

# On the Value of Energy Storage in Generation Cost Reduction

Yue Shen, Maxim Bichuch, and Enrique Mallada

**Abstract**—This work seeks to quantify the benefits of using energy storage toward the reduction of the energy generation cost of a power system. A two-fold optimization framework is provided where the first optimization problem seeks to find the optimal storage schedule that minimizes operational costs. Since the operational cost depends on the storage capacity, a second optimization problem is then formulated with the aim of finding the optimal storage capacity to be deployed. Although, in general, these problems are difficult to solve, we provide a lower bound on the cost savings for a parametrized family of demand profiles. The optimization framework is numerically illustrated using real-world demand data from ISO New England. Numerical results show that energy storage can reduce energy generation costs by at least 2.5%.

## I. INTRODUCTION

The electric power grid is undergoing one of the most fundamental transformations since its inception [1]. Technological development of renewable energy sources [2] coupled with the need to reduce carbon emissions is transforming the generation mix [3]. Alongside, the electrification of transportation is driving a rapid growth on global electricity demand [4]. Among the many challenges that this paradigm shift introduces is the lack of synchronism between the times when renewable energy is available and the time when energy demand is required. Energy storage is often seen as tentative solution towards addressing this challenge due to its ability to dispatch energy to shift energy availability across space, via deployment of distributed storage [5], and across time [6]. Thus the additional flexibility that storage provides, together with the steady decrease on build and installation prices, has stimulated the deployment of several grid-scale storage systems, e.g., [7] and [8].

However, despite the clear benefits that storage introduces, many questions regarding storage investment as well as efficient storage operation remain, to this day, unanswered. The key difficulty on this regard is that the cost of using storage is not a function of the instantaneous power, as it is the case for generators, where the cost can be mapped to the cost of the fuel consumed generate electricity. Instead, the cost of using storage indirectly arises from the unit degradation that is experienced during charging/discharging cycles [9].

Y. Shen and E. Mallada are with the ECE Department of Electrical and Computer Engineering and M. Bichuch is with the Department of Applied Mathematics and Statistics, Johns Hopkins University, 3400 North Charles St., Baltimore, MD 21218, USA {yshen50, mbichuch, mallada}@jhu.edu.

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This last observation is in contrast with the vast majority of existing works which formulate the cost of storage without detailed degradation model. Examples of this approach includes [10]–[12], which typically approach the storage operation by solving an optimization problem whose cost depend on fixed storage life-span. Only very recently, the question of how to optimally coordinate resources whose cost depend on instantaneous power with resources whose cost depend in energy trajectories, has started to be considered.

In [13], the problem of optimal coordination of limited-energy demand response and generation is considered. Similarly, [14], [15] looks considers the cost storage degradation and proposes an online algorithm with optimality guarantees.

In this paper, by perusing a path aligned to the works [14], [15], we seek to quantify the economic benefit of using storage arbitrage as means to reduce energy production costs. To this aim, we formulate a two-fold optimization framework aiming at: (a) finding the optimal storage operation that minimizes the total operational cost (including storage degradation cost); and (b) finding the optimal amount of storage that need to be deployed in the system to achieve the maximum benefit. Despite the complexity of such problems, we provide a sub-optimal policy for a parametrized family of demand trajectories that allow us quantify a lower bound on the potential operational savings.

The rest of the paper is organized as follows. Our problem setting, including the energy system model, as well as a general problem formulation of the optimal *operational problem* that seeks to optimally control storage to reduce operations costs, and the *planning problem*, that seeks to find the optimal storage deployment, are presented in section II. In Section III, we describe the proposed approach to quantify the potential benefits by reformulating the above-mentioned general problems. The analytical solutions to our reformulations are presented in Section IV and Section V. Finally, preliminary numerical analysis, and conclusions, are provided in sections VI and VII, respectively.

## II. PROBLEM FORMULATION

In this section, we describe the energy system that we seek to study and formalize the two problems that we consider in this paper.

### A. System Model

We use  $d(t) \geq 0$  to denote the net uncontrollable power demand –possibly including renewable– and  $p(t)$  the total aggregate power generation of a system operator at time  $t \in [t_0, t_f]$ . The total energy stored in the system at time  $t$  is

denoted by  $e(t)$ . The energy  $e(t)$  evolves according to

$$\dot{e}(t) = u(t), \quad (1)$$

where  $u$  is the rate of change of stored energy. We adopt the convention that  $u(t) > 0$  implies charging, whereas  $u(t) < 0$  means discharging. The total storage capacity is denoted by  $C$  and the maximum charging/discharging rate by  $r$ , i.e.,

$$-r \leq u(t) \leq r = \frac{C}{\epsilon}, \quad \text{and} \quad 0 \leq e(t) \leq C. \quad (2)$$

The ratio between  $\epsilon = \frac{C}{r}$  referred as technology parameter and it is aimed at representing different technological features of the storage. We further let  $T_{\text{ls}}$  denote the total lifespan of the energy system.

Finally, the net load of the energy system  $d(t) + u(t)$  is supplied by external power supply  $p(t)$ , i.e.,

$$p(t) = d(t) + u(t). \quad (3)$$

For simplicity, we assume in this paper that the charging/discharging process is lossless. A more realistic model that relaxes this assumption is subject of current research.

### B. Cost Model

We are interested in quantifying the benefits of using energy storage as a way to reduce the overall cost that the system incurs in meeting the demand  $d(t)$ .

1) *Generation Cost:* We model the aggregate generation cost using quadratic cost function  $L_g : \mathbb{R} \rightarrow \mathbb{R}$  for power supply, i.e.,

$$L_g(p) = \frac{a}{2}p^2 + bp, \quad (4)$$

where  $a$  and  $b$  are positive cost coefficients. Equation (4) represents either the generation cost derived from fuel consumption (see, e.g., [16]) or the integral of the inverse aggregate supply function derived empirically from the ISO (see, e.g., [17], [18]).

2) *Storage Cost:* Unlike generation cost that originates from cost of producing energy, storage cost is a result of battery degradation that occurs with each discharge cycle. To compute this cost we first let functional  $\mathcal{D}_i : C_{[t_0, t_f]}^\infty \rightarrow \mathbb{R}_{\geq 0}$  denote the  $i$ th Depth of Discharge (DoD),  $i \in \mathbb{N}^+$ , of the energy trajectory  $e \in C_{[t_0, t_f]}^\infty$ . Given the normalized DoD  $y \in [0, 1]$ , the storage degradation is given by the cycle depth stress function  $\Phi(y)$ , with  $\Phi : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ . This function quantifies the battery loss of life due to a cycle of depth  $y = \frac{D}{C}$  for a DoD  $D$  and capacity  $C$ .

Thus the total storage cost due to the energy trajectory  $e$  is represented by the cost functional  $\mathcal{L}_s : C_{[t_0, t_f]}^\infty \rightarrow \mathbb{R}_{\geq 0}$  given by

$$\mathcal{L}_s(e) = \sum_{i=1}^{\infty} \Phi\left(\frac{\mathcal{D}_i(e)}{C}\right) C\rho, \quad (5)$$

where  $\rho$  represents the one-time unit building cost.

### C. The Value of Storage

As mentioned before, we seek to quantify the benefit that storage brings to an ISO. We will investigate this benefit in two settings. We first consider the *operational problem* of how to optimally operate the available storage to minimize the system cost. Because such problem implicitly depends on the amount of storage available, we then move towards the *planning problem* of finding the optimal amount of storage that one needs to deploy.

1) *Operational Problem:* The first difficulty on seeking to optimally use storage arises from the difference in the argument between (4) and (5). While (4) is a function of the instantaneous power being generated, (5) depends on the entire energy trajectory, which implicitly depends on  $u(t)$ . We overcome this issue by expressing (4) in terms of the total energy production cost over the interval  $[t_0, t_f]$ . This is given by the generation cost functional  $\mathcal{L}_g : C_{[t_0, t_f]}^\infty \rightarrow \mathbb{R}_{\geq 0}$ , i.e.,

$$\begin{aligned} \mathcal{L}_g(p, t_0, t_f) &= \int_{t_0}^{t_f} L_g(p(t)) dt \\ &= \int_{t_0}^{t_f} \frac{a}{2} p(t)^2 + bp(t) dt. \end{aligned} \quad (6)$$

Using (6) and (5) one can formally define the optimal storage control problem as

$$\begin{aligned} J(C, [t_0, t_f]) &= \min_{u, e, p} \mathcal{L}_g(p, t_0, t_f) + \mathcal{L}_s(e, t_0, t_f), \quad (7a) \\ \text{s.t.} \quad &(1), (2), (3). \quad (7b) \end{aligned}$$

The optimal control problem (7) has the advantage of combining both, the cost of using the energy storage, together with the cost generation power in a common setting.

However, there are two main difficulties in using (7) towards quantifying the benefits of storage. Firstly, the solution will depend on the boundary conditions at  $t_0$  and  $t_f$ , which can make the interpretation of the benefits hard to assess. Secondly, the cost functional (5) is hard to evaluate and highly dependent on the demand  $d$ .

We overcome the dependence on the boundary conditions by consider the following average operational cost problem. The latter issue will be addressed in the next section.

#### Problem 1. (Average Operational Problem)

$$J(C) = \lim_{\substack{t_0 \rightarrow -\infty \\ t_f \rightarrow \infty}} \frac{1}{t_f - t_0} J(C, [t_0, t_f]) \quad (8)$$

Given a the tuple  $(C)$ , Problem 1 quantifies the average operational cost  $J(C)$ . This value can be used to measure the benefits of including storage by looking at difference in cost between  $J(0)$  and  $J(C)$ , i.e.,

$$B_f(C) = J(0) - J(C).$$

2) *Planning Problem*: We now turn to the formulation of the planning problem. More precisely, we seek to capture the effect of  $C$  on the operational cost and find the optimal storage capacity. However, because certain values of  $C$  may not be feasible, we will implicitly constraint them including a bound on the life-span  $T_{ls} \leq T_{ls, \max}$ . This leads to the following optimization problem.

**Problem 2.** (*Planning Problem*)

$$\min_C J(C), \quad (9a)$$

$$\text{s.t. } T_{ls}^{u^*}(C) \leq T_{\max}. \quad (9b)$$

where  $T_{ls}^{u^*}(C)$  is the life span of a storage of capacity  $C$  under the control  $u$ , and  $u^*$  is the optimal policy derived in Problem 1.

We finalize by noting that the relationship between  $T_{ls}$  and  $(C)$  is not straight forward, and only appears in cases where the optimal capacity in (9) leads to an optimal policy with very low storage degradation per unit of time that requires unrealistic life-spans.

### III. SOLUTION APPROACH

As mentioned before, problems 1 and 2 are either intractable due to the complexity of evaluating  $\mathcal{L}_s$  or uninformative of the overall benefits of using storage due to the dependence of (7) on boundary conditions. In order to overcome this limitation, we relax some of the constraints and seek to find an upper bound on a family sub-instances of such problems. This allows us to characterize the dependence of  $J(C)$  on the frequency and amplitude of intra-day demand cycles and, in this way, get an upper bound on the benefits that storage can introduce.

We focus on demand functions that capture the fluctuating demand behavior. We start by assuming a realistic case where power demand  $d$  is perturbation around certain baselines  $d_0 \geq 0$ . We further assume the perturbations around  $d_0$  is a periodic sinusoidal deviation with amplitude  $d_1 \leq d_0$ , i.e.

$$d(t) = d_0 + d_1 \sin(\omega_0 t). \quad (10)$$

#### A. Operational Problem Reformulation

Instead of considering explicitly the storage functional  $\mathcal{L}_s$  in (5), we use quadratic storage cost functional  $\mathcal{L}_q(\cdot; \gamma) : C_{[t_0, t_f]}^\infty \rightarrow \mathbb{R}_{\geq 0}$

$$\mathcal{L}_q(e, t_0, t_f; \gamma) = \int_{t_0}^{t_f} \frac{\gamma}{2} (e(t) - e_0)^2 dt \quad (11)$$

that penalizes the instantaneous stored energy deviation from a reference energy  $e_0$ . The penalty parameter  $\gamma > 0$  not only limits the amount of energy being used, thus limiting degradation, but it also implicitly constraints the control effort  $u$ . Thus we remove constraints (2) and solve instead the following auxiliary optimal control problem

$$\tilde{J}(\gamma, [t_0, t_f]) = \min_{u, e, p} \mathcal{L}_g(p, t_0, t_f) + \mathcal{L}_q(e, t_0, t_f; \gamma), \quad (12a)$$

$$\text{s.t. } p = d + u, \quad (12b)$$

$$\dot{e} = u, \quad (12c)$$

which after taking  $t_0 \rightarrow -\infty$  and  $t_f \rightarrow \infty$  leads to the following auxiliary operational problem.

**Problem 3.** (*Auxiliary Average Operational Problem*).

$$\tilde{J}(\gamma) = \lim_{\substack{t_0 \rightarrow -\infty \\ t_f \rightarrow \infty}} \frac{1}{t_f - t_0} \tilde{J}(\gamma, [t_0, t_f]). \quad (13a)$$

where  $d$  is defined by (10).

Although the value  $\tilde{J}(\gamma)$  does not have a specific economic meaning, we will show that the optimal solution of Problem 3 is pure sinusoidal around certain baselines with frequency  $\omega_0$ . The amplitudes of  $u(t)$  and  $e(t)$  are functions of  $\gamma$ , i.e.,

$$u(t, \gamma) = -u_1(\gamma) \sin(\omega_0 t), \quad (14a)$$

$$e(t, \gamma) = e_0 + e_1(\gamma) \cos(\omega_0 t). \quad (14b)$$

Note that for control and storage trajectories (14), every cycle is identical. Thus, for this particular choice of demand and class of problems it is possible to express the DoD for every cycle as:

$$D(\gamma) = 2e_1(\gamma). \quad (15)$$

as well as compute explicitly, the average generation cost

$$J_g(\gamma) = \frac{a}{4} (d_1 - u_1(\gamma))^2 + \frac{a}{2} d_0^2 + b d_0, \quad (16)$$

the average storage cost of the original storage model  $\mathcal{L}_s(e)$

$$J_s(\gamma, C) = \Phi(2e_1(\gamma)/C) C \rho \frac{\omega_0}{2\pi}, \quad (17)$$

and the total operational cost of the control policy (14):

$$J(\gamma, C) = J_g(\gamma) + J_s(\gamma, C). \quad (18)$$

As a result, since (14) is a feasible solution to Problem 1, it is an upper bound on the total operational cost.

#### B. Planning Problem Reformulation

Once we have explicit expressions for the long term average generation and storage cost as functions of  $\gamma$  and  $C$ , we can further reformulate the planning problem in the following form. Note that we will optimize it with respect to  $\gamma$  and  $C$  and the solution of this reformulated planning problem gives optimal storage capacity and penalty parameter that achieves an upper bound on the optimal planning problem Problem 2.

**Problem 4.** (*Reformulated Planning Problem*)

$$\min_{\gamma, C} J(\gamma, C), \quad (19a)$$

$$\text{s.t. } D(\gamma) \leq C, \quad (19b)$$

$$u_1(\gamma) \leq r = \frac{C}{\epsilon}, \quad (19c)$$

$$T_{ls}^{u(t, \gamma)}(C) \leq T_{\max}. \quad (19d)$$

where  $D(\gamma)$ ,  $J(\gamma, C)$ , and  $(u(t, \gamma), u_1(\gamma))$  are defined in (15), (18), and (14), respectively.

#### IV. OPERATIONAL PROBLEM

In this section we provide an analytical solution to the auxiliary problem as well as *Theorem 1* unveils the analytically optimal solution of the auxiliary average operational problem.

**Theorem 1.** *Given the operational problem (13), the optimal storage control in an infinite time horizon follows equation (14) with*

$$u_1(\gamma) = d_1 \frac{\omega_0^2}{\theta^2 + \omega_0^2}, \quad (20)$$

$$e_1(\gamma) = d_1 \frac{\omega_0}{\theta^2 + \omega_0^2}, \quad (21)$$

where  $\theta = \sqrt{\frac{\gamma}{a}}$ .

*Proof.* By substituting (10) and (12b) into (12a) and defining  $d_s(t) := d_1 \sin(\omega_0 t)$  as the sinusoidal part of demand (10), we can explicitly express the operational cost as

$$\begin{aligned} \mathcal{L}(p, e, t_0, t_f; \gamma) &= \mathcal{L}_g(p, t_0, t_f) + \mathcal{L}_q(e, t_0, t_f; \gamma), \\ &= \int_{t_0}^{t_f} \left( \frac{a}{2} d_0^2 + a d_0 (d_s(t) + u(t)) + \frac{a}{2} (d_s(t) + u(t))^2 \right. \\ &\quad \left. + b d_0 + b (d_s(t) + u(t)) + \frac{\gamma}{2} (e(t) - e_0)^2 \right) dt. \end{aligned} \quad (22)$$

Since the optimal control does not depend on the constant terms, we can drop the terms  $b d_0$  and  $\frac{a}{2} d_0^2$ , and further let  $p_s(t) := d_s(t) + u(t)$  and  $e_s(t) := e(t) - e_0$  to get

$$\mathcal{L}(p, e, t_0, t_f; \gamma) = \int_{t_0}^{t_f} \frac{a}{2} p_s(t)^2 + \beta p_s(t) + \frac{\gamma}{2} e_s(t)^2 dt, \quad (23)$$

where  $\beta := a d_0 + b$ .

By introducing a Lagrange multiplier  $\lambda$  for the storage dynamics (12c), the Euler-Lagrange equation demonstrates the optimal conditions for this problem:

$$a (d_s(t) + u(t)) + \beta + \lambda(t) = 0, \quad (24)$$

$$\dot{\lambda}(t) = -\gamma e_s(t). \quad (25)$$

Combining (24) and (25) eliminates  $\lambda$  and yields a second-order differential equation for which the optimal storage control needs to satisfy:

$$a \left( \dot{d}_s(t) + \dot{u}(t) \right) = \gamma e_s(t). \quad (26)$$

At the starting time  $t_0$ , let  $d_{t_0}^s := d_s(t_0)$ ,  $e_{t_0}^s := e_s(t_0)$

and  $\dot{e}_{t_0}^s := \dot{e}_s(t_0)$ . Then the closed-form solution to (26) is

$$\begin{aligned} e_s(t) &= \left( -d_1 \frac{\theta}{\theta^2 + \omega_0^2} \sin(\omega_0 t_0) + \frac{1}{\theta} (\dot{e}_{t_0}^s + d_{t_0}^s) \right) \sinh(\theta(t-t_0)) \\ &\quad + \left( -d_1 \frac{\omega_0}{\theta^2 + \omega_0^2} \cos(\omega_0 t_0) + e_{t_0}^s \right) \cosh(\theta(t-t_0)) \\ &\quad + d_1 \frac{\omega_0}{\theta^2 + \omega_0^2} \cos(\omega_0 t), \end{aligned} \quad (27a)$$

$$\begin{aligned} u(t) &= \left( -d_1 \frac{\theta^2}{\theta^2 + \omega_0^2} \sin(\omega_0 t_0) + (\dot{e}_{t_0}^s + d_{t_0}^s) \right) \cosh(\theta(t-t_0)) \\ &\quad + \theta \left( -d_1 \frac{\omega_0}{\theta^2 + \omega_0^2} \cos(\omega_0 t_0) + e_{t_0}^s \right) \sinh(\theta(t-t_0)) \\ &\quad - d_1 \frac{\omega_0^2}{\theta^2 + \omega_0^2} \sin(\omega_0 t), \end{aligned} \quad (27b)$$

where  $\theta = \sqrt{\frac{\gamma}{a}}$ ,  $\dot{e}_{t_0}^s$  and  $e_{t_0}^s$  are unknowns.

We further check the sufficient condition for optimality by solving the Jacobi Accessory Equation (28b) to (28d) with conditions in (28e) [19, p. 52]

$$L(u, e_s) = \frac{a}{2} (d_s + u)^2 + \beta (d_s + u) + \frac{\gamma}{2} (e_s)^2; \quad (28a)$$

$$R = \frac{1}{2} \left( \frac{\partial}{\partial^2 u} L(u, e_s) \right) \Bigg|_{\substack{d_s=d_s(t) \\ e_s=e_s(t) \\ u=u(t)}} = \frac{1}{2} a; \quad (28b)$$

$$\begin{aligned} Q &= \frac{1}{2} \left( \frac{\partial}{\partial^2 e_s} L(u, e_s) - \frac{d}{dt} \frac{\partial}{\partial u} \frac{\partial}{\partial e_s} L(u, e_s) \right) \Bigg|_{\substack{d_s=d_s(t) \\ e_s=e_s(t) \\ u=u(t)}} \\ &= \frac{1}{2} \gamma; \end{aligned} \quad (28c)$$

$$Qv(t) = \frac{d}{dt} (R\dot{v}(t)); \quad (28d)$$

$$v(t_0) = 0; \quad v(t) \neq 0. \quad (28e)$$

The solution is the given by

$$v(t) = c_1 e^{\sqrt{\frac{\gamma}{a}}(t-t_0)} - c_1. \quad (29)$$

for some constant  $c_1$ . Note that the solution  $v(t) \neq 0$  for all  $t > t_0$  leads to non-existence of conjugate points within the time interval  $[t_0, t_f]$ . This, together with the fact that  $R > 0$ , shows that the extremal in problem (13) is a strict minimum.

Now, we are able to derive the closed-form expressions of  $e_s(t)$  and  $u_s(t)$  with two remaining unknowns  $e_{t_0}^s$  and  $\dot{e}_{t_0}^s$ . Since  $e_s(t_0)$  and  $e_s(t_f)$  are constraint-free, by the transversality condition [19, p. 83]  $\lambda(t_0) = \lambda(t_f) = 0$ , it follows from (24) that

$$\dot{e}_{t_0}^s = u(t_0) = -\frac{\beta}{\alpha} - d_{t_0}^s, \quad (30a)$$

$$u(t_f) = -\frac{\beta}{\alpha} - d(t_f). \quad (30b)$$

Note that (30a) immediately gives rise to the  $\dot{e}_{t_0}^s$ . Then evaluating (27b) at  $t_f$  and plugging in the expression (30b) yields an algebraic equation in  $e_{t_0}^s$

Substituting both unknowns in (27) gives,

$$\begin{aligned}
e_s(t) = & \sinh(\theta(t-t_0)) \left( -d_1 \frac{\theta}{\theta^2 + \omega_0^2} \sin(\omega_0 t_0) - \frac{1\beta}{\theta a} \right) \\
& + \frac{\cosh(\theta(t-t_0))}{\sinh(\theta T_\Delta)} \left( \frac{1\beta}{\theta a} (\cosh(\theta T_\Delta) - 1) \right. \\
& \left. + d_1 \frac{\theta}{\theta^2 + \omega_0^2} (\sin(\omega_0 t_0) \cosh(\theta T_\Delta) - \sin(\omega_0 t_f)) \right) \\
& + d_1 \frac{\omega_0}{\theta^2 + \omega_0^2} \cos(\omega_0 t); \tag{31a}
\end{aligned}$$

$$\begin{aligned}
u(t) = & \cosh(\theta(t-t_0)) \left( -d_1 \frac{\theta^2}{\theta^2 + \omega_0^2} \sin(\omega_0 t_0) - \frac{\beta}{a} \right) \\
& + \frac{\sinh(\theta(t-t_0))}{\sinh(\theta T_\Delta)} \left( \frac{\beta}{a} (\cosh(\theta T_\Delta) - 1) \right. \\
& \left. + d_1 \frac{\theta^2}{\theta^2 + \omega_0^2} (\sin(\omega_0 t_0) \cosh(\theta T_\Delta) - \sin(\omega_0 t_f)) \right) \\
& - d_1 \frac{\omega_0^2}{\theta^2 + \omega_0^2} \sin(\omega_0 t); \tag{31b}
\end{aligned}$$

where  $T_\Delta := t_f - t_0$  is the total operational time.

After taking the limits  $t_0 \rightarrow -\infty$  and  $t_f \rightarrow \infty$ , the non-periodical terms in (31) that rely on the initial and final conditions vanish. Therefore, (31a) and (31b) lead to equation (14) where  $e_1(\gamma)$  and  $u_1(\gamma)$  are given by (20).

Finally, we observe that one can show that the average operational cost  $\frac{1}{t_f - t_0} \mathcal{L}(\cdot, \cdot, t_0, t_f; \gamma)$  is continuous and uniformly converges as  $t_0 \rightarrow -\infty$  and  $t_f \rightarrow \infty$  to the limit  $\lim_{\substack{t_0 \rightarrow -\infty \\ t_f \rightarrow \infty}} \frac{1}{t_f - t_0} \mathcal{L}(\cdot, \cdot, t_0, t_f; \gamma)$ . Therefore, the limiting policy obtained from (31) is an optimal solution of (13).  $\square$

**Corollary 1.** *Given the operational problem (13), the optimal average generation cost  $\mathcal{L}_s(e)$  and average storage cost  $\mathcal{L}_s(e)$  are expressed in (16) and (17), respectively.*

*Proof.* We find the average generation cost by substituting (31b) into the generation cost model  $\mathcal{L}_g(p)$  defined in (6) and then taking the average, i.e.

$$\begin{aligned}
J_g(\gamma, C) &= \lim_{\substack{t_0 \rightarrow -\infty \\ t_f \rightarrow \infty}} \frac{1}{t_f - t_0} \mathcal{L}_g(p, t_0, t_f) \\
&= \frac{a}{4} (d_1 + u_1(\gamma))^2 + \frac{a}{2} d_0 + b d_0. \tag{32}
\end{aligned}$$

Moreover, since we found the explicit expressions of  $\mathcal{D}(e)$  and  $r(\gamma)$  for every cycle as functions of  $\gamma$ , we can express the long term average storage cost by the original storage model  $\mathcal{L}_s(e)$  that we defined in (5), i.e.

$$\begin{aligned}
J_s(\gamma, C) &= \lim_{\substack{t_0 \rightarrow -\infty \\ t_f \rightarrow \infty}} \frac{1}{t_f - t_0} \sum_{i=1}^{\infty} \Phi(2e_1(\gamma)/C) C \rho \\
&= \Phi(2e_1(\gamma)/C) C \rho \frac{\omega_0}{2\pi}. \tag{33}
\end{aligned}$$

Thus, result follows.  $\square$

We can further use Theorem 1 to explicitly compute the life span of the storage under the control policy of (13). To do this we first require an explicit formula for the storage

degradation. Thus, we further assume the following storage degradation cost function experimentally derived in [9].

**Assumption 1.** *For a battery with capacity  $C$ , the battery degradation for a cycle with DoD  $D$  is approximately*

$$\Phi(D, C) = (k_1(D/C)^{k_2} + k_3)^{-1}, \tag{34}$$

with degradation parameters  $k_1$ ,  $k_2$  and  $k_3$  such that

$$k_1 > 0, \quad k_3 < 0, \tag{35a}$$

$$-1 < k_2 < 0, \tag{35b}$$

$$k_1 + k_3 > 0, \tag{35c}$$

$$k_1(1 + k_2) + k_3 \leq 0. \tag{35d}$$

**Corollary 2.** *Given the operational problem (13), the life span of a storage with capacity  $C$  under the optimal control  $u(t, \gamma)$  in Theorem 1 is*

$$T_{ls}^{u(t, \gamma)}(C) = (k_1(2e_1(\gamma)/C)^{k_2} + k_3) \frac{2\pi}{\omega_0}. \tag{36}$$

## V. PLANNING PROBLEM

We now leverage Lemma 1 and Theorem 2 to solve our operational problem (19).

**Lemma 1.** *Given the planning problem (19a)-(19d) with degradation model in Assumption 1, we can express the optimal cycle depth  $y^*$  as follows*

$$y^* = \begin{cases} \frac{2}{\epsilon \omega_0}, & \text{if } y_s > \frac{2}{\epsilon \omega_0}, \\ \hat{y}, & \text{if } y_s < \hat{y}, \\ y_s, & \text{o.w.} \end{cases} \tag{37a}$$

where  $y_s := \left( -\frac{k_3}{k_1(1+k_2)} \right)^{\frac{1}{k_2}} \in (0, 1]$  and  $\hat{y} := \left( (T_{max} \frac{\omega_0}{2\pi} - k_3) \frac{1}{k_1} \right)^{\frac{1}{k_2}} \leq 1$  are constants introduced by cost model (34) and the maximum life span constraint (19d), respectively.

*Proof.* By letting  $y = D/C$  be the normalized DoD, we can express battery degradation function (34) using

$$\Phi(y) = (k_1 y^{k_2} + k_3)^{-1}. \tag{38}$$

Moreover, we can further rewrite (17) such that the cost function (19a) is

$$\begin{aligned}
\tilde{J}(y, \gamma) &= J(C, \gamma) \\
&= J_g(\gamma) + \Phi(y) 2e_1(\gamma) y^{-1} \rho \frac{\omega_0}{2\pi}, \tag{39}
\end{aligned}$$

where  $y = 2e_1(\gamma)/C$ , with constraints on  $C$  (19b), (19c) and (19d) becoming:

$$0 < y \leq 1, \tag{40}$$

$$y \leq \frac{2}{\epsilon \omega_0}, \tag{41}$$

$$y \geq \left( (T_{max} \frac{\omega_0}{2\pi} - k_3) \frac{1}{k_1} \right)^{\frac{1}{k_2}}. \tag{42}$$

Clearly, for a solution to the problem to exist, we require the above equations to have a non-empty intersection, i.e.,

$$\left( (T_{max} \frac{\omega_0}{2\pi} - k_3) \frac{1}{k_1} \right)^{\frac{1}{k_2}} \leq 1, \quad (43)$$

$$\left( (T_{max} \frac{\omega_0}{2\pi} - k_3) \frac{1}{k_1} \right)^{\frac{1}{k_2}} \leq \frac{2}{\epsilon \omega_0}. \quad (44)$$

Under this assumption then, we can solve this optimization problem with respect to  $y$  and  $\gamma$ . To do this for a non-convex objective function, we first express the following stationary conditions:

$$\frac{\partial}{\partial y} \tilde{J} = -\frac{y^{k_2} k_1 (1 + k_2) + k_3}{y^2 (k_1 y^{k_2} + k_3)^2} \frac{\omega_0}{\pi} \frac{d_1 \rho}{\omega_0^2 + \frac{\gamma}{a}} = 0, \quad (45a)$$

$$\frac{\partial}{\partial \gamma} \tilde{J} = \frac{d_1 \omega_0^2 a \left( (d_1 \pi a y (k_1 y^{k_2} + k_3) - 2\rho) \gamma - 2\omega_0^2 a \rho \right)}{2y (k_1 y^{k_2} + k_3) \pi (\omega_0^2 a + \gamma)^3} = 0. \quad (45b)$$

Solving (45a) gives stationary  $y_s$  that is independent from  $\gamma$ :

$$y_s = \left( -\frac{k_3}{k_1(1+k_2)} \right)^{\frac{1}{k_2}}. \quad (46a)$$

Now, since  $k_1$ ,  $k_2$  and  $k_3$  satisfy (35), it follows that  $1 \geq y_s > 0$ , which leads to constraint (40) being always satisfied.

We then can claim that  $y_s$  is the unconstrained optimal cycle depth for all  $y \in (0, 1]$  and  $\gamma \in [0, \infty)$ . Indeed, we calculate

$$\frac{\partial}{\partial y} \tilde{J} < 0 \quad \text{for} \quad \gamma \geq 0, \quad 0 \leq y < y_s, \quad (47a)$$

$$\frac{\partial}{\partial y} \tilde{J} = 0 \quad \text{for} \quad \gamma \geq 0, \quad y = y_s, \quad (47b)$$

$$\frac{\partial}{\partial y} \tilde{J} > 0 \quad \text{for} \quad \gamma \geq 0, \quad 1 \geq y > y_s. \quad (47c)$$

With the upper and lower bound of  $y$ , (41) and (42), we see that if  $y_s$  violates the bounds, (e.g.  $y_s > \frac{2}{\epsilon \omega_0}$ ), then the optimal  $y = \frac{2}{\epsilon \omega_0}$ , and similar with the other bound (42).  $\square$

**Theorem 2.** *Given the planning problem (19a)-(19d) with degradation model in Assumption 1, the optimal solution is*

$$\gamma^* = \begin{cases} \infty, & \text{if } \gamma_s < 0, \\ \gamma_s, & \text{o.w.,} \end{cases} \quad (48)$$

$$C^* = 2e_1(\gamma^*)/y^*. \quad (49)$$

where  $\gamma_s := \frac{2\omega_0^2 a \rho}{d_1 \pi a y^* (k_1 (y^*)^{k_2} + k_3) - 2\rho}$  and  $e_1(\gamma)$  is defined in (21).

**Remark 1.** *When  $\gamma_s < 0$ , Theorem 2 states that  $\gamma^* = \infty$ , which implies that the optimal amount of storage is zero. This can occur in particular, when either  $y^*$  or  $d_1$  are small. In other words, when demand variations are not significant, using storage becomes inefficient.*

*Proof.* Using stationary condition (45b) and Lemma 1, one can solve the stationary  $\gamma_s$  as:

$$\gamma_s = \frac{2\omega_0^2 a \rho}{d_1 \pi a y^* (k_1 (y^*)^{k_2} + k_3) - 2\rho}. \quad (50)$$

Consider first the case  $\gamma_s > 0$ , which holds if and only if

$$d_1 \pi a y^* (k_1 (y^*)^{k_2} + k_3) - 2\rho > 0. \quad (51)$$

Then, we have

$$\frac{\partial}{\partial \gamma} \tilde{J} < 0 \quad \text{for} \quad 0 \leq \gamma < \gamma_s, \quad y = y^*, \quad (52a)$$

$$\frac{\partial}{\partial \gamma} \tilde{J} = 0 \quad \text{for} \quad \gamma = \gamma_s, \quad y = y^*, \quad (52b)$$

$$\frac{\partial}{\partial \gamma} \tilde{J} > 0 \quad \text{for} \quad \gamma > \gamma_s, \quad y = y^*. \quad (52c)$$

For the case that  $\gamma_s < 0$ , we have  $d_1 \pi a y^* (k_1 (y^*)^{k_2} + k_3) - 2\rho < 0$  that leads to  $\frac{\partial}{\partial \gamma} \tilde{J}|_{y=y^*} < 0$  for all positive  $\gamma$ . This implies that if  $\gamma_s < 0$  then the optimal  $\gamma$  is given by  $\gamma^* = \infty$ .

After changing variables back from  $y$ , and  $\gamma$  back to  $C$ , the results of Theorem 2 follows.  $\square$

## VI. NUMERICAL RESULTS

In this section, we use our analysis to shed light on the potential savings that can be achieved, by using real-world demand data from ISO New England [20] on (data: 07/17/2019). Based on the mentioned data, we choose  $d_0$  and  $d_1$  in (10), by selecting the DC and first harmonic of a one-day data. Thus,  $d_0 = 18091$  MW,  $d_1 = 4671$  MW, and  $\omega_0 = 0.26$  rad/hr, which corresponds to one cycle per day. Therefore the demand variation  $d_1 \cos(\omega_0 t)$  in (10) represents the demand fluctuations for the main daily cycle.

The storage degradation coefficients in our numerical test are  $k_1 = 1.4 \times 10^5$ ,  $k_2 = -0.5$ ,  $k_3 = -1.23 \times 10^5$  and the largest battery life spans  $T_{max} = 76$  years according to the LMO battery degradation test [9]. Note that these numbers satisfy our general assumptions on the degradation model (35).

The generation cost coefficients are selected to be  $a = 0.02$  and  $b = 16.24$  base on the energy system model [16]. Finally, the capacity-power ratio is set as  $\epsilon = 2$  by [21]. At last, by [9], the one-time unit building cost  $\rho = 209000$  USD/MWh.

We study the impact of varying the demand frequency  $\omega_0$  on the cost performance by varying  $\omega_0$  from 1 cycle per day ( $\omega_0 = 0.26$  rad/hr) to 144 cycle per day ( $\omega_0 = 37.66$  rad/hr). Note that the assumptions (43) and (44) are satisfied for the given frequency range. The storage capacity and the charging/discharging power in per-unit system and the cost composition are respectively plotted in the upper and lower panels of Fig 1, where the base is 4671MW.

Simulation results indicate cost saving between 2.77% – 2.56% can be achieved. Moreover, the storage is not used for both extremely low and extremely high frequencies (beyond the range of the plot). This is intuitive, since the storage operation will only incur extra cost when the demand is flat.

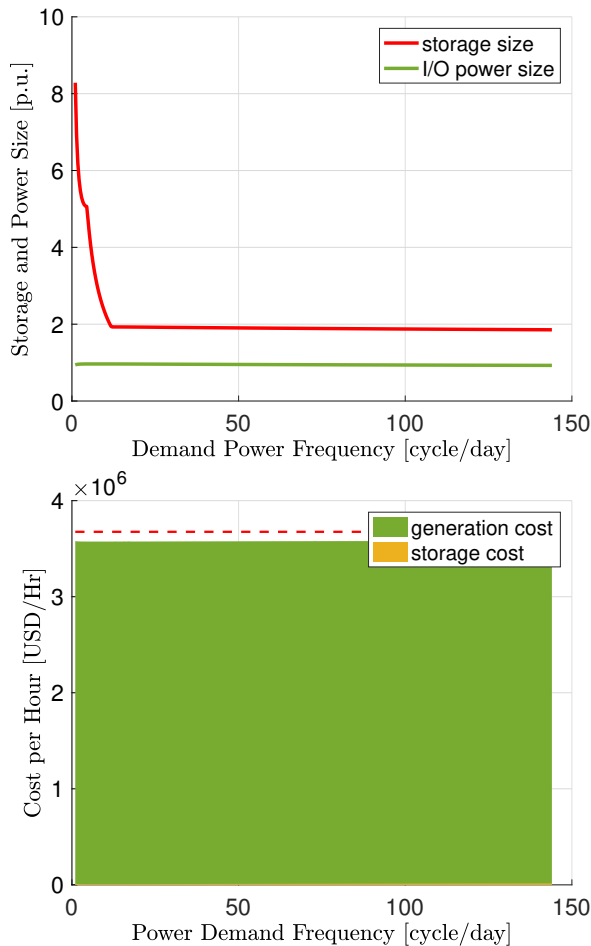


Fig. 1: The size of storage capacity and charging/discharging power in per unit system (upper) and cost composition (lower)

Similarly, if the demand variation are extremely frequent, the storage runs out of degrades quickly and the building cost increases. Our results also show that larger generation cost coefficient  $a$  leads to higher savings.

## VII. CONCLUSION

In this work, we propose an optimization framework that aims at estimating the operational cost benefits of using storage in an energy system as well as the optimal storage amount that should be deployed into the system. Analytical closed form solutions to this framework bring insight on the effect of the frequency of demand fluctuations and capacity on the overall system cost. Numerical examples are provided to illustrate our framework. Analysing the effect of charging/discharging inefficiency on this framework is an ongoing topic of research.

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