Optimal Coordination of Distribution System Resources under Uncertainty for Joint Energy and Ancillary Service Market Participation

Chengda Ji, Pengcheng You, Elijah J. Pivo, Yue Shen, Dennice F. Gayme, and Enrique Mallada

Abstract—We propose a stochastic optimization framework that co-optimizes the joint market participation of distribution system energy resources in real-time energy and ancillary service markets under uncertainty in solar generation, demand and market prices. The objective is to minimize the utility’s net cost by (1) utilizing the aggregate flexible resources of its set of feeders to arbitrage between real-time and day-ahead prices and (2) obtaining revenue through the provision of ancillary services. We mitigate the effect of system uncertainty by solving the problem using a receding horizon approach and adopt chance constraints to account for uncertainty in solar availability and demand. Price uncertainty is incorporated using a scenario-based approach. Our framework is illustrated on a real-world circuit under various levels of solar penetration, forecast uncertainty and risk. Our results highlight the importance of forecast quality as solar penetration increases. We also illustrate the trade-offs between either enforcing a very low probability of power-supply shortage (lower risk tolerance) or allowing higher forecast uncertainty (increased variance in solar predictions), and reducing the overall system costs.

I. INTRODUCTION

The number of distributed energy resources (DERs), such as rooftop solar panels, electric vehicles (EVs), home batteries, and smart household appliances, entering distribution networks is rapidly growing. These “behind-the-meter” DERs have gained popularity with consumers due to financial incentives and the desire to contribute to reducing greenhouse gases. Consumers are also seeking local control of electricity generation to mitigate the effect of power grid outages. This rapid increase in the penetration of DERs has led to unprecedented challenges in distribution network operations. Networks that have historically been dominated by predictable demand patterns must now deal with the uncertainty associated with solar energy, electric vehicle usage, and the rapidly changing make-up of resources and consumption patterns.

Alongside these challenges, appropriately leveraged DERs present opportunities for utilities to reduce overall costs and improve system reliability (e.g. voltage regulation). For instance, increasing energy production, storage, and load control in distribution networks can enable utilities to sell electricity in transmission network energy markets and to participate in ancillary service markets. In fact, [1] show that it is more cost effective for utilities to deal with uncertainty using DERs in the form of controllable loads rather than install a large lithium-ion battery storage facility. However, the coordination of these distribution system resources is challenging for the following reasons. The uncertainty in the availability of DERs complicates planning. The large number of DERs spread over a distribution network creates problems of scale for distributed control approaches. Privacy concerns arise if a utility or aggregator has access and control of individual usage patterns. In addition, ancillary services require extra energy to be put aside so that it is immediately available when called upon, which is challenging in the dynamic and uncertain environment of a distribution system with a high DER penetration.

A number of works have addressed the potential for DERs to improve distribution network reliability and enable their market participation. For example, [2] and [3] propose metrics of aggregate flexibility for DERs to quantify their capability of supplementing distribution network operation. However, these metrics are not designed to evaluate the economic efficiency for utilities. Strategies for DERs to participate in real-time energy markets are investigated in [4] and [5], whereas [6]–[8] co-optimize DER participation in both energy and ancillary service markets in a deterministic setup.

In particular, our previous work [6], introduces a hierarchical structure to manage aggregate solar generation and a set of lumped DER resources that are operated as virtual storage. The approach reduces both computational complexity and privacy concerns associated with a utility controlling individual DERs. That work investigated the top level of the hierarchy, the Grid Market Layer (GML), which coordinates secondary feeders abstracted as single load, solar generation and virtual storage elements. The feeders are controlled by the Feeder Operational Layer (FOL), which is described in [9] and [10]. While this work and that of [7], [8] represent important steps towards understanding the joint participation of distribution systems in transmission markets, the deterministic setting fails to fully characterize the inherent uncertainty in DERs.

This paper takes a further step toward addressing these questions using a stochastic optimization framework, that coordinates secondary feeders connected over a radial network, for joint participation in the energy and reserve markets. This framework enables hierarchical control of aggregate DERs with tractable computational overhead and non-intrusive management within a system that includes uncertainty in solar availability, demand and prices. In particular, chance
constraints are adopted to model the uncertainty in forecasts of demand and solar generation (see e.g., [11]). A scenario-based technique is applied to incorporate the uncertainty in price predictions. We further mitigate the effects of uncertainty by solving the problem in a receding horizon manner (see e.g., [9]).

Our approach is illustrated on a real-world circuit using trace-driven simulations across a range of solar penetration levels. We first test our system with deterministic formulations as a benchmark. We then illustrate the effects of forecast quality and risk tolerance on economic benefits and solar utilization percentage. In particular, the economic benefits is evaluated in terms of the utility’s net cost, i.e., the cost associated with energy procurement and generation minus the revenue from ancillary markets and the relative cost increase from the benchmark test. The solar utilization is defined as the percentage of solar used versus the total available. Our results demonstrate that the large risk tolerance and poor forecast quality leads to higher costs and lower solar utilization.

The remainder of this paper is organized as follows. Section II introduces the problem setup, which is followed by a description of the stochastic optimization framework in Section III. Next, numerical tests are run on a sub-network of a New Jersey (NJ) utility using traces of solar generation, demand and prices drawn from historical data in Section IV. This section also includes a sensitivity analysis. Finally, section V concludes the paper.

II. PROBLEM SETUP

In this section, we first describe the distribution network operational constraints. We then present the model for its participation in ancillary service markets using virtual storage.

A. Operational Constraints

We now provide the operational constraints for the deterministic version of the GML. We use $P_{f}^{P}(t)$ and $Q_{f}^{P}(t)$ to denote the real and reactive power generation on the feeder $f$ at time $t$, respectively. Each feeder can control its aggregate generation output within a range, i.e.,

$$P_{f}^{\min}(t) \leq P_{f}^{P}(t) \leq P_{f}^{\max}(t),$$

$$Q_{f}^{\min}(t) \leq Q_{f}^{P}(t) \leq Q_{f}^{\max}(t).$$

The virtual storage is modeled through its respective charge/discharge dynamics, its capacity level, and power constraints, which are given by

$$B_{f}(t+1) = B_{f}(t) - \delta_{t} R_{f}(t) + W_{f}(t),$$

$$B_{f}^{\min}(t) \leq B_{f}(t) \leq B_{f}^{\max}(t),$$

$$R_{f}^{\min}(t) \leq R_{f}(t) \leq R_{f}^{\max}(t).$$

Here $B_{f}(t)$ denotes the storage state of charge and $R_{f}(t)$ denotes the discharge rate of feeder $f$ at time $t$. Note that negative $R_{f}(t)$ means charging. $W_{f}(t)$ represents exogenous change in the storage state of charge due to, for instance, a storage resource such as an EV or set of EVs disconnecting from a feeder. $\delta_{t}$ is the time interval of each slot.

The secondary feeders are connected to the transmission network through transformer banks. We use $N$ to denote the set of banks in the distribution network with indices $i = 1, 2, \cdots, n$, and $T_{i}$ to denote the set of secondary feeders connected to bank $i$. The line impedance between banks $i$ and $j$ is $r_{i,j} + jx_{i,j}$. The real and reactive power $P_{i}(t), Q_{i}(t)$ on transformer bank $i$ at time $t$ is given by

$$P_{i}(t) = \sum_{f \in T_{i}} \left( P_{f}^{P}(t) - P_{f}^{P}(t) - R_{f}(t) \right),$$

$$Q_{i}(t) = \sum_{f \in T_{i}} (Q_{f}(t)),$$

$$S_{i}(t) \geq P_{i}(t)^{2} + Q_{i}(t)^{2},$$

where $P_{f}^{P}(t)$ is the demand of feeder $f$ at time $t$ and $S_{i}(t)$ is the capacity of bank $i$ at time $t$. The power flow between bank $i$ and $k$ is modeled by the DistFlow equations [12]:

$$P_{i,k}(t) = r_{i,k}l_{i,k}(t) + P_{k}(t) + \sum_{m : k \rightarrow m} P_{km}(t),$$

$$Q_{i,k}(t) = x_{i,k}l_{i,k}(t) + Q_{k}(t) + \sum_{m : k \rightarrow m} Q_{km}(t),$$

$$v_{k}(t) - v_{i}(t) = \left( \frac{P_{i,k}(t) + Q_{i,k}(t)}{V_{i}(t)} \right),$$

$$l_{i,k}(t) = P_{i,k}(t)^{2} + Q_{i,k}(t)^{2},$$

where $v_{i}(t) = |V_{i}(t)|^{2}$ is the squared voltage magnitude of bank $i$ at time $t$, and $l_{i,k}(t) = |I_{i,k}(t)|^{2}$ is the squared magnitude of the current flow from bank $i$ to bank $k$. $P_{i,k}(t)$ and $Q_{i,k}(t)$ are the real and reactive branch flows, respectively.

B. Participation in Ancillary Service Markets

We consider contingency reserve ancillary service markets [13] in which purveyors guarantee the availability energy for a fixed time period at a given interval of time after the market is cleared (e.g., a 10 minute reserve market ensures that energy is available in 10 minutes and this energy can be used over an hour interval). The market is characterized by the response time $\tau$, which specifies the time slots the energy must be available after a request, and a commitment time $\kappa$, which specifies the amount of real-time slots that the reserve should last. This behavior is modeled as follows.

$$P_{\text{rsrv}}^{c}(t) = \sum_{h = \max\{(t-\tau)-(k-1)+1,1\}}^{(t-\tau)} P_{\text{rsrv}}(h),$$

$$P_{\text{rsrv}}^{\min}(t) \leq P_{\text{rsrv}}(t) \leq P_{\text{rsrv}}^{\max}(t),$$

$$P_{\text{rsrv}}(t) = \delta_{t} \sum_{l = \max\{(t-\tau)-(k-1)+1,1\}}^{(t-\tau)} (l - (t-\tau) + k)P_{\text{rsrv}}(l),$$

where $P_{\text{rsrv}}^{c}(t)$ is the cumulative reserve power, and $B_{\text{rsrv}}(t)$ is the minimum amount of storage energy required. The
GML participates in the ancillary service market by setting aside virtual storage energy.

\[ B_{\text{rsrv}}(t) \leq \sum_{f} B_{f}(t). \]  

III. UNCERTAINTY MODEL AND STOCHASTIC OPTIMIZATION FRAMEWORK

In this section, we extend the model presented in Section II to account for uncertainty in solar generation, demand, and prices.

A. Uncertainty in Solar Generation and Demand

A chance constraint approach is adopted to handle the uncertainty in predictions of available solar generation and demand. That is, we constrain the probability that there is insufficient generation,

\[ \text{Prob}(P_f(t) \leq P_f^{\text{max}}(t)) \geq 1 - \epsilon, \]

where \( \epsilon \in (0, 1) \). \( 1 - \epsilon \) is referred to as the confidence level and \( \epsilon \) is a measure of risk tolerance.

We assume that the solar generation and demand predictions are normally distributed with mean \( \mu_f(t) \) and variance \( \Sigma_f(t) \) and \( \Sigma_d(t) \), respectively. The constraint (10) implies the solar and demand are linearly combined. Therefore, the uncertainty in solar and demand predictions can be accommodated together:

\[ P_f^{\text{max}} = \max \{ \Phi^{-1}(\epsilon)(\Sigma_f(t) + \Sigma_d(t))^{1/2} + \mu_f(t), P_f^{\text{min}}(t) \} \]  

where \( \Phi^{-1}(\cdot) \) is the inverse cumulative distribution function. We can then transform the constraints (11) and (6) into

\[ P_f^{\text{min}}(t) \leq P_f(t) \leq P_f^{\text{max}}(t) \]  

\[ P_f(t) = \sum_{f \in T_i} \left( \mu_f(t) - P_f(t) - R_f(t) \right). \]

B. Uncertainty in Prices

We propose a scenario-based approach to accommodate the uncertainty in price predictions. We consider a finite set of scenarios \( S \) extracted from historical data. For the price predictions that start from \( t_1 \) to \( t_f \), each scenario \( s \in S \) is given by

\[ s := \left\{ (\pi_s, \lambda^s_{1}(t), \alpha_s(t)) \mid t \in \{t_1, t_2, \cdots, t_f\} \right\}, \]

where \( \pi_s \) is the corresponding probability with \( \sum_{s \in S} \pi_s = 1 \), and \( \lambda^s_{1}(t) \) and \( \alpha_s(t) \) are the predictions of real-time energy market and ancillary service prices, respectively.

C. Receding Horizon Optimization Framework

A receding horizon optimization framework is implemented to calculate the optimal control trajectory. We use \( F \) to denote the set of feeders in the distribution network, and \( L \) to denote the set of lines that connect transformer banks.

We set a moving prediction window from \( t_i \) to \( t_i + T - 1 \) with all the scenarios in \( S \) accounted for. At each time step, we obtain the optimal control trajectory. We use \( U_s(t) \) to denote virtual storage energy.

We set a moving prediction window from \( t_i \) to \( t_i + T - 1 \), the decision variables in scenario \( s \in S \) are

\[ U_s(t) := \begin{cases} (P_f(t), Q^w_f(t), B_{f,s}(t), R_{f,s}(t)), & \forall f \in F, \forall i \in N, \forall (i, k) \in L, \end{cases} \]

and the optimization problem to be solved is

\[ \min \sum_{s \in S} \left[ \sum_{t=1}^{T} \delta \left( \lambda^s_{1}(t)(P_0(t) - P_d(t)) \right) \right] \]  

\[ + \lambda^s_{1}(t)(P_{d}(t) - \alpha_s(t)P_{rsrv,s}(t)) \sum_{f} f_{f,s}(P_f(t)) \]

s.t. \( \{ |f| \} \leq 5 \), \( \forall s \in S \)  

Network constraints: \( \{ 15 \} - \{ 12 \} \), \( \forall s \in S \)  

Ancillary service constraints: \( \{ 13 \} - \{ 16 \} \), \( \forall s \in S \)  

\[ P_0,s(t) = \sum_{i \in 0 \rightarrow i} P_0,i(t), \forall s \in S, \]  

\[ U_s(t_i) = U(t_i), \forall s \in S. \]

where bank 0 is connected to the transmission grid and \( P_0,s(t) \) is the net demand traded in the real-time energy market at time \( t \) in scenario \( s \). The constraint (22) enforces that the first-slot decision variables are scenario-invariant and given by \( U(t_i) \). The term \( f_{f,s}(P_f(t)) \) in (20) denotes the cost function associated with feeder \( f \) at time \( t \) and is given by

\[ f_{f,s}(P_f(t)) = \beta_f(P_f(t) - \bar{P}_f^{\text{max}}), \]

where \( \bar{P}_f^{\text{max}} - P_f(t) \) measures solar curtailment and the constant \( \beta_f < 0 \) represents its unit cost. We solve for the full trajectory \( U_s(t), t \in [t_i, t_i + T] \) and implement the control action at each time step \( t_i \). We then shift the prediction window to \( [t_i + 1, \cdots, t_i + T] \) and repeat the process. This iterative solution of the optimization problem mitigates uncertainty through the update in system state information at each time step.

IV. NUMERICAL STUDY

We now provide a set of numerical simulations of a distribution network consisting of three transformer banks, each with 4 feeders that is abstracted from an existing sub-network of a utility circuit in NJ. A schematic of the corresponding twelve-feeder circuit is shown in Figure 1. The GML co-optimizes participation in the real-time and 10 minute ancillary service market. The corresponding ancillary service response time is 10 minutes (\( \tau = 2 \)) and the commitment time is 1 hour (\( k = 12 \)).

We set the lower bounds for feeder generation, virtual storage capacity and reserve power all to be zero. For each virtual storage, \( B_{f}^{\text{max}}(t) = 0.25 \) MWWhr, and \( R_{f}^{\text{max}}(t) = -R_{f}^{\text{min}}(t) = 0.0833 \) MW. The resulting aggregate storage size is 3 MWWhr, which is equivalent to roughly 3000 electric water heaters [1]. It takes approximately 3 hours to fully discharge the aggregate virtual storage. The reactive power generation bounds are set to be \( Q_{f}^{\text{min}}(t) = -0.05P_{f}^{\text{max}}(t) \) at each time step.

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and \(Q_{\text{max}}(t) = 0.05P_{\text{max}}(t)\). The cost penalty of solar curtailment is set to be \(-15\$/MWhr. The time interval \(\delta_t\) is 5 minutes, which coincides with the real-time energy and ancillary service markets of the New York Independent System Operator (NYISO) [14].

The solar penetration is defined as the percentage of aggregate solar availability with respect to aggregate energy demand over a day. We consider solar penetrations of 25%, 50%, 75% to 100%.

The solar penetration level

\[ \text{ECONOMIC BENEFITS} \]

<table>
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<tr>
<th>Solar penetration level</th>
<th>Deterministic setup</th>
<th>Solar 25%</th>
<th>Solar 50%</th>
<th>Solar 75%</th>
<th>Solar 100%</th>
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</thead>
<tbody>
<tr>
<td>Chance constraint confidence</td>
<td>90%</td>
<td>23552.91 (3.61%)</td>
<td>16743.30 (6.99%)</td>
<td>9020.04 (17.7%)</td>
<td>1295.45 (971.65%)</td>
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<tr>
<td>Initial solar variance</td>
<td>0.01</td>
<td>24740.36 (4.41%)</td>
<td>17034.38 (9.65%)</td>
<td>9363.73 (22.19%)</td>
<td>1716.10 (1254.69%)</td>
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<tr>
<td>Solar utilization ratio</td>
<td>0.02</td>
<td>24834.93 (4.80%)</td>
<td>17294.92 (10.31%)</td>
<td>9798.35 (27.86%)</td>
<td>2318.71 (6.99%)</td>
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<tr>
<td>Solar utilization ratio</td>
<td>0.03</td>
<td>24972.55 (5.16%)</td>
<td>17511.76 (11.69%)</td>
<td>10149.05 (32.44%)</td>
<td>2801.70 (1985.14%)</td>
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</table>

\[ \text{SOLAR UTILIZATION RATIO} \]

<table>
<thead>
<tr>
<th>Solar penetration level</th>
<th>Deterministic setup</th>
<th>Solar 25%</th>
<th>Solar 50%</th>
<th>Solar 75%</th>
<th>Solar 100%</th>
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<tbody>
<tr>
<td>Chance constraint confidence</td>
<td>90%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>99.95%</td>
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<tr>
<td>Initial solar variance</td>
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<td>79.09%</td>
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<td>73.31%</td>
<td>74.68%</td>
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<tr>
<td>Solar utilization ratio</td>
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<td>56.44%</td>
<td>66.02%</td>
<td>68.64%</td>
<td>69.84%</td>
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</table>

Fig. 1. Schematic of the test system. Bank 0 is connected to the transmission systems. Each transformer bank has an apparent power capability of 35 MVA and base voltage at 69 kV. Feeders are depicted in circles.

A. Data Trace Overview

We generate predictions of the prices, solar generation and demand over a 24 hour time horizon. We adopt a two-stage prediction mechanism. The prediction horizon for the first stage is one hour. The price predictions for this stage are generated using a standard k-means clustering algorithm applied to the NYISO real-time (5-minute) data comprised of approximately 5,000 daily price trajectories.

We assume that the demand and solar generation prediction error is normally distributed [15] with linearly increasing variance over time, based on observations from historical data. The function describing the evolution of variance of the demand is

\[ \Sigma_d(t) = 0.01 + (t - t_i) \times 4.35E^{-4}, \]

where \(t_i\) is the first slot of the prediction horizon. At \(t_i\), the per unit prediction variance is 0.01 p.u., and one hour later at \(t = t_i + 1\) the variance is 0.015 p.u. The function describing the evolution of the solar generation variance is

\[ \Sigma_g(t) = \sigma_g(t) \times 1.25E^{-3}, \]

where \(\sigma_g\) is the initial variance at the first slot of the prediction horizon. Large initial variance represents poor forecast quality. These models indicate that the time derivatives of standard deviations, \(\Sigma_d(t)\) and \(\Sigma_g(t)\), approach zero given an infinite prediction horizon.

The second stage provides predictions for the remaining 23 hours. These are obtained by interpolating hourly day-ahead historical data into 5-minute time slots. In this stage, the variance of the demand and solar predictions are assumed to be fixed. The variance of demand is set to be 0.08 p.u., while the variance of solar generation is set to double the value at the end of the first stage, i.e., \(2 \times \Sigma_g(t_i + 11)\).

B. Numerical Results

We now demonstrate the performance of our stochastic optimization framework in terms of economic benefits and solar utilization. The economic benefits are measured by the utility’s net cost, and the solar utilization is the percent of solar availability that is used. We first consider a deterministic setup as a benchmark, in which we assume that full information is available as described in section III. The corresponding deterministic optimization problem is

\[ \min_{t_i} \sum_{t = t_i}^{t_i + T - 1} \delta_t \left( \lambda^{\text{da}}(t) P^{\text{da}}(t) + \lambda^{\text{d}}(t) (P_0(t) - P^{\text{da}}(t)) \right) \]
\[
\begin{align*}
&\sum_{t=t_0}^{t_f+T-1} \delta_t \left( \sum_f f_{f,t} P_r^f(t) - \alpha(t) P_{\text{srv}}(t) \right) \\
\text{s.t.} & \quad (1) - (16), \text{ and } P_0(t) = \sum_{i=0}^{n-1} P_i(t).
\end{align*}
\]

A stochastic setup is then studied to evaluate the effect of risk tolerance and forecast quality on the economic benefits and solar utilization. In order to compare the performance of different levels of risk tolerance and forecast quality, we set the nominal chance constraint confidence level to be 95% and initial solar prediction variance to be 0.01 p.u., respectively. We then vary the confidence level of the chance constraint to three values (90%, 95%, 99%). Next, we fix the chance constraint and vary solar variance over three initial prediction variances (0.01 p.u., 0.02 p.u., 0.03 p.u.).

In both setups, a second-order cone relaxation is implemented on the quadratic equality (12) based on the method proposed in [16] to facilitate efficient computation, i.e.,

\[l_{t,k}(t)v_i(t) \geq P_{t,k}(t)^2 + Q_{t,k}(t)^2.\]

The exactness of this relaxation is valid in all of our simulation runs.

Table II describes the utility’s net cost and relative cost increase, defined by

\[
\frac{\text{stochastic cost} - \text{deterministic cost}}{\text{deterministic cost}}
\]

for all of the cases. As expected, the results show that with higher solar penetration, the cost is reduced since more demand is satisfied by solar. Furthermore, a worse forecast (larger initial solar prediction variance) or a higher confidence level of chance constraints leads to a cost increase. It also indicates that the forecast quality and confidence level are more economically influential with high solar penetration.

Results for solar utilization are summarized in Table II. In the deterministic setup, the GML almost fully uses the solar so as to reduce the cost. In the stochastic setup, due to the imperfect prediction, solar is underutilized. Specifically, a less accurate forecast and a higher confidence level in the chance constraints (lower risk tolerance) both lead to more curtailment of solar generation. Another interesting observation is that under the same confidence level or forecast quality, increasing solar penetration decreases solar curtailment.

V. Conclusion

In this work, we propose an optimization framework to coordinate distribution system resources to reduce the utility’s net cost through arbitrage in energy markets and provision of reserve energy. The proposed framework can accommodate the uncertainty associated with the imperfect prediction in real-world scenarios. In particular, a chance constraint approach is adopted to deal with the uncertainty in solar generation and demand. Scenario-based optimization is then used to handle the uncertainty in prices, and a receding horizon prediction utilized to further mitigate the effects of uncertainty. This framework is illustrated on a real-world circuit. Simulation results indicate that the utility pays more when the confidence level goes up or the forecast quality becomes worse, and this sensitivity is more pronounced at higher penetration levels of solar. We also notice that when the solar penetration is high, the utility does not supply much energy to consumers, jeopardizing their traditional source of revenue.

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References