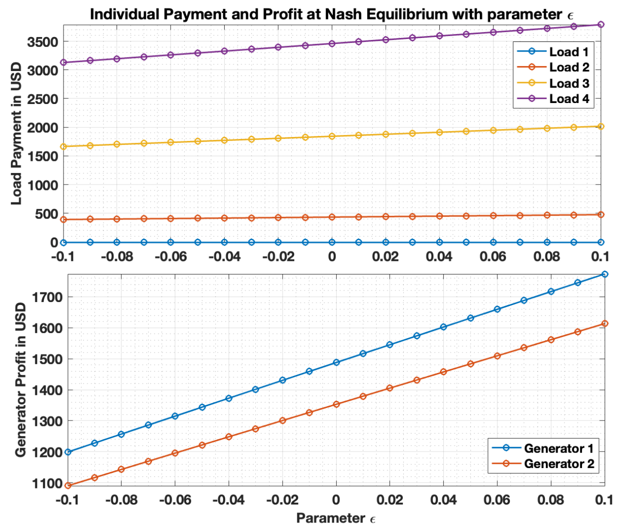

Figure 6: Individual load payment and generator profit w.r.t market threshold parameter ϵ at Nash Equilibrium.

6.2 Demand Uncertainty

To understand the impact of demand uncertainty, we assume that demand of each individual load l is denoted by $\tilde{d}_l := d_l + \delta_l$, $d_l^{DA} + d_l^{RT} = \tilde{d}_l\ \forall l \in \mathcal{L}$, where $\delta_l\ \forall l \in \mathcal{L}$ is a random variable. Assuming that default bids are used for generators in the day-ahead market, and intercept function bids are received in the real-time market, the market operator solves the dispatch problem (4) and (6) in the two stages, respectively. Each load l minimizes its expected payment

$$\rho_l(d_l^{DA}, \lambda^{DA}, \lambda^{RT}) := \mathbb{E}\left[\lambda^{DA} d_l^{DA} + \lambda^{RT}(\tilde{d}_l - d_l^{DA})\right] \tag{31}$$

where we take the expectation with respect to the uncorrelated random variable δ_l, $\forall l \in \mathcal{L}$. Each generator j assumes d^{DA}, d^{RT} as a parameter where $d^{RT} = \sum_l(\tilde{d}_l - d_l^{DA})$, and its individual problem remains the same. The following theorem characterizes the competitive equilibrium -

Theorem 6.3. *Assuming that $\mathbb{E}[\lambda^{RT}]$ is known to participants in the day-ahead market. The competitive equilibrium in the two-stage market mechanism exists, and given by:*

$$g_j^{DA} = \frac{1}{c_j}\frac{(d + \sum_l \mathbb{E}[\delta_l])}{\sum_j c_j^{-1}},\ g_j^{RT} = \frac{1}{c_j}\frac{\sum_l(\delta_l - \mathbb{E}[\delta_l])}{\sum_j c_j^{-1}}, \forall j \in \mathcal{G} \tag{32a}$$

$$d_l^{DA} + d_l^{RT} = \tilde{d}_l;\ d^{DA} = d + \sum_l \mathbb{E}[\delta_l],\ d^{RT} = \sum_l(\delta_l - E[\delta_l]) \tag{32b}$$

$$\lambda^{DA} = \frac{(d + \sum_l \mathbb{E}[\delta_l])}{\sum_j c_j^{-1}},\ \lambda^{RT} = \frac{(d + \sum_l \delta_l)}{\sum_j c_j^{-1}} \tag{32c}$$

We provide the proof the theorem in Appendix H. The competitive equilibrium aligns with the social planner (1). At the equilibrium mean demand is allocated in the day-ahead market with uncertain deviations handled in the real-time market (32a). We next characterize the Nash equilibrium.

Theorem 6.4. *If there are at least two generators participating in the market, i.e., $|\mathcal{G}| \geq 2$, then the two-stage Nash equilibrium exists and is uniquely given by:*

$$g_j^{DA} = \frac{1}{c_j}\frac{1}{\sum_j c_j^{-1}}\left(1-\frac{1}{(L+1)}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_j c_j^{-1}}\right)\left(d+\sum_l \mathbb{E}[\delta_l]\right) \quad (33a)$$

$$g_j^{RT}=\frac{C_j^{-1}}{\sum_j C_j^{-1}}\left(\frac{(d+\sum_l \mathbb{E}[\delta_l])}{(L+1)}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_j c_j^{-1}}-\sum_l(\mathbb{E}[\delta_l]-\delta_l)\right) \quad (33b)$$

$$d_l^{DA}=d_l+\mathbb{E}[\delta_l]+\left(\frac{d+\sum_l \mathbb{E}[\delta_l]}{(L+1)}-d_l-\mathbb{E}[\delta_l]\right)\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_j c_j^{-1}} \quad (33c)$$

$$d_l^{RT}=\left(d_l+\mathbb{E}[\delta_l]-\frac{d+\sum_l \mathbb{E}[\delta_l]}{(L+1)}\right)\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_j c_j^{-1}}-(E[\delta_l]-\delta_l) \quad (33d)$$

$$\lambda^{DA}=\frac{1}{\sum_j c_j^{-1}}\left(1-\frac{1}{(L+1)}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_j c_j^{-1}}\right)\left(d+\sum_l \mathbb{E}[\delta_l]\right), \quad (33e)$$

$$\lambda^{RT}=\frac{1}{\sum_j C_j^{-1}}\left(\frac{(d+\sum_l \mathbb{E}[\delta_l])}{(L+1)}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_j c_j^{-1}}-\sum_l(\mathbb{E}[\delta_l]-\delta_l)\right) + \frac{1}{\sum_j c_j^{-1}}\left(1-\frac{1}{(L+1)}\frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_j c_j^{-1}}\right)\left(d+\sum_l \mathbb{E}[\delta_l]\right) \quad (33f)$$

where $C_j = \frac{1}{b(G-1)} + c_j$.

We provide the proof the theorem in Appendix I. Interestingly, the generation dispatch, demand allocation and prices in the day-ahead stage at the market equilibrium (33a), (33c), and (33e) only depend on the mean of the distribution of the random variable δ_l. The demand allocation, prices and generator dispatch in real-time now become random quantities at the time of day-ahead settlement, yet they are realized for the real-time settlement. Moreover, the Nash equilibrium (33) aligns in expectation with the Nash equilibrium (21), but the expected payment and expected revenue of participants still increases relatively, with the variance of the distribution, meaning uncertainty penalizes the demand by increasing its payment in the market.

We next compare the aggregate profit of generators and the aggregate payment of loads at the Nash equilibrium in Theorem 6.4. For this, we keep the same cost coefficient and parameter b, but for ease of analysis, we fix the error parameter $\epsilon = 0$. Moreover, we assume that $\delta_l = \delta d_l$ where $\delta \sim N(0, \sigma^2)$ is a random variable sampled $10,000$ times from a normal distribution with mean 0 and different variances. In Figure. 7 we plot the ratio of aggregate profit (payment) at Nash equilibrium between the case with random demand and without random demand, i.e. $\delta_l = 0, \forall l \in \mathcal{L}$. Although the mean ratio is approximately the same, the confidence interval grows as variance increases. The uncertainty does not significantly impact the equilibrium of the market, provided that it has zero bias.

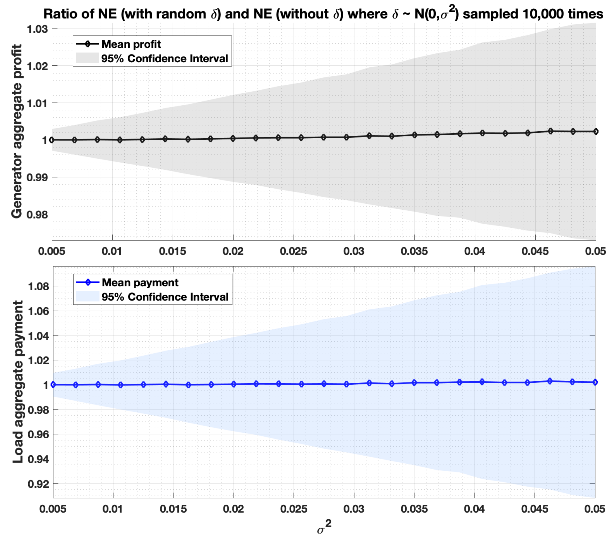

Figure 7: Ratio of Nash Equilibrium (NE) with δ and Nash Equilibrium (NE) without δ for aggregate individual profit/payment w.r.t σ^2; $\delta \sim \mathcal{N}(0, \sigma^2)$ sampled 10,000 times.

7 CONCLUSIONS

In this paper, we model the competition of generators and loads across two clearing stages comprising a day-ahead market and the real-time market. Following proposed industry policies, e.g., CAISO, we assume that the generators bids are substituted by default bids in the day-ahead market, and model generators as prosumers in the real-time stage that fulfills the positive and negative deviations from the day-ahead bids. Generators bid as both, supplier and consumer, in real-time, while the load allocates its demand across the two stages. By focusing on generator real-time bids based on supply function intercept, we study the market equilibrium of the two-stage sequential game. The resulting competitive equilibrium aligns with the social planner problem, and is thus considered to be efficient. However, when participants are strategic, a Nash equilibrium analysis shows that generators have limited opportunities to manipulate market prices as they act as a follower in a generalized Stackelberg-Nash game where demand acts as a leader. Numerical case studies illustrate the impact of market parameters and reveals how heterogeneity across generator cost hurts loads ability to benefit from price manipulation. We further show that, unlike slope bidding that fails to lead to an equilibrium for most settings, the intercept bidding mechanism can guarantee the existence of a Nash equilibrium, irrespectively of the number of market participants. Finally, in case of market uncertainty, at the Nash Equilibrium overestimation of generation cost, and demand uncertainty increases demand payments and generators revenue, i.e., loads are penalized in the presence of market uncertainty.

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A PROOF OF THEOREM 3.2

Under price-taking behaviour, the individual problem for loads (16) is a linear program with the closed-form solution given by:

$$\begin{cases} d_l^{DA} = \infty, d_l^{RT} = -\infty, d_l^{DA} + d_l^{RT} = d_l, \text{ if } \lambda^{DA} < \lambda^{RT} \\ d_l^{DA} = -\infty, d_l^{RT} = \infty, d_l^{DA} + d_l^{RT} = d_l, \text{ if } \lambda^{DA} > \lambda^{RT} \\ d_l^{DA} + d_l^{RT} = d_l, \quad \text{if } \lambda^{DA} = \lambda^{RT} \end{cases} \tag{34}$$

where loads prefer the lower price in the market. Also the market-clearing in the day-ahead market (4) require the following KKT conditions

$$c_j g_j^{DA} = \lambda^{DA}, \ \sum_{j \in \mathcal{G}} g_j^{DA} = d^{DA} \tag{35}$$

whereas the market-clearing in the real-time market (6) requires:

$$\frac{1}{b} g_j^{RT} + \frac{1}{b} \beta_j = \lambda^{RT}, \ \sum_{j \in \mathcal{G}} g_j^{RT} = d^{RT} \tag{36}$$

Further solving the individual bidding problem for generators in real-time market (13) by taking the derivative of the concave profit function, we get

$$-\lambda^{RT} + c_j(g_j^{DA} + b\lambda^{RT} - \beta_j) = 0 \tag{37a}$$

$$\implies -\lambda^{RT} + c_j(g_j^{DA} + g_j^{RT}) = 0$$

$$\implies \sum_{j \in \mathcal{G}} \frac{1}{c_j} \lambda^{RT} = \sum_{j \in \mathcal{G}} (g_j^{DA} + g_j^{RT}) = (d^{DA} + d^{RT}) = d$$

$$\implies \lambda^{RT} = \frac{d}{\sum_{j \in \mathcal{G}} c_j^{-1}} \tag{37b}$$

where we substitute (35) and (36) in (37a). Also from the day-ahead market clearing equations (35) we have

$$\lambda^{DA} = \frac{d^{DA}}{\sum_{j \in \mathcal{G}} c_j^{-1}} \tag{38}$$

At the competitive equilibrium the conditions (34),(35),(36),(37a) holds simultaneously and this is only possible if the market price are equal in the two-stages, i.e.,

$$\lambda^{RT} = \lambda^{DA} = \frac{d}{\sum_{j \in \mathcal{G}} c_j^{-1}}$$

Using (37b) andd (38) for such a price in the market implies

$$d^{DA} = d, \ d^{RT} = 0; \ d_l^{DA} + d_l^{RT} = d_l, \forall l \in \mathcal{L}$$

and

$$g_j^{DA} = \frac{1}{c_j} \frac{d}{\sum_{j \in \mathcal{G}} c_j^{-1}}, \ g_j^{RT} = 0, \ \forall j \in \mathcal{G}$$

Thus the competitive equilibrium exists.

B PROOF OF THEOREM 3.4

Given the parameter $(g_j^{DA}, d - d^{DA})$ from market-clearing in the day-ahead market, each generator j maximizes their profit (17) for the optimal decision β_j with complete knowledge of the market clearing in the real-time stage as characterized below:

$$\sum_{j \in \mathcal{G}} g_j^{RT} = d^{RT} \implies \sum_{j \in \mathcal{G}} (b\lambda^{RT} - \beta_j) = d^{RT} \implies \lambda^{RT} = \frac{d^{RT} + \beta^{\mathcal{G}}}{bG} \tag{39}$$

where $\beta^{\mathcal{G}} = \sum_{j \in \mathcal{G}} \beta_j$. Substituting (39) in the individual problem (10) gives the concave strategic individual problem of generators:

$$\max_{\beta_j \geq 0} \left(\frac{d^{RT} + \beta^{\mathcal{G}}}{bG} \right) \left(b \frac{d^{RT} + \beta^{\mathcal{G}}}{bG} - \beta_j \right) + \lambda^{DA} g_j^{DA}$$
$$- \frac{c_j}{2} \left(g_j^{DA} + b \left(\frac{d^{RT} + \beta^{\mathcal{G}}}{bG} \right) - \beta_j \right)^2 \tag{40}$$

Hence, taking the derivative of (40) with respect to bid β_j we get:

$$\frac{\partial \pi_j}{\partial b_j} = 0$$

$$\implies \frac{1}{bG} \left(\frac{d^{RT} + \beta^{\mathcal{G}}}{G} - \beta_j \right) - \frac{G-1}{G} \left(\frac{d^{RT} + \beta^{\mathcal{G}}}{bG} \right)$$
$$+ c_j \left(g_j^{DA} + \frac{d^{RT} + \beta^{\mathcal{G}}}{G} - \beta_j \right) \frac{G-1}{G} = 0$$

$$\implies \frac{1}{bG} \left(g_j^{RT} \right) - \frac{G-1}{G} \left(\lambda^{RT} \right) + c_j \left(g_j^{DA} + g_j^{RT} \right) \frac{G-1}{G} = 0$$

$$\implies \frac{1}{b(G-1)} g_j^{RT} - \lambda^{RT} + c_j \left(g_j^{DA} + g_j^{RT} \right) = 0 \tag{41}$$

where we substitute (5) and (39). The equation (41) is the required KKT condition of the convex dispatch problem (20), with λ^{RT} as the dual variable of the constraint (20b).

C PROOF OF THEOREM 3.5

Using the market-price in the real-time stage λ^{RT} as given by the KKT conditions (41) we get,

$$g_j^{RT} = \frac{\lambda^{RT} - c_j g_j^{DA}}{C_j} \implies \sum_{j \in \mathcal{G}} g_j^{RT} = \sum_{j \in \mathcal{G}} \frac{\lambda^{RT} - c_j g_j^{DA}}{C_j}$$

$$\implies d^{RT} = \sum_{j \in \mathcal{G}} \frac{\lambda^{RT} - c_j g_j^{DA}}{C_j} \implies \lambda^{RT} = \frac{d^{RT} + \sum_{j \in \mathcal{G}} \frac{c_j g_j^{DA}}{C_j}}{\sum_{j \in \mathcal{G}} C_j^{-1}} \tag{42}$$

where $C_j = \frac{1}{b(G-1)} + c_j$ and we use (6b) in the second equality equation. Substituting (42) in (41) we get

$$g_j^{RT} = \frac{d^{RT} + \sum_{j \in \mathcal{G}} \frac{c_j g_j^{DA}}{C_j}}{\sum_{j \in \mathcal{G}} C_j^{-1} C_j} - \frac{c_j g_j^{DA}}{C_j} \tag{43}$$

From the market-clearing in the day-ahead stage (4) we have the following relation

$$\lambda^{DA} = c_j g_j^{DA} \implies g_j^{DA} = \frac{1}{c_j} \lambda^{DA}, \text{and} \ \sum_{j \in \mathcal{G}} g_j^{DA} = d^{DA}$$

$$\implies \lambda^{DA} = \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}} \implies g_j^{DA} = \frac{1}{c_j} \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}} \tag{44}$$

Substituting (44) in the expression (42) and (43) we get

$$\lambda^{RT} = \frac{d^{RT}}{\sum_{j \in \mathcal{G}} C_j^{-1}} + \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}}, \ g_j^{RT} = \frac{1}{C_j} \frac{d^{RT}}{\sum_{j \in \mathcal{G}} C_j^{-1}} \tag{45}$$

Substituting (44) and (45) in the individual problem of load l (18) we get

$$\min_{d_l^{DA}} \frac{d^{DA}}{\sum_{j\in G} c_j^{-1}} d_l^{DA} + \left(\frac{d-d^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}} \right)(d_l - d_l^{DA}) \quad (46)$$

Therefore taking the derivative of the convex individual problem (46) wrt d_l^{DA} we get,

$$\frac{d^{DA}+d_l^{DA}}{\sum_{j\in G} c_j^{-1}} - \frac{d-d^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} - \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}} + \frac{d_l - d_l^{DA}}{\sum_{j=1}^{G} c_j^{-1}} - \frac{d_l - d_l^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} = 0$$

$$\implies -\frac{d-d^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d_l}{\sum_{j=1}^{G} c_j^{-1}} - \frac{d_l}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d_l^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} = 0$$

$$\implies \sum_{l\in\mathcal{L}} \left(-\frac{d-d^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d_l}{\sum_{j=1}^{G} c_j^{-1}} - \frac{d_l}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d_l^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} \right) = 0$$

$$\implies d^{DA} = \left(1 - \frac{1}{L+1} \frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^{G} c_j^{-1}} \right) d \quad (47a)$$

Therefore we get unique Nash equilibrium (21)

D PROOF OF THEOREM 4.1

From the discussion in the proof of Theorem 3.5 we get

$$\lambda^{DA} = \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \implies g_j^{DA} = \frac{1}{c_j} \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \quad (48)$$

$$\lambda^{RT} = \frac{d^{RT}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ g_j^{RT} = \frac{1}{C_j} \frac{d^{RT}}{\sum_{j\in\mathcal{G}} C_j^{-1}} \quad (49)$$

where $\sum_{j\in\mathcal{G}} C_j^{-1} = \sum_{j\in\mathcal{G}} \frac{1}{C_j}$, $C_j = \frac{1}{b(G-1)} + c_j$ and $\sum_{j\in\mathcal{G}} c_j^{-1} = \sum_{j=1}^{G} \frac{1}{c_j}$. Similarly under price-taking behaviour, the individual problem for loads (14) is a linear program with solution given by:

$$\begin{cases} d_l^{DA} = \infty, d_l^{RT} = -\infty, d_l^{DA} + d_l^{RT} = d_l, \text{ if } \lambda^{DA} < \lambda^{RT} \\ d_l^{DA} = -\infty, d_l^{RT} = \infty, d_l^{DA} + d_l^{RT} = d_l, \text{ if } \lambda^{DA} > \lambda^{RT} \\ d_l^{DA} + d_l^{RT} = d_l, \text{ if } \lambda^{DA} = \lambda^{RT}. \end{cases} \quad (50)$$

where loads prefer the lower price in the market. At the Nash equilibrium the conditions (6b),(7b),(48),(49), and (50) holds simultaneously and this is only possible if the market price are equal in the two-stages, i.e.

$$\lambda^{RT} = \lambda^{DA} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}$$

Using (48) and (49) for such a price in the market implies

$$d^{DA} = d,\ d^{RT} = 0;\ d_l^{DA} + d_l^{RT} = d_l, \forall l \in \mathcal{L}$$

and

$$g_j^{DA} = \frac{1}{c_j} \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ g_j^{RT} = 0,\ \forall j \in \mathcal{G}$$

Thus the Nash equilibrium aligns with the competitive equilibrium.

E PROOF OF THEOREM 4.2

Taking the derivative of the individual concave profit function (13) we get

$$-\lambda^{RT} + c_j(g_j^{DA} + b\lambda^{RT} - \beta_j) = 0 \quad (51a)$$

$$\implies -\lambda^{RT} + c_j(g_j^{DA} + g_j^{RT}) = 0$$

$$\implies \sum_{j\in\mathcal{G}} \frac{1}{c_j}\lambda^{RT} = \sum_{j\in\mathcal{G}} (g_j^{DA} + g_j^{RT}) = (d^{DA} + d^{RT}) = d$$

$$\implies \lambda^{RT} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \sum_{j\in\mathcal{G}} c_j^{-1} = \sum_{j\in\mathcal{G}} \frac{1}{c_j} \quad (51b)$$

where we substitute (35) and (36) in (51a). Also from the day-ahead market clearing equations (35) we have

$$\lambda^{DA} = \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \sum_{j\in\mathcal{G}} c_j^{-1} = \sum_{j\in\mathcal{G}} \frac{1}{c_j} \quad (52)$$

$$\implies g_j^{DA} = \frac{1}{c_j} \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \quad (53)$$

Substituting (51b) and (53) in (51a) we get,

$$\begin{aligned} \beta_j &= \frac{1}{c_j}\left(-\lambda^{RT} + c_j(b\lambda^{RT} + g_j^{DA}) \right) \\ &= \frac{1}{c_j}\left(-\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} + c_j\left(b\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} + \frac{1}{c_j}\frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \right) \right) \\ &= \frac{1}{c_j}\left(\frac{(bc_j - 1)d}{\sum_{j\in\mathcal{G}} c_j^{-1}} + \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \right) \end{aligned} \quad (54)$$

This implies that

$$\begin{aligned} g_j^{RT} &= b\lambda^{RT} - \beta_j = b\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} - \frac{1}{c_j}\left(\frac{(bc_j - 1)d}{\sum_{j\in\mathcal{G}} c_j^{-1}} + \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \right) \\ &= \frac{1}{c_j} \frac{d - d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \end{aligned} \quad (55)$$

Substituting (51b) and (52) in the individual problem of load l (18) we get

$$\min_{d_l^{DA}} \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} d_l^{DA} + \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}} (d_l - d_l^{DA})$$

$$\iff \min_{d_l^{DA}} d^{DA} d_l^{DA} + d(d_l - d_l^{DA}) \quad (56a)$$

taking the derivative of this convex optimization problem (56a) we get

$$d^{DA} + d_l^{DA} - d = 0 \implies \sum_{l\in\mathcal{L}} \left(d^{DA} + d_l^{DA} - d \right) = 0$$

$$\implies d^{DA} = \frac{L}{L+1} d \implies d^{RT} = \frac{1}{L+1} d \quad (57)$$

This implies

$$g_j^{DA} = \frac{L}{L+1} \frac{1}{c_j} \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, g_j^{RT} = \frac{1}{L+1} \frac{1}{c_j} \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, \forall j \in \mathcal{G}$$

$$d_l^{DA} = \frac{1}{L+1} d,\ d_l^{RT} = d_l - \frac{1}{L+1} d,\ \forall l \in \mathcal{L}$$

$$\beta_j = \left(b - \frac{1}{L+1}\frac{1}{c_j}\right)\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}},\ \forall j \in \mathcal{G}$$
$$\lambda^{DA} = \frac{L}{L+1}\frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}, \lambda^{RT} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}$$

F PROOF OF THEOREM 6.1

Under price-taking behaviour, the closed-form solution of the individual problem for loads (16) is given by (34). Also the market-clearing in the day-ahead market (28) require the following KKT conditions

$$(1+\epsilon)c_j g_j^{DA} = \lambda^{DA},\ \sum_{j\in\mathcal{G}} g_j^{DA} = d^{DA} \tag{58}$$

whereas the market-clearing in the real-time market (6) requires:

$$\frac{1}{b}g_j^{RT} + \frac{1}{b}\beta_j = \lambda^{RT},\ \sum_{j\in\mathcal{G}} g_j^{RT} = d^{RT} \tag{59}$$

Using (37b) we have

$$\lambda^{RT} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}$$

Also from the day-ahead market clearing equations (58) we have

$$\lambda^{DA} = \frac{(1+\epsilon)d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{60}$$

At the competitive equilibrium the conditions (34),(37a),(58),(59) holds simultaneously and this is only possible if the market price are equal in the two-stages, i.e.,

$$\lambda^{RT} = \lambda^{DA} = \frac{d}{\sum_{j\in\mathcal{G}} c_j^{-1}}$$

Using (37b) andd (60) for such a price in the market implies

$$d_l^{DA} + d_l^{RT} = d_l,\ \forall l \in \mathcal{L}; d^{DA} = \frac{1}{1+\epsilon}d; d^{RT} = \left(1 - \frac{1}{1+\epsilon}\right)d$$

and

$$g_j^{DA} = \frac{1}{c_j}\frac{1}{1+\epsilon}\frac{d}{\sum_{j=1}^G c_j^{-1}},\ g_j^{RT} = \frac{1}{c_j}\left(1 - \frac{1}{1+\epsilon}\right)\frac{d}{\sum_{j=1}^G c_j^{-1}}$$

Thus the competitive equilibrium exists.

G PROOF OF THEOREM 6.2

Using (42),(43), we have the generator dispatch and prices in the real-time market as

$$g_j^{RT} = \frac{d^{RT} + \sum_{j\in\mathcal{G}} \frac{c_j g_j^{DA}}{C_j}}{\sum_{j\in\mathcal{G}} C_j^{-1} C_j} - \frac{c_j g_j^{DA}}{C_j} \tag{61a}$$

$$\lambda^{RT} = \frac{d^{RT} + \sum_{j\in\mathcal{G}} \frac{c_j g_j^{DA}}{C_j}}{\sum_{j\in\mathcal{G}} C_j^{-1}} \tag{61b}$$

From the market-clearing in the day-ahead stage (58) we have the following relation

$$\lambda^{DA} = \frac{(1+\epsilon)d^{DA}}{\sum_{j=1}^G c_j^{-1}} \implies g_j^{DA} = \frac{1}{c_j}\frac{d^{DA}}{\sum_{j=1}^G c_j^{-1}} \tag{62}$$

Substituting (62) in the expression (61) we get

$$\lambda^{RT} = \frac{d^{RT}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d^{DA}}{\sum_{j=1}^G c_j^{-1}},\ g_j^{RT} = \frac{1}{C_j}\frac{d^{RT}}{\sum_{j\in\mathcal{G}} C_j^{-1}} \tag{63}$$

Substituting (62) and (63) in the individual problem of load l (18) we get

$$\min_{d_l^{DA}} \frac{(1+\epsilon)d^{DA}}{\sum_{j\in G} c_j^{-1}} d_l^{DA} + \left(\frac{d - d^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d^{DA}}{\sum_{j=1}^G c_j^{-1}}\right)(d_l - d_l^{DA}) \tag{64}$$

Therefore taking the derivative of the convex individual problem (64) wrt d_l^{DA} we get,

$$\epsilon\frac{d^{DA} + d_l^{DA}}{\sum_{j\in G} c_j^{-1}} - \frac{d - d^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d_l}{\sum_{j=1}^G c_j^{-1}} - \frac{d_l}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d_l^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} = 0$$

Summing over $l \in \mathcal{L}$ we get

$$d^{DA} = \frac{\sum_{j=1}^G c_j^{-1} - \frac{1}{L+1}\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_{j=1}^G c_j^{-1} + \epsilon\sum_{j\in\mathcal{G}} C_j^{-1}} d^T$$

Therefore we get unique Nash equilibrium (30)

H PROOF OF THEOREM 6.3

Under price-taking behaviour, the individual problem for loads (31) is a linear program with the closed-form solution given by:

$$\begin{cases} d_l^{DA} = \infty, d_l^{RT} = -\infty, d_l^{DA} + d_l^{RT} = \tilde{d}_l, \text{ if } \lambda^{DA} < \mathbb{E}[\lambda^{RT}] \\ d_l^{DA} = -\infty, d_l^{RT} = \infty, d_l^{DA} + d_l^{RT} = \tilde{d}_l, \text{ if } \lambda^{DA} > \mathbb{E}[\lambda^{RT}] \\ d_l^{DA} + d_l^{RT} = \tilde{d}_l, \quad \text{if } \lambda^{DA} = \mathbb{E}[\lambda^{RT}] \end{cases} \tag{66}$$

where loads prefer the lower price in the market. Also the market-clearing in the day-ahead market (4) require the following KKT conditions

$$c_j g_j^{DA} = \lambda^{DA},\ \sum_{j\in\mathcal{G}} g_j^{DA} = d^{DA} \tag{67}$$

whereas the market-clearing in the real-time market (6) requires:

$$\frac{1}{b}g_j^{RT} + \frac{1}{b}\beta_j = \lambda^{RT},\ \sum_{j\in\mathcal{G}} g_j^{RT} = d^{RT} \tag{68}$$

Further solving the individual bidding problem for generators in real-time market (13), we get

$$-\lambda^{RT} + c_j(g_j^{DA} + b\lambda^{RT} - \beta_j) = 0 \tag{69a}$$
$$\implies -\lambda^{RT} + c_j(g_j^{DA} + g_j^{RT}) = 0$$
$$\implies \sum_{j\in\mathcal{G}} \frac{1}{c_j}\lambda^{RT} = \sum_{j\in\mathcal{G}} (g_j^{DA} + g_j^{RT}) = (d^{DA} + d^{RT}) = d + \sum_l \delta_l$$
$$\implies \lambda^{RT} = \frac{d + \sum_l \delta_l}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{69b}$$

where we substitute (67) and (68) in (69a). Also from the day-ahead market clearing equations (67) we have

$$\lambda^{DA} = \frac{d^{DA}}{\sum_{j\in\mathcal{G}} c_j^{-1}} \tag{70}$$

At the competitive equilibrium the conditions (66),(67),(68),(69a) holds simultaneously and this is only possible if the market price are equal in the two-stages, i.e.,

$$\mathbb{E}[\lambda^{RT}] = \lambda^{DA} = \frac{d + \sum_l \mathbb{E}[\delta_l]}{\sum_{j\in\mathcal{G}} c_j^{-1}}$$

Using (69b) and (70) for such a price in the market implies

$$d^{DA} = d + \sum_l \mathbb{E}[\delta_l],\ d^{RT} = \sum_l (\delta_l - \mathbb{E}[\delta_l])\ ; d_l^{DA} + d_l^{RT} = \tilde{d}_l, \forall l \in \mathcal{L}$$

and

$$g_j^{DA} = \frac{1}{c_j} \frac{d + \sum_l \mathbb{E}[\delta_l]}{\sum_{j\in\mathcal{G}} c_j^{-1}}\ \forall j \in \mathcal{G}$$

Thus the competitive equilibrium exists.

I PROOF OF THEOREM 6.4

Using (42),(43), we have the generator dispatch and prices in the real-time market as

$$g_j^{RT} = \frac{d^{RT} + \sum_{j\in\mathcal{G}} \frac{c_j g_j^{DA}}{C_j}}{\sum_{j\in\mathcal{G}} C_j^{-1} C_j} - \frac{c_j g_j^{DA}}{C_j} \tag{71a}$$

$$\lambda^{RT} = \frac{d^{RT} + \sum_{j\in\mathcal{G}} \frac{c_j g_j^{DA}}{C_j}}{\sum_{j\in\mathcal{G}} C_j^{-1}} \tag{71b}$$

From the market-clearing in the day-ahead stage (44) we have the following relation

$$\lambda^{DA} = \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}} \implies g_j^{DA} = \frac{1}{c_j} \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}} \tag{72}$$

Substituting (44) in the expression (71) we get

$$\lambda^{RT} = \frac{d^{RT}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}},\ g_j^{RT} = \frac{1}{C_j} \frac{d^{RT}}{\sum_{j\in\mathcal{G}} C_j^{-1}} \tag{73}$$

Using (44) and (73) to minimize individual payment of load l (31) we get

$$\min_{d_l^{DA}} \mathbb{E}\left[\frac{d^{DA} d_l^{DA}}{\sum_{j\in G} c_j^{-1}} + \left(\frac{d + \sum_l \delta_l - d^{DA}}{\sum_{j\in\mathcal{G}} C_j^{-1}} + \frac{d^{DA}}{\sum_{j=1}^{G} c_j^{-1}} \right) (d_l + \delta_l - d_l^{DA}) \right] \tag{74}$$

Taking the derivative of the convex individual problem (74) w.r.t d_l^{DA} and summing over $l \in \mathcal{L}$ we get,

$$d^{DA} = \left(1 - \frac{1}{(L+1)} \frac{\sum_{j\in\mathcal{G}} C_j^{-1}}{\sum_j c_j^{-1}} \right) (d + \sum_l \mathbb{E}[\delta_l]) \tag{75a}$$

Hence we get unique Nash equilibrium (33).