Minimum-Time Charging of Energy Storage in Microgrids via Approximate Conic Relaxation

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Abstract— **We study the problem of maximizing energy transfer to a load in a DC microgrid while respecting constraints on bus voltages and currents, and accounting for the impact of neighboring constant power loads. Both the objective and dynamics give rise to indefinite quadratic terms, resulting in a non-convex optimization problem. Through change of variables and relaxations we develop a closely related second-order cone program. The problem retains the same feasible set as the original problem but utilizes a linear approximation of the non-convex objective. We demonstrate how this can be used to design approximately optimal charging profiles for periodic pulsed loads in real time.**

I. INTRODUCTION

Pulsed loads are electrical loads that consume large amounts of energy near instantaneously. Classical examples of technologies that use pulsed power include particle accelerators and lasers. More recently, within the marine and aerospace communities, pulsed power loads are being installed on ships and aircraft. Examples include electromagnetic launch and recovery systems, solid-state radars, and high-energy lasers. These loads are stressing to the host power systems which have limited generating capacity and are not designed to accommodate rapid power variations. To mitigate this stress, pulsed loads are often supported by an energy storage device such as a capacitor bank or flywheel [1].

While energy buffers provide the means for reducing the transient demand placed on the supplying generators, it is often desirable to charge these devices as quickly as possible to allow repetitive use of the pulsed load. Due to the limited voltage regulation capabilities of the generators, rapid variations in current draw can easily lead to voltage sags, which violate operational specifications, and can lead to equipment damage or loads shutting down. This problem is exacerbated by neighboring high-bandwidth power electronic loads which are common in such microgrids and act as constant power loads. As voltage drops, these loads consume more current, leading to further voltage sag and possible instability [2].

Power management algorithms in ship and aircraft microgrids are typically centralized with full control of the system configuration and scheduling of loads. As such, it is possible to coordinate the sources and loads to charge an energy storage device. We use the term *charging profile* to denote a system trajectory that, through a coordination of the sources and loads, charges an energy storage device. An optimal charging profile is one that achieves a given energy demand E^* in minimum time while keeping the system state within operational specifications.

The problem of determining an optimal charging profile was introduced in [3]. There the authors derived a closed-form expression for the charging profile that minimizes the power ramp rate while ensuring a given energy transfer. The profile is seen to be a paraboloid. While straight forward, the solution makes no account of system dynamics or constraints placed on power, voltage, or current. In [4] the authors apply linear model predictive control (MPC) techniques to coordinate power supplied to ramped loads in a shipboard application. Energy and power constraints are addressed but circuit dynamics (voltage, current) are neglected. Results are shown using ideal sources (negligible impedance) to prevent the possibility of voltage sags. In [5], the authors develop a feedback-linearization control algorithm for smoothly charging an energy storage device while minimizing frequency deviations on an AC microgrid. The algorithm is based on a heuristic trapezoidal power profile consisting of five stages. The algorithm neglects the dynamics of neighboring loads and does not ensure constraints on system voltage and current are respected.

In this work, we develop an optimization-based approach for determining the maximum amount of energy that can be transferred to an energy storage device over a finite time duration. We account for the dynamics of both the generator and connected constant power loads, ensuring voltage and current constraints are satisfied. In its original form, the problem is a quadratically-constrained quadratic program (QCQP) with an indefinite objective and indefinite constraints. Through change of variables and relaxations we develop a closely related second-order cone program. The problem retains the same feasible set as the original problem but utilizes a linear approximation of the non-convex objective. We demonstrate how this can be used to find the minimum-time charging profile for an energy storage device that must be periodically operated.

The rest of the paper is organized as follows. Section II introduces the power system model utilized. Section III poses the maximum energy transfer problem and develops an SOCP-based solution. Section IV leverages this result to find the minimum-time to charge an energy storage device subject to periodicity constraints. Section V concludes the paper and discusses future directions.

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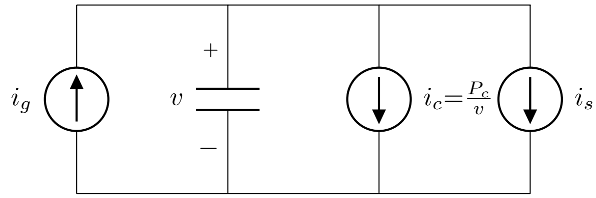

Fig. 1. Microgrid Model

II. MODEL DESCRIPTION

We consider a DC source supplying a constant power load and an energy storage device as shown in Figure 1. The source model is composed of a bus capacitor C and a current source i_g driven by a proportional-integral voltage regulator with error integral term e. The regulator tracks the commanded voltage $\bar{v}$. This serves as a simple model of a synchronous generator-rectifier.

The source supplies two loads which are current sinks driven by appropriate control laws. The first is an energy storage device with current draw i_s. The second represents the aggregate behavior of connected constant power loads with total power consumption P_c and current draw i_c. This is representative of power electronic loads which have current regulation bandwidths orders of magnitude faster than the voltage regulation capabilities of synchronous generators.

The continuous-time dynamics are given by:

$$\begin{aligned} C\frac{dv}{dt} &= (k_p(\bar{v}-v)+k_i e) - i_s - i_c\,, \\ \frac{de}{dt} &= \bar{v} - v\,. \end{aligned} \tag{1}$$

The generator and constant power load currents are:

$$i_g = (k_p(\bar{v}-v)+k_i e), \qquad i_c = \frac{P_c}{v}\,.$$

Remark. We use this model to simplify the presentation. However, the results developed herein are easily modified to accomodate more complicated linear source models and the presence of linear loads.

III. PROBLEM STATEMENT

Our objective is to maximize the energy transferred to the energy storage device over a finite time horizon. It is assumed that both the generator's commanded voltage and the energy bank's current are control signals available to us. This is realistic for microgrids on ships and aircraft which typically have centralized control. However, either of the control signals can be eliminated by appending equality constraints in the optimization problem (e.g. $\bar{v} = c$ where c is some constant).

We assume the dynamics are appropriately discretized with time step Δt, yielding the model

$$x_{k+1} = Ax_k + B_v\bar{v}_k + B_i(i_{s,k}+i_{c,k}) \tag{2}$$

$$v_k i_{c,k} = P_c \tag{3}$$

where $x_k = \begin{bmatrix} v_k & e_k \end{bmatrix}^T$. Subscript k denotes the time index. Note that we leave the constant power load current as an input with an associated bilinear constraint.

Let T_f the denote the time horizon and $N = \frac{T_f}{\Delta t}$ be the number of steps. Using Euler integration to calculate energy from power, the maximum energy transfer problem is then written as:

$$\begin{aligned} \min \quad & -\Delta t \sum_{k=0}^{N-1} v_k i_{s,k} \\ \text{s.t.} \quad & x_{k+1} = Ax_k + B_v\bar{v}_k + B_i(i_{s,k}+i_{c,k}), \\ & v_k i_{c,k} = P_c, \\ & F\begin{bmatrix} x \\ i_s + i_c \\ \bar{v} \end{bmatrix} \le g \end{aligned} \tag{4}$$

All constraints with terms containing subscript k apply to all time indices $k = 0, \ldots, N$. Here the last inequality is used to capture any linear constraints on the states and control inputs. To support our change of variables to come, we assume current constraints apply to the total load current. It is readily seen that both the objective and the bilinear power constraint are indefinite. The resulting optimization problem is a non-convex QCQP that is NP-hard in general [6].

Remark. Maximizing power transfer over a finite horizon falls within the category of economic model predictive control [7]. Economic MPC seeks to maximize general performance indices in place of traditional tracking problems. Indefinite objectives frequently arise involving the product of input controls and output signals [8]. In some instances, mostly arising from dissipativity properties, the resulting problem is convex once projected onto the dynamic constraints [9]. For the problem at hand, if the control is limited to i_s and no constant power load is present the system dynamics are dissipative with respect to input i_s and output v (i.e. $i_{s,k}v_k \ge 0 \;\forall\; i_{s,k} \in \mathbb{R}$). By projecting the quadratic program onto the linear dynamic constraints (often referred to as condensed MPC), the indefinite quadratic cost function becomes convex. However, once $\bar{v}$ is introduced as a second input, the condensed MPC formulation is no longer convex. Further, it cannot address the nonlinear constant power load dynamics.

In the following development, we assume that the bus voltage v is constrained (via F and g) to be strictly positive (such that i_c is well-defined). Further we assume the energy storage device and constant power load are only allowed non-negative power consumption. This represents the common case of not allowing reverse power flow into the source. Given the stated positivity assumption on v, this translates to the constraints $i_c \ge 0, i_s \ge 0$.

We now introduce a change of variables to eliminate the bilinear power constraint. Let $i = i_s + i_c$. Given the non-negative power consumption constraint on i_s, the total power consumed will be at least P_c. The resulting problem is rewritten as:

$$
\begin{aligned}
\min \quad & -\Delta t \sum_{k=0}^{N-1} (v_k i_k - P_c) \\
\text{s.t.} \quad & x_{k+1} = A x_k + B_v \bar{v}_k + B_i i_k, \\
& v_k i_{c,k} \geq P_c, \\
& F \begin{bmatrix} x \\ i \\ \bar{v} \end{bmatrix} \leq g
\end{aligned} \tag{5}
$$

Define $z = \sqrt{vi}$, representing the square root of the total load power. Given our stated assumption, vi is always non-negative and therefore z is real. For reasons that will become apparent, we write this equality constraint as two inequalities.

$$
\begin{aligned}
\min \quad & -\Delta t \sum_{k=0}^{N-1} (z_k^2 - P_c) \\
\text{s.t.} \quad & x_{k+1} = A x_k + B_v \bar{v}_k + B_i i_k, \\
& z_k \geq \sqrt{P_c}, \\
& v_k i_k \leq z_k^2 \leq v_k i_k, \\
& F \begin{bmatrix} x \\ i \\ \bar{v} \end{bmatrix} \leq g
\end{aligned} \tag{6}
$$

Consider now the following relaxed problem in which we drop the lower bound on z_k^2.

$$
\begin{aligned}
\min \quad & -\Delta t \sum_{k=0}^{N-1} (z_k^2 - P_c) \\
\text{s.t.} \quad & x_{k+1} = A x_k + B_v \bar{v}_k + B_i i_k, \\
& z_k \geq \sqrt{P_c}, \\
& z_k^2 \leq v_k i_k, \\
& F \begin{bmatrix} x \\ i \\ \bar{v} \end{bmatrix} \leq g
\end{aligned} \tag{7}
$$

Lemma 1. *Solutions of Problem (7) satisfy $z_k^2 = v_k i_k$ for all $k = 0, \ldots, N$.*

Proof. We show this by contradiction. Let $S^* = \{v^*, e^*, i^*, \bar{v}^*, z^*\}$ be an optimal solution of Problem (7) with cost J^*. Assume that for some k, the inequality is strict ($z_k^{*2} < v_k^* i_k^*$). Let $S = \{v^*, e^*, i^*, \bar{v}^*, \tilde{z}\}$ with $\tilde{z}_k^2 = v_k^* i_k^*$. Note that $\tilde{z}_k > z_k^* \geq 0$. Thus S is a feasible solution as the inequality $P_c \leq \tilde{z}^2$ holds and the remaining inequalities do not involve $\tilde{z}$. The resulting cost is $J = J^* + \Delta t (z_k^2 - \tilde{z}_k^2)$ and therefore $J < J^*$. This contradicts S^* being an optimal solution. Therefore solutions of Problem (7) have $z_k^2 = v_k i_k$ for all k. □

Lemma 1 allows us to drop the constraint $z_k^2 \geq v_k$ without introducing spurious solutions. However, the remaining quadratic inequality is indefinite. Given our stated assumptions on positive power flow, we show that this inequality constraint can be represented by a convex second-order cone constraint.

Lemma 2. *When $v_k + i_k \geq 0$, the power constraint $z_k^2 \leq v_k i_k$ can be rewritten as the following second-order cone constraint:*

$$
\left\| \begin{matrix} 0.5(-v_k + i_k) \\ z_k \end{matrix} \right\|_2 \leq \frac{1}{2}(v_k + i_k) \tag{8}
$$

Proof. Let $w = \begin{bmatrix} z_k & v_k & i_k \end{bmatrix}^T$. We first rewrite the constraint in homogeneous form:

$$
w^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 \end{bmatrix} w \leq 0 \tag{9}
$$

The matrix has eigenvalues $\{-\frac{1}{2}, \frac{1}{2}, 1\}$ with associated unit eigenvectors $q_1 = \frac{1}{\sqrt{2}}[0\ -1\ -1]^T$, $q_2 = \frac{1}{\sqrt{2}}[0\ -1\ 1]^T$, $q_3 = [1\ 0\ 0]^T$.

Then, the inequality (9) is equivalently written as:

$$
\frac{1}{2}(q_2^T w)^2 + 1(q_3^T w)^2 \leq \frac{1}{2}(q_1^T w)^2. \tag{10}
$$

Note that $q_1^T w = -\frac{1}{\sqrt{2}}(v_k + i_k)$ is non-positive for $v_k + i_k \geq 0$. Therefore, by accounting for the sign of $q_1^T w$, we can take the square root on both sides of (10) to obtain [10]:

$$
\sqrt{\frac{1}{2}(q_2^T w)^2 + 1(q_3^T w)^2} \leq -\sqrt{\frac{1}{2}} q_1^T w\,. \tag{11}
$$

Finally, substituting the eigenvectors we obtain the stated second-order cone constraint (8). □

Taken together, lemmas 1 and 2 provide convex representations of the feasible set for Problem (4). The only remaining source of non-convexity is the concave objective $J(z) = -\Delta t \sum_{k=0}^{N-1} (z_k^2 - P_c)$. We now develop a surrogate linear objective which approximates the concave objective. For clarity, we drop P_c from the objective as it is constant.

Let z^* be the optimal solution to (7). Consider a Taylor expansion of the cost function with $\tilde{z} = z - z^*$:

$$
J(\tilde{z}) = -\Delta t \sum_{k=0}^{N-1} z_k^{*2} + 2 z_k^* \tilde{z}_k + h.o.t.
$$

Assume there is a maximum power P_{max} that can be transferred in the microgrid.[1] If we maximized energy transfer over an infinite horizon, the system would be operated at P_{max} for almost all time indices ($z_k^{*2} = P_{max}$). Neglecting higher order terms, the linear objective function would be:

$$
J(\tilde{z}) = -\Delta t \sum_{k=0}^{\infty} P_{max} + 2\sqrt{P_{max}} \tilde{z}_k
$$

Minimizing this objective simply involves maximizing the sum of $\tilde{z}$ as all terms are weighted equally. This motivates

[1] This may be directly imposed or be an indirect consequence of constraints placed on voltage and current.

us to replace our original objective with this linear objective. Our final problem formulation is a second-order cone program and therefore convex:

$$
\begin{aligned}
\min \quad & -\sum_{k=0}^{N-1} z_k \\
\text{s.t.} \quad & x_{k+1} = A x_k + B_v \bar{v}_k + B_i i_k, \\
& z_k \geq \sqrt{P_c}, \\
& \left\| \begin{matrix} 0.5(-v_k + i_k) \\ z_k \end{matrix} \right\|_2 \leq 0.5(v_k + i_k), \\
& F \begin{bmatrix} x \\ i \\ \bar{v} \end{bmatrix} \leq g
\end{aligned}
\tag{12}
$$

Theorem 1. *Under the constraints $v > 0, i_s \geq 0, P_c \geq 0$, Problem (4) is feasible if and only if Problem (12) is feasible. Further, solutions to Problem (12) provide sub-optimal solutions to Problem (4).*

Proof. The feasibility proof follows immediately from Lemmas 1 and 2 which showed that, despite enlarging the feasible set, optimal solutions still satisfied the constraints of Problem (4). Sub-optimality follows from the use of a surrogate objective in place of the original objective. □

IV. Minimum-Time Charging of Periodic Loads

Problem (12) provides a convex method for obtaining sub-optimal solutions to the original non-convex problem (4) when the bus voltage and load current are constrained to be strictly positive and non-negative respectively. The linear inequality constraint pair (F, g) affords us flexibility to impose additional constraints on the problem. In the following we demonstrate how this can be used to design charging profiles for loads that are repetitively exercised.

Consider a periodic pulsed load that requires a minimum stored energy E^* prior to use. Our objective is to find the minimum time required to charge this load with the constraint that the system starts and ends at the same operating point ($v_0 = v_N, i_0 = i_N$). This periodic constraint allows the resulting charge profile to be repeatedly executed. In addition to the energy storage device, a constant power load ($P_c = 300kW$) is connected. We impose minimum and maximum constraints on the voltages and currents along with constraints on their rate of change. These boundaries can be used to indirectly impose limits on the rate at which power is varied. Finally we impose a voltage-dependent upper limit on the maximum total current. This approximates a maximum power limit of $500kW$. The dynamics are given by (1) discretized with a time step $\Delta t = 0.05s$ using a zero-order hold. Table I lists the parameters. The controller tuning achieves a voltage regulation bandwidth of 8 Hz, representative of a synchronous generator-rectifier in a DC microgrid.

$$
\begin{aligned}
\min \quad & -\sum_{k=0}^{N-1} z_k \\
\text{s.t.} \quad & x_{k+1} = A x_k + B_v \bar{v}_k + B_i i_k, \\
& z_k \geq \sqrt{P_c}, \\
& \left\| \begin{matrix} 0.5(-v_k + i_k) \\ z_k \end{matrix} \right\|_2 \leq 0.5(v_k + i_k), \\
& v_0 = v_N = v_{nom}, \quad \bar{v}_0 = \bar{v}_N = v_{nom}, \\
& e_0 = \frac{P_c}{v_0 k_i}, \quad i_0 = i_N = \frac{P_c}{v_0}, \\
& v_{min} \leq v_k \leq v_{max}, \quad v_{min} \leq \bar{v}_k \leq v_{max}, \\
& i_{min} \leq i_k \leq i_{max}, \quad i_k \leq s_1(v_k - s_2) + i_{max}, \\
& -\Delta v_{max} \leq (\Delta t)^{-1}(v_{k+1} - v_k) \leq \Delta v_{max}, \\
& -\Delta v_{max} \leq (\Delta t)^{-1}(\bar{v}_{k+1} - \bar{v}_k) \leq \Delta v_{max}, \\
& -\Delta i_{max} \leq (\Delta t)^{-1}(i_{k+1} - i_k) \leq \Delta i_{max}
\end{aligned}
\tag{13}
$$

Remark. In normal power system operation we would likely only constrain the voltage command (and not the voltage) as the two would be nearly equivalent. Here we are aggressively varying both the voltage command and current draw. Given the generator's limited regulation bandwidth, the voltage command and actual command are clearly not the same as seen in Figure 5. To ensure we respect operational constraints we limit both.

Remark. We do not impose periodicity constraints on the voltage error integral term e as this is a virtual (vice physical) state which can be reset by the control algorithm. The initial constraint on e represents an equilibrium condition. Periodicity is imposed on the current i as it is a physical signal which cannot be discontinuous.

As the maximum energy transfer monotonically increases with the time duration $T = N\Delta t$, determining the minimum time to achieve a given energy demand E^* can be solved via bisection. To show solution trends, we instead exhaustively vary the time duration from 1s to 50s in 1 second increments and solve Problem (13) for each case.

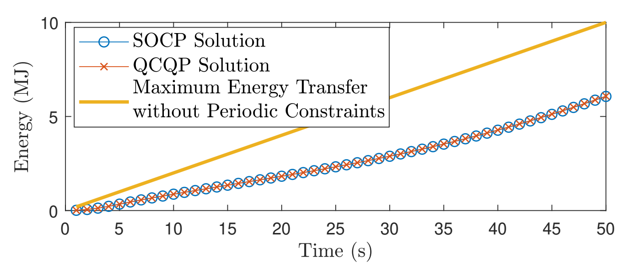

Fig. 2. Maximum Energy Transfer for Various Time Durations

Figure 2 shows the energy transferred to the storage device for each solution. Note that we are plotting the original objective here ($\Delta t \sum_{k=0}^{N-1} v_k i_{s,k}$) and not the linear surrogate objective. For reference we also plot the maximum energy transfer possible if the system had no constraints on initial or

TABLE I
MICROGRID PARAMETERS

C	k_p	k_i	P_c	v_{nom}	v_{min}	v_{max}	Δv_{max}	i_{min}	i_{max}	Δi_{max}	s_1	s_2
0.01F	0.5	0.025	300kW	1000V	800V	1200V	80V/s	0A	525A	500A/s	-0.4375 A/V	952.4V

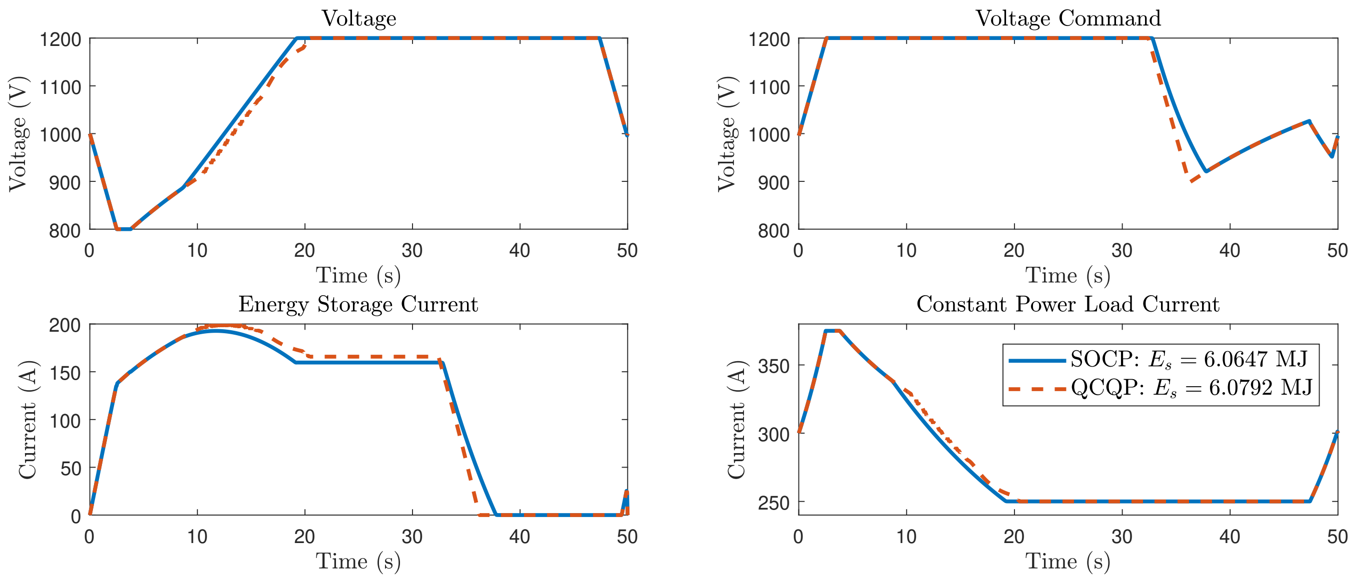

Fig. 3. QCQP and SOCP Solutions for 50s Charging Profile

final states. In this scenario the system would be operated at the maximum power of 500kW indefinitely. Of this, 200kW would be going to the energy storage device, which is the line we plot. For comparison, we also solve the non-convex QCQP using IPOPT 3.11.0 [11] with gradients and Hessians supplied by CasADi [12]. For horizon lengths up to 10 seconds we confirmed with YALMIP's global solver BMIBNB that IPOPT is indeed finding the global solution (to within 0.1%). Solutions to longer horizon lengths were not proven global due to excessive runtimes. With respect to the original objective, our surrogate objective gave results that were at most 0.26% sub-optimal relative to the local solution returned by IPOPT.

Figure 3 plots the solutions obtained for a 50 second charging duration. Despite the different objectives, the profiles are very similar with the QCQP formulation transferring slightly more net energy. Figure 4 plots the phase-portrait of voltage and current for the 50s charging profile obtained from the SOCP formulation. The trajectory begins at $(1000V, 300A)$ and moves clockwise. The current increases while the voltage drops, eventually reaching both the voltage and current lower limits. The current is then held constant while the voltage recovers. The current is then decreased until we hit the maximum voltage limit $1200V$. The current continues to decrease to $1200V, 250A$ at which point the energy storage device is no longer drawing current (only the constant power load is). We move along the minimum power bound $300kW$, and then experience a short additional power draw which returns us to our starting condition.

Figure 5 plots the time profiles of trajectories ranging

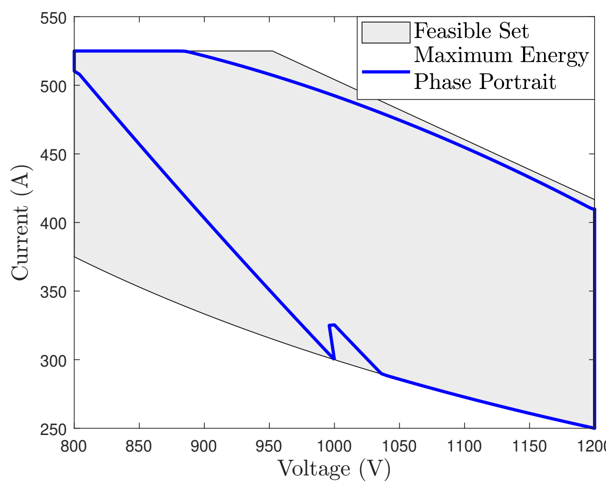

Fig. 4. Maximum Energy Transfer for 50s Charging Profile

from 10s to 50s. While trends are evident in the voltage and energy storage commands, they are not simple parabolic or trapezoidal shapes as proposed in [3] and [5] respectively. This supports the need for optimization-based approaches to the design of charging profiles, rather than relying on pre-specified stages.

Figure 6 plots the percentage error in the equality constraint $z_k^2 = v_k i_k$. This numerically validates Lemma 1, showing that the solutions satisfy $z_k^2 = v_k i_k$ to within solver tolerances.

Figure 7 plots the solver runtimes for the SOCP and QCQP

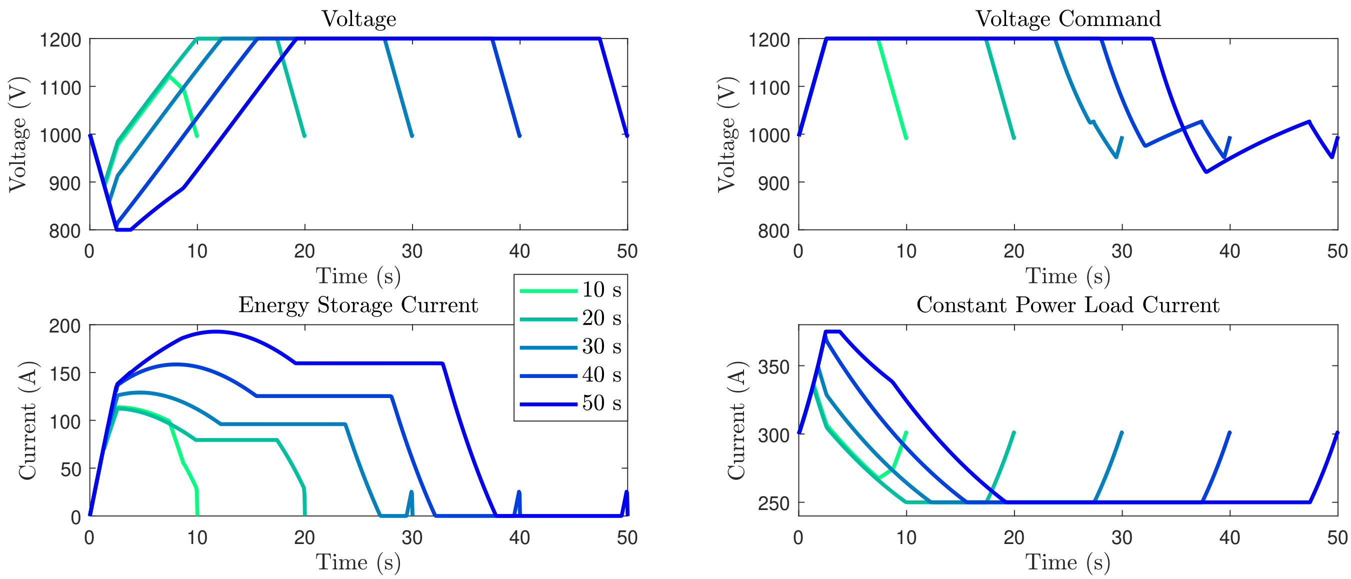

Fig. 5. SOCP Solution for Various Charging Durations

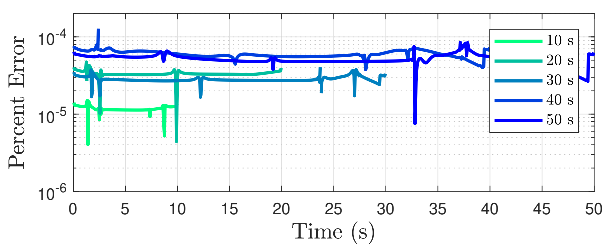

Fig. 6. Equality Constraint Percent Error $(100 \times \frac{z_k^2 - v_k i_k}{v_k i_k})$

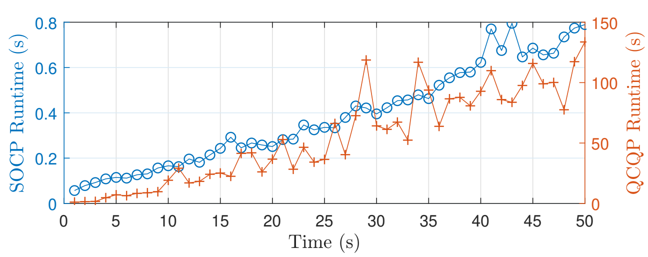

Fig. 7. Solver Runtime: SOCP (CPLEX) vs. QCQP (IPOPT)

formulations of Problem (13). Solutions were obtained on a laptop with an Intel Core i7-4800MQ CPU. On average, the SOCP formulation is solved 87x faster than the non-convex QCQP formulation.

V. Conclusions

This work considered the maximum energy transfer problem in a microgrid with constant power loads. In its original form, the problem is a non-convex QCQP. A convex approach based on second-order cone programming was developed and leveraged to design periodic charging profiles. Future work includes extending this result to AC microgrids and further characterizing the conditions under which the linear objective is a good surrogate for the concave objective.

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