Accurate Reduced Order Models for Heterogeneous Coherent Generators

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***Abstract*— We introduce a novel framework to approximate the aggregate frequency dynamics of coherent generators. By leveraging recent results on dynamics concentration of tightly connected networks, we develop a hierarchy of reduced order models –based on frequency weighted balanced truncation– that accurately approximate the aggregate system response. Our results outperform existing aggregation techniques and can be shown to monotonically improve the approximation as the hierarchy order increases.**

I. Introduction

Accurately modeling generator frequency response to power disturbances is essential for assessing frequency control performance in power grids. Techniques for deriving reduced order approximations of large-scale power networks based on *coherence* and *aggregation* have been investigated for decades [1]. Generally, a group of generators is considered coherent if their bus frequencies exhibit a similar response when subject to power disturbances. A widely used modeling technique is to subsequently aggregate the response of coherent generators into a single effective machine.

In past decades, various methods for identifying coherent group of generators have been introduced [2]–[6]. The Linear Simulation Method [7] groups generators whose maximum difference in time-domain response is within some tolerance. Similarly, [3] develops a clustering algorithm based on the pairwise maximum difference in time-domain response. The Weak Coupling Method [6] quantifies strength of coupling between two areas to iteratively determine the boundaries of coherent generator groups. The Two Time Scale Method [4], [5] computes the eigen basis matrix associated with the electromechanical modes in the linearized network: generators with similar entries on the basis matrix with respect to low frequency oscillatory modes are considered coherent.

Once all generators are grouped by coherence, each group can be aggregated into a single effective machine. Previous work [8]–[13] has demonstrated that the best choice of inertial and damping coefficients for the effective generator is obtained by adding among all the corresponding generator parameters. However, in the presence of turbine dynamics, the proper choice of turbine time constants is unclear. Optimization-based approaches [9], [10] minimize an error function to choose the time constant of the effective generator. Other approaches use the average [11], or the weighted harmonic mean [12] of time constants of generators in the coherent group. However, these methods cannot in general achieve high accuracy in capturing the coherent frequency response. Moreover, the aggregation techniques mentioned above are proposed for coherent synchronous generators, while more realistic scenarios generally include both synchronous generators and grid-forming inverters [14], [15] in a coherent group.

In this paper, we leverage new results on characterizing coherence in tightly-connected networks [16] to propose a general framework for aggregation of coherent generators. We show that for n coherent generators with transfer function $g_i(s),\ i = 1, \cdots, n$, the aggregate coherent dynamics are accurately approximated by $\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$. In particular, we show that $\hat{g}(s)$ is a natural characterization of the coherent dynamics in the sense that, as the algebraic connectivity of the network increases, the response of the coherent group is asymptotically $\hat{g}(s)$. In the case of heterogeneous turbine dynamics, the aggregate dynamics $\hat{g}(s)$ can be as high order as the network size n, then the aggregation of generators essentially asks for a low-order approximation of $\hat{g}(s)$. We propose a hierarchy of reduced order models, based on frequency weighted balanced truncation, which not only offers as reduced model a single effective generator, but also higher-order reduction models with significantly improved accuracy.

Our result shows that aggregation of coherent generators can be regarded as finding a low-order approximation of $\hat{g}(s)$. In the case of high-order $\hat{g}(s)$, the conventional approaches [9], [10], [12] are too restrictive, where the approximation model is given by a single effective generator with proper time constant and all other parameters chosen as their aggregate value. Our proposed models suggests two improvements by enforcing less constraints: 1) Increase the order of the approximation model; in particular for a 2nd order generator model, a 3rd order reduced model for $\hat{g}(s)$ is almost accurate; 2) Model reduction on the closed-loop dynamics $\hat{g}(s)$ rather than on the turbine dynamics. Additionally, our models can still be interpreted as a generator model with appropriate structure and parameters. Lastly, the aggregation techniques introduced in this paper apply to any linear model of generators, allowing us to obtain accurate aggregate higher order generator models.

The rest of the paper is organized as follows. In Section II, we provide the theoretical justification of the coherent dynamics $\hat{g}(s)$. In Section III, we propose reduced order models for $\hat{g}(s)$ by frequency weighted balanced truncation. We then show via numerical illustrations that the proposed models can achieve accurate approximation (Section IV). Lastly, we conclude this paper with more discussions on the implications of our current results.

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II. Aggregate Dynamics of Coherent Generators

Consider a group of n generators, indexed by $i = 1, \cdots, n$, dynamically coupled through an AC network. Assuming the network is in steady-state, the block diagram of the linearized system around this operating point is shown in Fig.1.

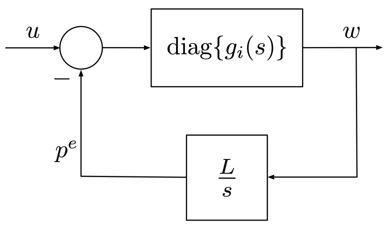

Fig. 1. Block Diagram of Linearized Power Networks

For generator i, the transfer function from net power deviation at its generator axis to its angular frequency deviation w_i, relative to their equilibrium values, is given by $g_i(s)$. The net power deviation at generator i, includes disturbance u_i reflecting variations in mechanical power or local load, minus the electrical power p_i^e drawn from the network.

The network power fluctuations p^e are given by a linearized (lossless) DC model of the power flow equation $p^e(s) = \frac{1}{s} L w(s)$. Here L is the Laplacian matrix of an undirected weighted graph, with its elements given by $L_{ij} = \frac{\partial}{\partial \theta_j} \sum_{k=1}^{n} |V_i||V_k| b_{ik} \sin(\theta_i - \theta_k)\Big|_{\theta=\theta_0}$, where θ_0 are angles at steady state, $|V_i|$ is the voltage magnitude at bus i and b_{ij} is the line susceptance. Without loss of generality, we assume the steady state angular difference $\theta_{0i} - \theta_{0j}$ across each line is smaller than $\frac{\pi}{2}$. Moreover, because L is a symmetric real Laplacian, its eigenvalues are given by $0 = \lambda_1(L) \leq \lambda_2(L) \leq \cdots \leq \lambda_n(L)$. The overall linearized frequency dynamics of the generators is given by

$$w_i(s) = g_i(s)(u_i(s) - p_i^e(s)), \quad i = 1, \cdots, n, \tag{1a}$$

$$p^e(s) = \frac{1}{s} L w(s). \tag{1b}$$

In this section, we are interested in characterizing the dynamic response of coherent generators to system disturbances, which we term here *coherent dynamics*. With this aim, we seek conditions on the network (1) under which the entire set of generators behave coherently. The same approach can be used on subgroups of generators.

To motivate our results, we follow the typical assumption, which is to impose an equal response $w_i(s) = \hat{w}(s)$ at the output of the coherent generators [8]–[10], to derive a closed form expression for the coherent dynamics; the theory that justifies the result of this derivation is then provided in Section II-A. By assuming $w_i(s) = \hat{w}(s)$, it is possible to sum over all equations in (1a) to get

$$\left(\sum_{i=1}^{n} g_i^{-1}(s) \right) \hat{w}(s) = \sum_{i=1}^{n} u_i(s) - \sum_{i=1}^{n} p_i^e(s). \tag{2}$$

Notice that the last term $\sum_{i=1}^{n} p_i^e(s) = \mathbb{1}^T \frac{L}{s} \mathbb{1} \hat{w}(s) = 0$ since $\mathbb{1} = [1, \cdots, 1]^T$ is an eigenvector of $\lambda_1(L) = 0$. Then the aggregate model for the coherent group is given by

$$\hat{w}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1} \sum_{i=1}^{n} u_i(s). \tag{3}$$

From (3), the coherent group of generators is aggregated into a single effective machine with its transfer function given by

$$\hat{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1}. \tag{4}$$

While insightful, equation (4) is not properly substantiated. In what follows we provide a principled justification for using (4) as our model for the coherent dynamics by leveraging recent results on coherence of tightly connected networks [16].

A. Coherence in Tightly Connected Networks

We now lay down the basic theory that justifies the use of (4) as an accurate descriptor of the dynamics of coherent generators. Our analysis will highlight the role of the algebraic connectivity $\lambda_2(L)$ of the network as a direct indicator of how coherent a group of generators is.

For the network shown in Fig.1, the transfer matrix from the disturbance u to the frequency deviation w is given by

$$T(s) = (I_n + \mathrm{diag}\{g_i(s)\} L/s)^{-1} \mathrm{diag}\{g_i(s)\}, \tag{5}$$

where I_n is the $n \times n$ identity matrix. To justify the coherent response of generators, we show that the transfer matrix $T(s)$ converges, as algebraic connectivity $\lambda_2(L)$ increases, to one where all entries are given by $\hat{g}(s)$.

We make the following assumptions: 1) $T(s)$ is stable; 2) all $g_i(s)$ are minimum phase systems; 3) $\hat{g}(s)$ in (4) is stable. For generators that satisfy these assumptions, we have the following result.

Theorem 1. *Given the assumptions above, the following holds for any $\eta_0 > 0$:*

$$\lim_{\lambda_2(L) \to +\infty} \sup_{\eta \in [-\eta_0, \eta_0]} \left\| T(j\eta) - \hat{g}(j\eta) \mathbb{1}\mathbb{1}^T \right\| = 0,$$

where $j = \sqrt{-1}$ and $\mathbb{1} \in \mathbb{R}^n$ is the vector of all ones.

Due to space constraints, we refer to [17] for the proof. The analysis relies on the fact that $T(s)$ is close to $\hat{g}(s)\mathbb{1}\mathbb{1}^T$ if the *effective algebraic connectivity* $\left| \frac{\lambda_2(L)}{s} \right|$ is large. For any frequency band $[-j\eta_0, j\eta_0]$ on the imaginary axis, the effective algebraic connectivity is lower bounded by $\frac{\lambda_2(L)}{\eta_0}$, hence one can make sure $T(s)$ is arbitrarily close to $\hat{g}(s)\mathbb{1}\mathbb{1}^T$ on this frequency band by increasing $\lambda_2(L)$.

The transfer matrix $\hat{g}(s)\mathbb{1}\mathbb{1}^T$ can be interpreted as follows. Given any arbitrary disturbance $u(s)$, the frequency response to such disturbance is given by

$$w(s) = \hat{g}(s)\mathbb{1}\mathbb{1}^T u(s) = \left(\hat{g}(s) \sum_{i=1}^{n} u_i(s) \right) \mathbb{1}. \tag{6}$$

In other words, every bus frequency reacts to the aggregate disturbance $\sum_i u_i(s)$ based on the response $\hat{g}(s)$. As a result, for any disturbance limited over band $[0, \eta_0]$, the response of the network $T(s)u(s)$ is approximated by the one in (6). Therefore generator networks with large algebraic connectivity should be considered coherent and $\hat{g}(s)$ gives the coherent dynamics.

B. Aggregate Dynamics for Different Generator Models

Having characterized how the *coherent dynamics* given by $\hat{g}(s)$ represent the network's aggregate behavior, from now on we will use with no distinction the terms "aggregate" and "coherent" dynamics. Now we look into the explicit forms these dynamics take for different generator models.

Example 1. *Generators with 1st order model, of two types:*

1) For synchronous generators [13], $g_i(s) = \frac{1}{m_i s + d_i}$, *where* m_i, d_i *are the inertia and damping of generator* i, *respectively. The coherent dynamics are* $\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d}}$, *where* $\hat{m} = \sum_{i=1}^{n} m_i$ *and* $\hat{d} = \sum_{i=1}^{n} d_i$.

2) For droop-controlled inverters [14], $g_i(s) = \frac{k_{P,i}}{\tau_{P,i}s+1}$, *where* $k_{P,i}$ *and* $\tau_{P,i}$ *are the droop coefficient and the filter time constant of the active power measurement, respectively. The coherent dynamics are* $\hat{g}(s) = \frac{\hat{k}_P}{\hat{\tau}_P s+1}$, *where* $\hat{k}_P = \left(\sum_{i=1}^{n} k_{P,i}^{-1}\right)^{-1}$, $\hat{\tau}_P = \hat{k}_P \left(\sum_{i=1}^{n} \tau_{P,i}/k_{P,i}\right)$.

Notice that both dynamics are of the same form; by suitable reparameterization, we may use the "swing" model $g_i(s) = \frac{1}{m_i s + d_i}$ to model both types of generators.

The aggregate model given in Example 1 is consistent with the conventional approach of choosing inertia $\hat{m}$ and damping $\hat{d}$ as the respective sums over all generators. Theorem 1 explains why such a choice is indeed appropriate.

The aggregation is more complicated when considering generators with turbine droop control:

Example 2. *Synchronous generators given by the swing model with turbine droop [13]*

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s+1}}, \tag{7}$$

where r_i^{-1} *and* τ_i *are the droop coefficient and turbine time constant of generator* i, *respectively. The coherent dynamics are given by*

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s+1}}. \tag{8}$$

This example illustrates, in particular, the difficulty in aggregating generators with heterogeneous turbine time constants. When all generators have the same turbine time constant $\tau_i = \hat{\tau}$, then $\hat{g}(s)$ in (8) reduces to the typical effective machine model of the form (7) with parameters $(\hat{m}, \hat{d}, \hat{r}^{-1}, \hat{\tau})$, where $\hat{r}^{-1} = \sum_{i=1}^{n} r_i^{-1}$, i.e. the aggregation model is still obtained by choosing parameters as the respective sums of their individual values. However, if the τ_i are heterogeneous, then $\hat{g}(s)$ is a high-order transfer function and cannot be accurately represented by a single generator model. The aggregation of generators essentially requires a low-order approximation of $\hat{g}(s)$.

C. Aggregate Dynamics for Mixture of Generators

We have shown the aggregate dynamics for generators of three different types. When a mixture of these different types is present[1], we propose (7) to be a general representation of the three types; in particular, the first order models can be regarded as (7) with $r_i^{-1} = 0$. Therefore, (8) provides a general representation of the aggregate dynamics resulting from a mixture of generators. Again, high-order coherent dynamics arise when the network includes heterogeneous turbines.

III. Reduced Order Model for Coherent Generators with Heterogeneous Turbines

As shown in the previous section, the coherent dynamics $\hat{g}(s)$ are of high-order if the coherent group has generators with different turbine time constants. This suggests that substituting $\hat{g}(s)$ with an equivalent machine of the same order as each $g_i(s)$ may lead to substantial approximation error. In this section we propose instead a hierarchy of reduction models with increasing order, based on balanced realization theory [18], such that eventually an accurate reduction model is obtained as the order of the reduction increases. We further explore other avenues of improvement by applying the reduction methodology over the coherent dynamics itself, instead of the standard approach of applying a reduction only on the turbines [9], [10], [12].

In this paper, we use frequency weighted balanced truncation [19] to approximate $\hat{g}(s)$. Frequency weighted balanced truncation identifies the most significant dynamics with respect to particular LTI frequency weight by computing the weighted Hankel singular values, the square root of eigenvalues of $X_c Y_o$, where X_c and Y_o are the frequency weighted controllability and observability gramians of the system to be reduced. In many cases, the Hankel singular values decay fast, allowing us to accurately approximate high-order systems. More importantly, the reduction procedure favors approximation accuracy in certain frequency range specified by the frequency weights.

Due to space constraints, we refer to [17] for the detailed procedure of frequency weighted balanced truncation. It suffices to regard this model reduction method as a tool that, given a SISO proper transfer function $G(s)$, a frequency weight $W(s)$, and a number k, returns a transfer function

$$\tilde{G}_k(s) = \frac{b_{k-1}s^{k-1} + \cdots + b_1 s + b_0}{a_k s^k + \cdots + a_1 s + a_0}, \tag{9}$$

guaranteed to be stable [19], and such that the weighted error $\sup_{\eta\in\mathbb{R}} |W(j\eta)(G(j\eta) - \tilde{G}_k(j\eta))|$ is upper bounded, with an upper bound decreasing to zero with the order k. For our purposes, $W(s)$ must have high gain in the low frequency range, so that the DC gains of the original and reduced dynamics are approximately matched, i.e., $G(0) \simeq \tilde{G}(0)$. We propose two model reduction approaches for high-order $\hat{g}(s)$ in (8) based on frequency weighted balanced truncation.

[1]Generally, when considering a mixture of synchronous generators and grid-forming inverters, our network model is valid only when synchronous generators make up a significant portion of the composition.

A. Model Reduction on Turbine Dynamics

Our first model is based on applying balanced truncation to the turbine aggregate. Essentially, $\hat{g}(s)$ in (8) is of high order because it has high-order turbine dynamics $\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s+1}$; we seek to replace it with a reduced order model. This is akin to the existing literature [9], [10] which replaces an aggregate of turbines in parallel by a first order turbine model with parameters obtained by minimizing certain error functions.

We denote the aggregate turbine dynamics as $\hat{g}_t(s) := \sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s+1}$. We also denote the $(k-1)$-th reduction model of $\hat{g}_t(s)$ by frequency-weighted balanced truncation as $\tilde{g}_{t,k-1}(s)$. Then the k-th order reduction model of $\hat{g}(s)$ is given by

$$\tilde{g}_k^{tb}(s) = \frac{1}{\hat{m}s + \hat{d} + \tilde{g}_{t,k-1}(s)}, \tag{10}$$

with, again, $\hat{m} = \sum_{i=1}^{n} m_i$, $\hat{d} = \sum_{i=1}^{n} d_i$. We highlight two special instances of relevance for our numerical illustration.

1) 2nd order reduction model: When $k = 2$, the reduced model $\tilde{g}_{t,1}(s)$ can be interpreted as a first order turbine model

$$\tilde{g}_{t,1}(s) = \frac{b_0}{a_1 s + a_0} = \frac{\tilde{r}^{-1}}{\tilde{\tau}s + 1},$$

with parameters $(\tilde{r}^{-1}, \tilde{\tau})$ chosen by the weighted balanced truncation method. Then the overall reduction model $\tilde{g}_2^{tb}(s)$ is second order, which is a single generator model.

Unlike [9], [10], there is a DC gain mismatch between $\tilde{g}_2^{tb}(s)$ and the original $\hat{g}(s)$ since $\tilde{r}^{-1} \neq \hat{r}^{-1} = \sum_{i=1}^{n} r_i^{-1}$. Later in the simulation section, by choosing a proper frequency weight $W(s)$, we effectively make the DC gain mismatch negligible. Unfortunately, as we will see in the numerical section, $k = 2$ may not suffice to accurately approximate the coherent dynamics.

2) 3rd order reduction model: To obtain a more accurate reduced order model, one may consider $k = 3$ as the next suitable option. In fact, according to numerical observations, a 2nd order turbine model $\tilde{g}_{t,2}(s)$, i.e., $k = 3$, is sufficient to give an almost exact approximation of $\hat{g}_t(s)$.

We can also interpret $\tilde{g}_{t,2}(s)$, by means of partial fraction expansion, i.e.,

$$\tilde{g}_{t,2}(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} = \frac{\tilde{r}_1^{-1}}{\tilde{\tau}_1 s + 1} + \frac{\tilde{r}_2^{-1}}{\tilde{\tau}_2 s + 1},$$

assuming the poles are real. Then the reduced turbine dynamics $\tilde{g}_{t,2}(s)$ can be interpreted as two first order turbines in parallel with parameters $(\tilde{r}_1^{-1}, \tilde{\tau}_1)$ and $(\tilde{r}_2^{-1}, \tilde{\tau}_2)$.

B. Model Reduction on Closed-loop Coherent Dynamics

Our second proposal is: instead of reducing the turbine dynamics (10), to apply weighted balanced truncation directly on $\hat{g}(s)$. Thus, we denote $\tilde{g}_k^{cl}(s)$ as the k-th order reduction model, via frequency weighted balanced truncation, of the coherent dynamics $\hat{g}(s)$. Again, DC gain mismatch can be made negligible by properly choosing $W(s)$.

As compared to Section III-A, the reduced model might not be easy to interpret in practice. Nevertheless, the procedure described below often leads to such an interpretation.

1) 2nd order reduction model: When $k = 2$, we wish to interpret $\tilde{g}_2^{cl}(s)$ in terms of a single generator with a first order turbine of the form in (7), with parameters $(\tilde{m}, \tilde{d}, \tilde{r}^{-1}, \tilde{\tau})$. Given

$$\tilde{g}_2^{cl}(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} =: \frac{N(s)}{D(s)},$$

obtained via the proposed method: write the polynomial division $D(s) = Q(s)N(s) + R$, where $Q(s), R$ are quotient and remainder, respectively. This leads to the expression

$$\tilde{g}_2^{cl}(s) = \frac{N(s)}{Q(s)N(s) + R} = \frac{1}{Q(s) + \frac{R}{N(s)}}.$$

Here the first order polynomial $Q(s)$ can be matched to $\tilde{m}s + \tilde{d}$, and $\frac{R}{N(s)}$ to $\frac{\tilde{r}^{-1}}{\tilde{\tau}s+1}$. Provided the obtained constants $(\tilde{m}, \tilde{d}, \tilde{r}^{-1}, \tilde{\tau})$ are positive, the interpretation follows.

2) 3rd order reduction model: Similarly, when $k = 3$, the reduced model is $\tilde{g}_3^{cl}(s) = \frac{N(s)}{D(s)}$, with $N(s)$ of 2nd order and $D(s)$ of 3rd order. The polynomial division $D(s) = Q(s)N(s) + R(s)$, still gives a first order quotient $Q(s)$, which is interpreted as $\tilde{m}s + \tilde{d}$; the second order transfer function $\frac{R(s)}{N(s)}$ can be expressed, by partial fraction expansion, as two first order turbines in parallel, provided the obtained constants remain positive. We explore this in the examples studied below.

IV. Numerical Simulations

We now evaluate the reduction methodologies proposed in the previous section, and compare their performance with the solutions proposed in [9], [10]. In our comparison, we consider 5 generators forming a coherent group[2]. All parameters are expressed in a common base of 100 MVA.

The test case: 5 generators, $\hat{m} = 0.0683(\text{s}^2/\text{rad})$, $\hat{d} = 0.0107$. The turbine and droop parameters of each generator are listed in Table I. In all comparisons, a step change of -0.1 p.u. is used.

TABLE I

Droop control parameters of generators in test case

Parameter \ Index	1	2	3	4	5
droop r_i^{-1} (p.u.)	0.0218	0.0256	0.0236	0.0255	0.0192
time constant τ_i (s)	9.08	5.26	2.29	7.97	3.24

Remark. In the test case, we only aggregate 5 generators and report all parameters explicitly in order to give more insights on how the distribution of time constant τ_i affects our approximations. It is worth noting that similar behavior is observed when reducing coherent groups with a much larger number of generators. In particular, the accuracy found below with 3rd order reduced models is also observed in these higher order problems.

[2]More specifically, we assume sufficiently strong network coupling among these generators such that the frequency responses are coherent. The numerical simulation will only illustrate the approximation accuracy with respect to the coherent response rather than individual ones.

As mentioned in the previous section, one of the drawbacks of the balanced truncation method is that it does not match the DC gain of the original system, which leads to an error on the steady-state frequency. In our simulation, the DC gain mismatch is effectively cancelled by picking proper frequency weights for different reduced models. Due to space constraints, we refer to [17] for the comparison between reduced models with and without frequency weights.

A. Effect of Reduction Order k in Accuracy

We now evaluate the effect of the order of the reduction in the accuracy. That is, we compare 2nd and 3rd order balanced truncation on the turbine dynamics, $\tilde{g}_2^{tb}(s)$ (BT2-tb), $\tilde{g}_3^{tb}(s)$ (BT3-tb), as well as balanced truncation on the closed-loop coherent dynamics $\tilde{g}_2^{cl}(s)$ (BT2-cl), $\tilde{g}_3^{cl}(s)$ (BT3-cl). The frequency weights are given by $W_{tb}(s) = \frac{s+3\cdot10^{-2}}{s+10^{-4}}$ and $W_{cl}(s) = \frac{s+8\cdot10^{-2}}{s+10^{-4}}$, respectively. The step response and step response error with respect to $\hat{g}(s)$ are shown in Fig. 2.

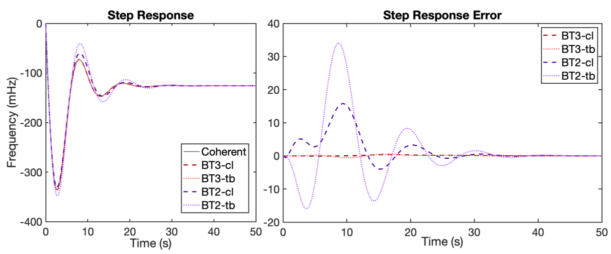

Fig. 2. Comparison of all reduced order models by balanced truncation

It is clear that, compared to 2nd order models, 3rd order reduced models give a very accurate approximation of $\hat{g}(s)$. While it is not surprising that approximation models with higher order ($k = 3$) outperform models with lower order ($k = 2$), we highlight that with only a 3rd order model one can accurately approximate the entire aggregate response.

Moreover, when we examine the transfer function given by $\tilde{g}_3^{tb}(s)$ (from input u in p.u. to output w in rad/s), we find an interesting interpretation. That is, the turbine model for $\tilde{g}_3^{tb}(s)$ is given by

$$\tilde{g}_{t,2}(s) = \frac{0.02664s + 0.00566}{s^2 + 0.5046s + 0.04891},$$

which, after doing partial fraction expansion, gives

$$\tilde{g}_{t,2}(s) = \frac{0.0473}{2.6759s + 1} + \frac{0.0684}{7.64s + 1}.$$

The latter can be viewed as two turbines (one fast turbine and one slow turbine) in parallel, and the choices of droop coefficients for these two turbines reflects the aggregate droop coefficients of fast turbines (generators 3 and 5) and slow turbines (generators 1,2, and 4), respectively, in $\hat{g}(s)$.

B. Reduction on Turbines vs. Closed-loop Dynamics

Another interesting observation that can also be derived from Fig. 2 is that balanced truncation on the closed-loop is more accurate than balanced truncation on the turbine. To get a more straightforward comparison, we list in Table II the approximation errors of all 4 models in Fig 2 using the following metrics: 1) $\mathcal{L}_2$-norm of step response error[3] $e(t)$ (in rad/s$^{1/2}$): $(\int_0^{+\infty} |e(t)|^2 dt)^{1/2}$; 2) $\mathcal{L}_\infty$-norm of $e(t)$ (in rad/s): $\max_{t\geq0} |e(t)|$; 3) $\mathcal{H}_\infty$-norm difference between reduced and original models (from input u in p.u. to output w in rad/s).

TABLE II

APPROXIMATION ERRORS OF REDUCED ORDER MODELS

Model \ Metric	$\mathcal{L}_2$ diff. (rad/s$^{1/2}$)	$\mathcal{L}_\infty$ diff. (rad/s)	$\mathcal{H}_\infty$ diff.
Guggilam [10]	7.2956	3.8287	10.2748
Germond [9]	3.9594	1.9974	5.1431
BT2-tb	4.3737	2.1454	7.5879
BT2-cl	2.0376	0.9934	2.0381
BT3-tb	0.0967	0.0361	0.1315
BT3-cl	**0.0704**	**0.0249**	**0.0317**

We observe from Table II that for a given the reduction order, balanced truncation on the closed-loop dynamics ($\tilde{g}_2^{cl}(s)$, $\tilde{g}_3^{cl}(s)$) has smaller approximation error than balanced truncation on turbine dynamics ($\tilde{g}_2^{tb}(s)$, $\tilde{g}_3^{tb}(s)$) *across all metrics*. Such observation seems to be true in general. For instance, Fig. 3 shows a similar trend by plotting the same configuration (metrics and models) of Table II for different values of of the aggregate inertia $\hat{m}$, while keeping all other parameters the same.

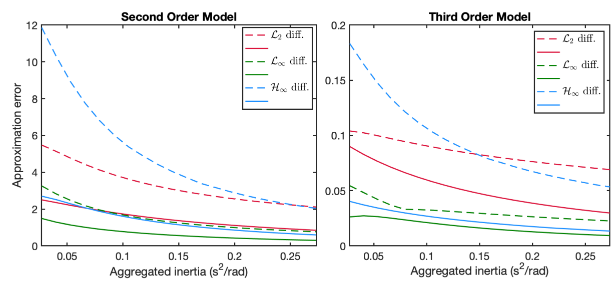

Fig. 3. Approximation errors of second order models (left) and third order models (right) by balanced truncation. Different metrics are shown in different colors. Approximation errors of reduced order models $\tilde{g}_2^{tb}(s)$, $\tilde{g}_3^{tb}(s)$ by reduction on turbine dynamics are shown in dashed lines; Approximation errors of reduced order models $\tilde{g}_2^{cl}(s)$, $\tilde{g}_3^{cl}(s)$ by reduction on closed-loop dynamics are shown in solid lines. The approximation errors are in their respective unit shown in Table.II

It can be seen from Fig. 3 that reduction on closed-loop dynamics improves the approximation in every metric, uniformly, for a wide range of aggregate inertia $\hat{m}$ values. The main reason is that, when applying reduction on closed-loop dynamics, the algorithm has the flexibility to choose the corresponding values of inertia and damping to be different from the aggregate ones in order to better approximate the response. More precisely, from the reduced model we obtain

$$\tilde{g}_2^{cl}(s) = \frac{4.9733s + 1}{(0.06715s + 0.01464)(4.9733s + 1) + 0.1118},$$

[3] For reduced order models obtained via frequency weighted balanced truncation, there exists an extremely small but non-zero DC gain mismatch that makes the $\mathcal{L}_2$-norm unbounded. We resolve this issue by simply scaling our reduced order models to have exactly the same DC gain as $\hat{g}(s)$.

from which we can get the equivalent swing and turbine model as

swing model: $\dfrac{1}{0.06715s+0.01464}$, turbine: $\dfrac{0.1118}{4.9733s+1}$.

The equivalent inertia and damping are $\tilde{m} = 0.06715$ and $\tilde{d} = 0.01464$, which are different from the aggregate values $\hat{m}, \hat{d}$. Therefore, when compared to reduction on turbine dynamics, reduction on closed-loop dynamics is essentially less constrained on the parameter space, thus achieving smaller approximation errors.

C. Comparison with Existing Methods

Lastly, we compare reduced order models via balanced truncation on the closed-loop dynamics, $\tilde{g}_2^{cl}(s)$, $\tilde{g}_3^{cl}(s)$, with the solutions proposed in [9], [10]. The step responses and the approximation errors are shown in Fig. 4 and Table. II.

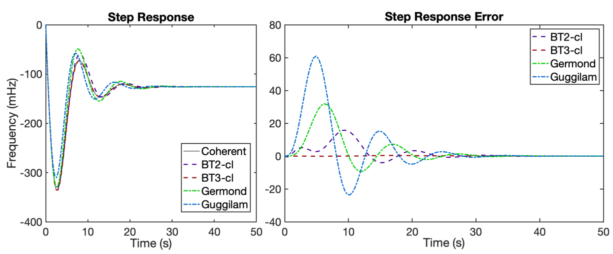

Fig. 4. Comparison with existing reduced order models

In the comparison, $\tilde{g}_3^{cl}(s)$ outperforms all other reduced order models and it is the most **accurate reduced order model** of $\hat{g}(s)$. It is also worth noting that $\tilde{g}_2^{cl}(s)$ has the least approximation error among all 2nd order models. In general, such results suggest us that to improve the accuracy of reduced order model of coherent dynamics of generators $\hat{g}(s)$, we should consider: 1) increasing the complexity (order) of the reduction model; 2) reduction on closed-loop dynamics instead of on turbine dynamics.

V. Conclusion and Future Work

This paper proposes a novel method to derive reduced order models for coherent generators. We derive a novel characterization of the aggregate response of coherent generators, i.e., $\hat{g}(s) = \left(\sum_{i=1}^n g_i^{-1}(s)\right)^{-1}$. We show that this aggregate dynamics $\hat{g}(s)$ is asymptotically accurate as the coupling between generators (characterized via $\lambda_2(L)$) increases. Our characterization not only explains why methods to aggregate generators with homogeneous time constants are accurate, but also explains the difficulties of aggregating generators with heterogeneous turbine time constants, i.e., when the coherent dynamics $\hat{g}(s)$ becomes a high-order transfer function. We solve this problem by leveraging tools from control theory to develop a methodology that finds accurate reduced order models of $\hat{g}(s)$. For $\{g_i(s)\}_{i=1}^n$ given by the 2nd order generator models, the numerical simulations show that 3rd order models based on frequency weighted balanced truncation on closed-loop dynamics are sufficient to accurately recover $\hat{g}(s)$.

There are many possible extensions to the existing results. Firstly, it has been shown in [13] that, whenever all the generator transfer functions $\{g_i(s)\}_{i=1}^n$ are proportional to each other, $\hat{g}(s)$ is a perfect descriptor of of the Center of Inertia (COI) frequency $\bar{w} = \left(\sum_{i=1}^n m_i w_i\right) / \left(\sum_{i=1}^n m_i\right)$. It is currently an on-going effort to show that $\hat{g}(s)$ is a reasonable approximation of the dynamics of COI frequency $\bar{w}$ even when the proportionality condition fails. Secondly, further experimentation with higher-order generator models as well as an extension of our analysis to multiple groups of coherent generators is a subject of future research.

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