A Unified Framework for Frequency Control and Congestion Management

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***Abstract*—The existing frequency control framework in power systems is challenged by lower inertia and more volatile power injections. We propose a new framework for frequency control and congestion management. We formulate an optimization problem that rebalances power, restores the nominal frequency, restores inter-area flows and maintains line flows below their limits in a way that minimizes the control cost. The cost can be squared deviations from the reference generations, minimizing the disruption from the last optimal dispatch. Our control thus maintains system security without interfering with the market operation. By deriving a primal-dual algorithm to solve this optimization, we design a completely decentralized primary frequency control without the need for explicit communication among the participating agents, and a distributed unified control which integrates primary and secondary frequency control and congestion management. Simulations show that the unified control not only achieves all the desired control goals in system equilibrium, but also improves the transient compared to traditional control schemes.**

***Index Terms*—Congestion management, distributed control, frequency control.**

I. Introduction

The current framework of frequency control and congestion management operates at three different timescales. The primary frequency control restores power balance in about 30 secs. The secondary control restores the nominal frequency in about 5–10 mins by bringing in additional generation. Secondary control of an interconnected system also restores inter-area flows through the use of Area Control Error (ACE). Finally the tertiary control consists of economic dispatch combined with congestion management. This is achieved by solving the Security Constrained Optimal Power Flow (SCOPF) problem at regular intervals ranging from 5 mins (PJM) to 30 mins (Great Britain) or one hour. SCOPF usually takes into account $N-1$ contingency condition. The penetration of distributed and renewable generation reduces system inertia, threatening the foundation of the current control especially as the systems become more volatile due to intermittent renewables and active demand. This motivates a rethinking of frequency control in a low-inertia world from the first principle.

In this paper we present a new approach which not only integrates primary and secondary frequency control but also performs congestion management at the frequency control timescale. This new framework is distributed, scalable, and can exploit active loads and grid-friendly appliances [1]. It is based on our earlier work on load-side frequency control [2]–[4]. We explain in this paper how the ideas there can be applied to generators; in the full version [5] we will combine both generator and load control.

We consider a network modeled by linearized swing dynamics at generator buses, power flow dynamics on the branches, and a measure of control effort to agents (generators and loads) when they participate in the control. We formulate the controller design as an optimization problem that rebalances power (primary frequency control), restores the nominal frequency and inter-area flows (secondary frequency control), and maintains line flows below their limits (congestion management) in a way that minimizes the control effort (disutility to the agents participating in control). We design our controllers so that the closed-loop system is a distributed primal-dual algorithm for solving the optimization problem and its Lagrangian dual. In other words we explicitly exploit the network dynamics to help carry out our primal-dual algorithm over the power network in real time. This allows a completely decentralized primary frequency control without the need for explicit communication among the agents, and a distributed unified control where communication is only required between neighboring agents. The asymptotic stability of the primal-dual algorithm suggests that the proposed control is stable over an arbitrary network. We demonstrate in simulations that, compared to the traditional schemes, the unified control not only implements congestion management at a faster timescale,

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but also has a better transient.

The rest of the paper is organized as follows. We explain in Section II our network model and main design methodology. We describe our primary frequency control (PFC) in Section III and a unified control (UC) in Section IV. We use a two-bus example in Section V to illustrate how PFC and UC achieve their design goals. We present an itemized derivation of our controllers in Section VI. We present our simulations of UC in Section VII. We conclude in Section VIII with implications of this work. Proofs are omitted due to space limitation but can be found in [5].

II. A NEW APPROACH TO FREQUENCY CONTROL

A. Notations

Let $\mathbb{R}$ be the set of real numbers and $\mathbb{N}$ the set of natural numbers. Given a finite set $S \subset \mathbb{N}$ we use $|S|$ to denote its cardinality. For a set of scalar numbers $\{a_i \in \mathbb{R} \mid i \in S\}$, we use a_S to denote the column vector of the a_i's; we usually drop the subscript S when S is clear from the context. For two vectors $a \in \mathbb{R}^{|S|}$ and $b \in \mathbb{R}^{|S'|}$, $(a, b) \in \mathbb{R}^{|S|+|S'|}$ is a column vector. Given any matrix A, we denote its transpose by A^T, and its i-th row by A_i. We use A_S to denote the submatrix of A composed only of the rows A_i for $i \in S$. The diagonal matrix of a sequence $\{a_i,\ i \in S\}$ is represented by $\text{diag}(a_S) = \text{diag}(a_i,\ i \in S)$, or a_S for short when its meaning is clear. Finally, we use $1(0)$ to denote the vector/matrix of all ones (zeros), whose dimension is understood from the context.

We consider a classical power network model [6] which is represented by a *directed* graph $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N} := \{1, \ldots, |\mathcal{N}|\}$ is the set of buses (or control areas, depending on the granularity of the model), and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is the set of lines. A line is denoted interchangeably by $e \in \mathcal{E}$, or $ij \in \mathcal{E}$ if it is directed from buses i to j. The set of the neighbors of bus i is defined as $N(i) := \{j \in \mathcal{N} | ij \in \mathcal{E} \text{ or } ji \in \mathcal{E}\}$ We partition the buses $\mathcal{N} = \mathcal{G} \cup \mathcal{L}$ where $\mathcal{G}$ and $\mathcal{L}$ are the set of generator (high inertia) and load (zero inertia) buses respectively. Assume $(\mathcal{N}, \mathcal{E})$ is connected. The main variables and parameters are summarized in Table I.

TABLE I: Notations.

$\theta := (\theta_i,\ i \in \mathcal{N})$	*deviations* of bus voltage angles from their nominal values with respect to the rotating frame at the nominal frequency.
$\omega := (\omega_i,\ i \in \mathcal{N})$	*deviations* of bus frequencies from their nominal value
$r := (r_i,\ i \in \mathcal{N})$	simultaneous step changes in power injections on an arbitrary subset of the buses
$p_{\mathcal{G}}^M := (p_i^M,\ i \in \mathcal{G})$	mechanic power injections at generators
$p_{\mathcal{G}} := (p_i,\ i \in \mathcal{G})$	control commands at generators
$T_{\mathcal{G}} := \text{diag}(T_i,\ i \in \mathcal{G})$	time constants that characterize time delay of turbines
$D_i\omega_i,\ i \in \mathcal{N}$	aggregate power of generator friction and damping as well as frequency-dependent (uncontrollable) loads like induction motors ($D_i > 0$). Define $D_{\mathcal{G}} := \text{diag}(D_i, i \in \mathcal{G})$ and $D_{\mathcal{L}} = \text{diag}(D_i, i \in \mathcal{L})$.
$M_i > 0, i \in \mathcal{G}$	inertia constant of generators; define $M_{\mathcal{G}} := \text{diag}(M_i, i \in \mathcal{G})$. For convenience, also define $M_i := 0, i \in \mathcal{L}$ at load buses.
$P := (P_{ij},\ ij \in \mathcal{E})$	active line power flows. For convenience, define $P_{ji} := -P_{ij}$ for $ij \in \mathcal{E}$ even though $ji \notin \mathcal{E}$ since the graph is directed.
$C \in \mathbb{R}^{\lvert\mathcal{N}\rvert \times \lvert\mathcal{E}\rvert}$	incidence matrix: $C_{j,e} = 1$ if line $e = jk$ is from bus j to some bus k, $C_{j,e} = -1$ if line $e = ij$ is from some bus i to bus j, and $C_{j,e} = 0$ otherwise
$B := \text{diag}(B_{ij},\ ij \in \mathcal{E})$	line parameters that depend on line susceptances, voltage magnitudes (assumed fixed) and reference phase angles. For convenience, define $B_{ji} := B_{ij}$ even though $ji \notin \mathcal{E}$ since the graph is directed.

B. Network model

The power network is modeled as

$$\dot{\theta} = \omega \tag{1a}$$
$$M_{\mathcal{G}}\dot{\omega}_{\mathcal{G}} = r_{\mathcal{G}} + p_{\mathcal{G}}^M - D_{\mathcal{G}}\omega_{\mathcal{G}} - C_{\mathcal{G}}P \tag{1b}$$
$$0 = r_{\mathcal{L}} - D_{\mathcal{L}}\omega_{\mathcal{L}} - C_{\mathcal{L}}P \tag{1c}$$
$$P = BC^T\theta \tag{1d}$$
$$T_{\mathcal{G}}\dot{p}_{\mathcal{G}}^M = -p_{\mathcal{G}}^M + p_{\mathcal{G}}. \tag{1e}$$

Here (1b) is the set of generator swing equations in vector form for generator buses, (1c) is the set of the power balance equations for load buses, and (1d) is the set of DC power flow equations in vector form. The generators are modeled as (1e), where we only consider turbine dynamics and ignore governor dynamics since the time constants of governors are usually much smaller than turbines [6]. We also ignore the traditional droop control since it is a special case of the redesigned primary frequency control $p_{\mathcal{G}}$ below.

We denote the state of the system (1) by $x(t) := (\theta(t), \omega(t), P(t), p_{\mathcal{G}}(t), p_{\mathcal{G}}^M(t))$.

Definition 1. *A state* $x^* := (\theta^*, \omega^*, P^*, p_{\mathcal{G}}^*, p_{\mathcal{G}}^{M,*}) \in \mathbb{R}^{2|\mathcal{N}|+|\mathcal{E}|+2|\mathcal{G}|}$ *is an* equilibrium (point) *of* (1) *if* x^* *satisfies* (1) *with the right-hand sides of* (1a)–(1b), (1e) *being zero.*

An equilibrium x^* always has $\omega^* = 0$, meaning that all bus frequencies are at their nominal value.

C. Design methodology

Suppose the system (1) is in an equilibrium before time 0 and there is disturbance r at time 0 on an arbitrary subset of the buses. Our goal is to design a feedback controller

$$p_{\mathcal{G}}(t) \quad := \quad p_{\mathcal{G}}(x(t)) \tag{2}$$

to drive the system back to a new equilibrium point. The design requirements are:

R1. *Asymptotic stability.* The closed-loop system (1) (2) will always converge to a (new) equilibrium point x^*.

R2. *Control goals.* The equilibrium x^* rebalances power and stabilizes bus frequencies (primary frequency control), restores bus frequencies and inter-area flows to their nominal values (secondary frequency control), and ensures line limits are satisfied (congestion management).

Our design approach consists of three steps:

D1. Formalize the control goals R2 as an optimization problem.

D2. Derive the controller $p_{\mathcal{G}}(x(t))$ as a primal-dual algorithm, or its variant, to solve the optimization problem in step D1.

D3. Prove that the requirements R1 and R2 are satisfied.

This design approach offers *three advantages*. First the objective function and the constraints of the optimization problem in D1 are designed to inherit the decentralized structure inherent in system (1). A first-order primal-dual algorithm then often takes the form of a distributed computation, leading to a controller $p_{\mathcal{G}}(x(t))$ with a distributed structure. Second the potentially difficult search for a Lyapunov function for stability proof is often facilitated by the Lagrangian function of the optimization problem designed in step D1. Finally by proving that every equilibrium point of the closed-loop system (1)(2) solves the optimization problem defined in step D1, the controller design automatically satisfies the requirement R2.

We allow a generator to perform primary frequency control only, unified control (primary + secondary + congestion management), or neither.[1] In practice one often has to iterate between steps D1–D3.

III. Primary frequency control

In this section we specify the optimization problem for the primary frequency control, design a decentralized controller, and discuss how it satisfies our requirements R1 and R2.

A. Control goals

Primary frequency control cannot in general drive the system to an equilibrium of (1), but only to a point where frequencies are synchronized $\omega_i^* = \omega_j^*$, not necessarily zero. This motivates the following definition.

Definition 2. *A state* $x^* := (\theta^*, \omega^*, P^*, p_{\mathcal{G}}^*, p_{\mathcal{G}}^{M,*}) \in \mathbb{R}^{2|\mathcal{N}|+|\mathcal{E}|+2|\mathcal{G}|}$ *is called a* frequency synchronized solution *of* (1) *if and only if all of the following are satisfied.*

(i) $\tilde{x}^* := (\tilde{\theta}^*(t), \omega^*, P^*, p_{\mathcal{G}}^*, p_{\mathcal{G}}^{M,*})$, *where* $\tilde{\theta}^*(t) := \omega^* t + \theta^*$, *is a solution of* (1).

(ii) *Right-hand sides of* (1b), (1e) *are zero.*

(iii) $\omega^* \in \text{span}\{1_{\mathcal{N}}\}$.

Conditions (i) and (ii) say that x^* satisfies (1) with $\dot{\omega}_{\mathcal{G}}$, $\dot{p}_{\mathcal{G}}^M$ equal to zero but $\dot{\theta}$ not necessarily zero. Condition (iii) is equivalent to $\omega_i^* = \omega_j^*$ for all $i, j \in \mathcal{N}$. Conditions (ii)(iii) are necessary conditions for (i). Clearly a frequency synchronized solution x^* that also has $\omega^* = 0$ is an equilibrium of (1) with $\dot{\theta} \equiv 0$.

The goal of a primary frequency controller $p_{\mathcal{G}}(x(t))$ is to drive the system (1) to a frequency synchronized solution x^* after any disturbance $r \in \mathbb{R}^{|\mathcal{N}|}$. In general frequency synchronized solutions are nonunique and we want to design the controller $p_{\mathcal{G}}(x(t))$ such that at the resulting x^*, the corresponding $(\theta^*, \omega^*, P^*, p_{\mathcal{G}}^*)$ solves the following optimization problem.

Primary Frequency Control (PFC):

$$\min_{\theta,\omega,P,p_{\mathcal{G}}} \quad \sum_{i\in\mathcal{G}} c_i(p_i) + \sum_{i\in\mathcal{N}} \frac{D_i(\omega_i)^2}{2} \tag{3a}$$

$$\text{subject to} \quad r_{\mathcal{G}} + p_{\mathcal{G}} - D_{\mathcal{G}}\omega_{\mathcal{G}} - C_{\mathcal{G}}P = 0 \tag{3b}$$

$$r_{\mathcal{L}} - D_{\mathcal{L}}\omega_{\mathcal{L}} - C_{\mathcal{L}}P = 0 \tag{3c}$$

$$P = BC^T\theta \tag{3d}$$

$$\underline{p}_i^{\text{PFC}} \le p_i \le \overline{p}_i^{\text{PFC}} \qquad \forall i \in \mathcal{G}. \tag{3e}$$

This problem formalizes the goal of primary frequency control: PFC (3) rebalances power and stabilizes bus frequencies at a common value (not necessarily nominal) in a way that minimizes the sum of the total effort c_i for controlling the generators $i \in \mathcal{G}$ and a quadratic penalty on the steady-state frequency deviation, subject to the capacity limits (3e) of generator reserves for primary frequency control. Indeed the control costs c_i minimize deviations from the reference generation dispatch and therefore PFC disrupts as little as possible the last optimal dispatch from tertiary frequency control. The constraints (3b)–(3d) ensures that, if the closed system converges to a frequency-synchronized solution x^* that solves PFC (3), then x^* satisfies the power balance conditions (1b)(1c) and the power flow equation (1d) with $\dot{\omega}_{\mathcal{G}} \equiv 0$.

We now describe our feedback controller $p_{\mathcal{G}}(x(t))$ and informally show that it indeed drives the system (1) to an optimal solution of PFC (3).

B. Decentralized control

We design primary frequency control as

$$p_i(t) = p_i(\omega_i(t)) := \begin{cases} \overline{p}_i^{\text{PFC}} & \text{if } \omega_i(t) < -c_i'(\overline{p}_i^{\text{PFC}}) \\ \underline{p}_i^{\text{PFC}} & \text{if } \omega_i(t) > -c_i'(\underline{p}_i^{\text{PFC}}) \\ (c_i')^{-1}(-\omega_i(t)) & \text{otherwise} \end{cases} \tag{4}$$

for all time $t \ge 0$ and all generators $i \in \mathcal{G}$. As in the current primary frequency control, the design is completely decentralized where each generator i only needs to measure its local frequency deviation $\omega_i(t)$ to decide its control $p_i(t)$.

Theorem 1. *The closed-loop system* (1)(4) *satisfies:*

(i) *There is a frequency synchronized solution* $x^* := (\theta^*, \omega^*, P^*, p_{\mathcal{G}}^*, p_{\mathcal{G}}^{M,*})$ *where* $(\theta^*, \omega^*, P^*, p_{\mathcal{G}}^*)$ *is the unique optimal for PFC* (3)[2] *and* ω^* *is also the unique optimal for the dual of PFC* (3).

(ii) *Suppose the control effort of all generators* $i \in \mathcal{G}$ *are* $c_i(p_i) = p_i^2/(2\alpha_i)$ *for arbitrary constants* $\alpha_i > 0$, *and the capacity constraints* (3e) *are not binding. Then every trajectory* $x(t) := (\theta(t), \omega(t), P(t), p_{\mathcal{G}}(t), p_{\mathcal{G}}^M(t))$ *converges to the frequency synchronized solution in* (i) *as* $t \to \infty$.

Theorem 1 says that our design requirements R1 and R2 for primary frequency control are met. It also has three interesting implications. First it implies that the local frequency deviation

[1] For ease of exposition, however, the discussion in this paper assumes either all generators perform the primary control or all performs the unified control. The results however can easily extend to the case where a generator may run either one of them or neither.

[2] For uniqueness of θ^*, we consider θ to be invariant under a rigid rotation of all the angles [7].

$\omega_i(t)$ at each generator conveys exactly the right information about the power imbalance for the generators themselves to make globally optimal decisions based on their local marginal cost functions c_i'. This allows a completely decentralized solution without explicit the need for communication among generators. Second when the measure c_i of control efforts are quadratic, our PFC controller (4) reduces to the traditional linear frequency-droop control at generators. An advantage of the proposed control (4) is that it can be applied also to load control with more general cost functions c_i.[3] It allows incremental deployment where generators still use the traditional droop control while the loads utilize the proposed PFC (4). Finally Theorem 1 has an interesting reverse-engineering implication first observed in [8] [2]: the traditional frequency-droop control is optimal over an arbitrary power network in the sense of solving the underlying optimization problem (3) with quadratic c_i. Moreover the *closed-loop* system defined by (1)(4) is derived as (a variant of) the standard first-order primal-dual algorithm to solve PFC (3) and its Lagrangian dual. Therefore the generator swing dynamics automatically carries out our primal-dual algorithm over the network in real time. In this sense, our design explicitly exploits the system dynamic (1) to help achieve our control goals.

IV. Unified control

In this section we specify the optimization problem for the unified control (primary + secondary + congestion management), design a distributed controller, and discuss how it satisfies our design requirements R1 and R2.

A. Control goals

Our unified controller $p_{\mathcal{G}}$ drives the system (1) to an equilibrium $x^* := (\theta^*, \omega^*, P^*, p_{\mathcal{G}}^*, p_{\mathcal{G}}^{M,*})$ that solves the following optimization problem, after any disturbance $r \in \mathbb{R}^{|\mathcal{N}|}$:

Unified Control (UC):

$$\min_{\theta, P, p_{\mathcal{G}}} \quad \sum_{i\in\mathcal{G}} c_i(p_i) \tag{5a}$$

$$\text{subject to} \quad r_{\mathcal{G}} + p_{\mathcal{G}} - C_{\mathcal{G}} P = 0 \tag{5b}$$

$$r_{\mathcal{L}} - C_{\mathcal{L}} P = 0 \tag{5c}$$

$$P = BC^T\theta \tag{5d}$$

$$ECP = 0 \tag{5e}$$

$$\underline{P}_{ij} \le P_{ij} \le \overline{P}_{ij} \qquad \forall ij \in \mathcal{E} \tag{5f}$$

$$\underline{p}_i^{\text{UC}} \le p_i \le \overline{p}_i^{\text{UC}} \qquad \forall i \in \mathcal{G}. \tag{5g}$$

The problem UC (5) formalizes the goals of the unified control. It rebalances power, restores bus frequencies and inter-area flows to their nominal values, and ensures the line limits are satisfied in a way that minimizes the total effort c_i of generator control subject to power flow equations (5d) and capacity limits (5g) of generator reserves. Specifically the constraints (5b)(5c) rebalance the power and stabilize the bus frequencies. Moreover we will design the controller $p_{\mathcal{G}}$ so that it also restores the frequencies to their nominal value. Congestion management is implemented by enforcing line flow limits (5f). To fulfill the zero area control error (ACE) requirement [6], the net inter-area flows are restored to the nominal values by imposing (5e), as follows. The power network is partitioned into a set $\mathcal{K}$ of control areas (subgraphs). Define $E \in \{0,1\}^{|\mathcal{K}|\times|\mathcal{N}|}$ such that $E_{k,i} = 1$ if bus i is in area k, and $E_{k,i} = 0$ otherwise. Therefore the k-th component of the vector $ECP \in \mathbb{R}^{|\mathcal{K}|}$ is the total net power flow from area k to the other areas.

Again the control costs c_i minimize deviations from the reference generation dispatch and therefore UC disrupts as little as possible the last optimal dispatch from tertiary frequency control. Once UC attains its goals, SCOPF could be solved to calculate a new economically-optimal dispatch. This means that UC maintains system security without interfering with the market operation.

B. Distributed control

Our controller involves internal variables and is specified by:

$$\dot{\lambda}_i = K_i^{\lambda}\Big(M_i\dot{\omega}_i + D_i\omega_i + \sum_{j\in N(i)}\big(P_{ij} - B_{ij}(\phi_i - \phi_j)\big)\Big) \tag{6a}$$

$$\dot{\pi}_k = K_k^{\pi}\Big(\textstyle\sum_{i\in\mathcal{N}(k), j\notin\mathcal{N}(k), \text{and } j\in N(i)} B_{ij}(\phi_i - \phi_j)\Big) \tag{6b}$$

$$\dot{\rho}_{ij}^{+} = K_{ij}^{\rho^+}\big[B_{ij}(\phi_i - \phi_j) - \overline{P}_{ij}\big]_{\rho_{ij}^+}^{+} \tag{6c}$$

$$\dot{\rho}_{ij}^{-} = K_{ij}^{\rho^-}\big[\underline{P}_{ij} - B_{ij}(\phi_i - \phi_j)\big]_{\rho_{ij}^-}^{+} \tag{6d}$$

$$\dot{\phi}_i = K_i^{\phi}\Big(\textstyle\sum_{j\in N(i)} B_{ij}(\lambda_i - \lambda_j) - \sum_{e\in\mathcal{E}} C_{i,e}B_e(\rho_e^+ - \rho_e^-) - \sum_{j\in N(i) \text{ and } k(j)\ne k(i)} B_{ij}\big(\pi_{k(i)} - \pi_{k(j)}\big)\Big) \tag{6e}$$

$$p_i = p_i(\omega_i + \lambda_i) \tag{6f}$$

with (6a)(6e) for all buses $i \in \mathcal{N}$, (6b) for all areas $k \in \mathcal{K}$, (6c)(6d) for all lines $ij \in \mathcal{E}$, and (6f) for all generators $i \in \mathcal{G}$. The diagonal matrices K^{λ}, K^{π}, K^{ρ^+}, K^{ρ^-} and K^{ϕ} are positive constant gains. The expression $[a]_u^+$ in (6c)(6d) are defined for $a \in \mathbb{R}$, $u \ge 0$ as

$$[a]_u^+ := \begin{cases} a & \text{if } u > 0 \text{ or } a > 0 \\ 0 & \text{otherwise} \end{cases}$$

such that $\rho_{ij}^+ \ge 0$ and $\rho_{ij}^- \ge 0$ are always satisfied along the trajectory if they start from any nonnegative initial values. We use $\mathcal{N}(k)$ in (6b) to denote all the buses in area k, and $k(i)$ in (6e) to denote the area where bus i is. The function $p_i(\cdot)$ in (6f) is defined in the same way as (4) except that $\underline{p}_i^{\text{PFC}}$, $\overline{p}_i^{\text{PFC}}$ are replaced by $\underline{p}_i^{\text{UC}}$, $\overline{p}_i^{\text{UC}}$.

We make three remarks on the controller structure. First, the local nature of the design (6) is in striking contrast with traditional AGC and congestion management through SCOPF where global information is used to centrally compute control decisions. For instance, to update its state λ_i in (6a), bus i only needs to measure its local frequency deviation ω_i and branch power flow P_{ij} incident on i, and receive the local

[3] Theorem 1(ii) holds for general convex functions c_i given an additional assumption on their curvatures, if we also have load control. See [5].

states ϕ_j from all its neighbors j. UC (6) can therefore be implemented with a real-time message passing protocol between neighboring buses. Second, the only state whose update requires nonlocal information is π_k in (6b) which is an area-wide state variable for each area. Its update requires the angle difference $\phi_i - \phi_j$ across each line $ij \in \mathcal{E}$ that connects a boundary bus in area k with its neighbor outside area k. The update (6b) can be carried out either centrally in each area or in a distributed manner. In the centralized approach, a designated agent (e.g., area control center) collects $\phi_i(t)-\phi_j(t)$ from all boundary buses in area k, updates $\pi_k(t)$ and communicates $\pi_k(t)$ to all buses i in area k for them to update $\phi_i(t)$ using (6e). In the distributed approach, additional computation agents are created (tie-line agents), one for each side of a boundary edge e that connects two separate areas. Within each area k, these tie-line agents communicate among each other through an arbitrary (connected) communication graph. Each agent in an area k maintains two (new) local variables $\pi_e^k(t)$ and $\gamma_e^k(t)$ and exchanges $(\pi_e^k(t), \gamma_e^k(t))$ with neighboring tie-line agents within area k and with agents which are its direct neighbors outside area k; see [4, Section VI-B] for details. Third, delays in the communication above have a substantial impact on stability of the closed-loop system (1)(6). See [9] for an analysis of delay robustness. Fourth, besides state information that is exchanged in real time, each bus i also needs to know local network parameters M_i, D_i, and B_{ij} between all its neighbors and itself. See [4, Section V] for convergence of (1)(6) with uncertain parameters D_i.

Theorem 2. *The closed-loop system* (1)(6) *satisfies:*

(i) *There is an equilibrium* $x^* := (\theta^*, \omega^*, P^*, p_{\mathcal{G}}^*, p_{\mathcal{G}}^{M,*})$ *where* $(\theta^*, P^*, p_{\mathcal{G}}^*)$ *is the unique optimal for UC* (5).

(ii) *Suppose the control effort of all generators* $i \in \mathcal{G}$ *are* $c_i(p_i) = p_i^2/(2\alpha_i)$ *for arbitrary constants* $\alpha_i > 0$, *and the capacity constraints* (5g) *are not binding. Then every trajectory* $x(t) := (\theta(t), \omega(t), P(t), p_{\mathcal{G}}(t), p_{\mathcal{G}}^M(t))$ *converges to the equilibrium in* (i) *as* $t \to \infty$.

Theorem 2 confirms that our controller (6) meets not only our design requirement R1 but also R2. Before we illustrate how this is achieved using a two-bus example in Section V, we remark on the important role of the auxiliary variables $\phi_i(t)$ in UC (6). The variables $\phi_i(t)$ should be interpreted as *virtual phase angles* on buses i. The quantities

$$B_{ij}(\phi_i(t) - \phi_j(t))$$

should be interpreted as *virtual flows* on branches ij (cf. $P_{ij}(t) = B_{ij}(\theta_i(t) - \theta_j(t))$), because, when all $i \in \mathcal{N}$ are generator buses, the virtual flows converge to the actual branch power flows in equilibrium, i.e.,

$$P_{ij}^* \quad = \quad B_{ij}(\phi_i^* - \phi_j^*).$$

Virtual flow is an important concept introduced in [4] to enforce line flow limits as well as scheduled inter-area flows (see below).

V. Two-bus example

In this section we illustrate our controllers PFC (4) and UC (6) on an example with only two generator buses $\mathcal{N} = \mathcal{G} = \{1, 2\}$, and explain how they achieve our design requirement R2 in equilibrium.

We assume that each generator has a quadratic cost function $c_i(p_i) = p_i^2/(2\alpha_i)$, $i = 1, 2$, that measures their control effort and that each bus belongs to a different area ($k = i \in \{1, 2\}$). To simplify the notation, we remove generator capacity constraints (3e) and (5g).

The network dynamics for this simple example are described by: for $i = 1, 2$,

$$\dot{\theta}_i \;=\; \omega_i \tag{7a}$$

$$M_i\dot{\omega}_i \;=\; r_i + p_i^M - D_i\omega_i - B_{12}(\theta_i - \theta_j) \qquad i \neq j \tag{7b}$$

$$T_i\dot{p}_i^M \;=\; -p_i^M + p_i. \tag{7c}$$

The PFC controller (4) is given by

$$p_i(t) \quad := \quad -\alpha_i\omega_i(t), \qquad i = 1, 2. \tag{8}$$

Hence the PFC controller (4) reduces to the standard frequency-droop control when the cost functions c_i are quadratic. In other words PFC is a generalization of droop control for arbitrary convex cost functions. Theorem 1 implies that in equilibrium the generation changes scaled by their cost parameters α_i converge to the *same* value which is the equilibrium frequency deviation:

$$\frac{p_i^*}{\alpha_i} \quad = \quad -\omega^*, \qquad i = 1, 2. \tag{9}$$

It can be shown that (9) is the key requirement for the Karus-Kuhn-Tucker optimality condition [10] for PFC (3), and therefore the equilibrium of the closed-loop system (7)(8) solves PFC (3).

We next explain our UC controller (6) and illustrate how it achieves its goals R2 in equilibrium, in three steps starting with the simplest case where the nominal frequency is restored but the requirements on the inter-area flow and congestion management are not enforced.

In this case the controller (6) reduces to: for $i = 1, 2$,

$$\dot{\lambda}_i \quad = \quad K_i^{\lambda}\Big(M_i\dot{\omega}_i + D_i\omega_i + \big(P_{12} - B_{12}(\phi_i - \phi_j)\big)\Big) \qquad i \neq j \tag{10a}$$

$$\dot{\phi}_i \quad = \quad K_i^{\phi}\Big(B_{12}(\lambda_i - \lambda_j)\Big) \qquad i \neq j \tag{10b}$$

$$p_i \quad = \quad -\alpha_i(\omega_i + \lambda_i). \tag{10c}$$

At an equilibrium of the closed-loop system (7)(10), we have $\dot{\theta}_i = \dot{\omega}_i = \dot{\lambda}_i = \dot{\phi}_i = 0$. This implies $\omega_i^* = 0$ from (7a) and $P_{12}^* = B_{12}(\phi_1^* - \phi_2^*)$ from (10a). This means, respectively, the nominal frequency is restored and the virtual flow is equal to the real line flow. Moreover (10b) implies that $\lambda_1^* = \lambda_2^* =: \lambda^*$ and hence, since $\omega_i^* = 0$, we have

$$\frac{p_i^*}{\alpha_i} \quad = \quad -\lambda^* \qquad i = 1, 2$$

i.e., the scaled generation changes of both generators again converge to a common value as in (9). This implies that the equilibrium is optimal, in the absence of inter-area constraints and congestion management.

Next, to restore inter-area flows to their scheduled values we replace (10b) with

$$\begin{aligned}\dot{\phi}_i &= K_i^{\phi}\Big(B_{12}(\lambda_i - \lambda_j) - B_{12}\big(\pi_i - \pi_j\big)\Big) && i \neq j \\ \dot{\pi}_i &= K_i^{\pi}\Big(B_{12}(\phi_i - \phi_j)\Big) && i \neq j\end{aligned}$$

for $i = 1, 2$. In equilibrium we have $\dot{\pi} = 0$ yielding $B_{12}(\phi_1^* - \phi_2^*) = 0$, i.e., the inter-area *virtual* flow is restored to its scheduled value. Since in equilibrium the deviation $P_{12}^* = B_{12}(\phi_1^* - \phi_2^*) = 0$, the actual inter-area flow is also restored to its scheduled value.

Finally for congestion management, we introduce the additional states ρ_{12}^+ and ρ_{12}^- that penalize when the virtual flow $B_{12}(\phi_i - \phi_j)$ exceeds the line limits $\overline{P}_{12}$ and $\underline{P}_{12}$ respectively. This is achieved by replacing (10b) with

$$\begin{aligned}\dot{\phi}_i &= K_i^{\phi}\Big(B_{12}(\lambda_i - \lambda_j) - C_{i,12}B_{12}(\rho_{12}^+ - \rho_{12}^-) \\ &\qquad - B_{12}\big(\pi_i - \pi_j\big)\Big) && i \neq j \\ \dot{\pi}_i &= K_i^{\pi}\Big(B_{12}(\phi_i - \phi_j)\Big) && i \neq j \\ \dot{\rho}_{12}^+ &= K_{12}^{\rho^+}\left[B_{12}(\phi_1 - \phi_2) - \overline{P}_{12}\right]_{\rho_{12}^+}^+ \\ \dot{\rho}_{12}^- &= K_{12}^{\rho^-}\left[\underline{P}_{12} - B_{12}(\phi_1 - \phi_2)\right]_{\rho_{12}^-}^+ .\end{aligned}$$

The condition $\dot{\rho}_{12}^+ = \dot{\rho}_{12}^- = 0$ in equilibrium enforces $\underline{P}_{12} \leq P_{12}^* \leq \overline{P}_{12}$.

VI. Controller derivation

We outline how we derive the PFC controller (4) to illustrate our design methodology. The crux of our derivation is to construct primal-dual dynamics of a properly designed Lagrangian and modify it to match the network dynamics:

- *Step 1:* Remove θ and constraint (3d) from the optimization problem PFC (3) and build the Lagrangian by relaxing non-local constraints (3b) and (3c)

$$\begin{aligned}L(\omega, P, p_{\mathcal{G}}, \nu) = \sum_{i\in\mathcal{G}} c_i(p_i) + \sum_{i\in\mathcal{N}} \frac{D_i(\omega_i)^2}{2} + \nu_{\mathcal{G}}^T(r_{\mathcal{G}} + p_{\mathcal{G}} \\ - D_{\mathcal{G}}\omega_{\mathcal{G}} - C_{\mathcal{G}}P) + \nu_{\mathcal{L}}^T(r_{\mathcal{L}} - D_{\mathcal{L}}\omega_{\mathcal{L}} - C_{\mathcal{L}}P)\end{aligned}$$

  where $\nu_{\mathcal{G}}$ and $\nu_{\mathcal{L}}$ are the Lagrange multipliers of (3b) and (3c) respectively.
- *Step 2:* Minimize $L(\omega, P, p_{\mathcal{G}}, \nu)$ with respect to ω and $p_{\mathcal{G}}$ subject to generator constraints (3e), i.e.

$$L(P, \nu) = \min_{\omega, p_{\mathcal{G}} \in [\underline{p}^{PFC}, \overline{p}^{PFC}]} L(\omega, P, p_{\mathcal{G}}, \nu). \tag{13}$$

  It is easy to verify that the minimum is achieved when $\omega = \nu$ and $p_{\mathcal{G}}$ satisfies (4). We therefore substitute w.l.o.g. ν with ω in (13) to get $L(P, \omega) := L(P, \nu)|_{\nu=\omega}$.
- *Step 3:* Maximize (13) with respect to $\nu_{\mathcal{L}} = \omega_{\mathcal{L}}$, i.e.

$$L(P, \omega_{\mathcal{G}}) = \max_{\omega_{\mathcal{L}}} L(P, \omega)$$

  which is achieved if and only if the optimal solution $\omega_{\mathcal{L}}^*(P, \omega_{\mathcal{G}})$ satisfies (1c).
- *Step 4:* Compute the continuous time primal-dual algorithm of $L(P, \omega_{\mathcal{G}})$ with specific choice of gains

$$\dot{P} := -B[\frac{\partial}{\partial P} L(P, \omega_{\mathcal{G}})]^T = B(C^T\omega)|_{\omega_{\mathcal{L}} = \omega_{\mathcal{L}}^*(P, \omega_{\mathcal{G}})} \tag{14a}$$

$$\begin{aligned}\dot{\omega}_{\mathcal{G}} &:= M_{\mathcal{G}}^{-1}[\frac{\partial}{\partial \omega_{\mathcal{G}}} L(P, \omega_{\mathcal{G}})]^T \\ &= M_{\mathcal{G}}^{-1}(r_{\mathcal{G}} + p_{\mathcal{G}}(\omega_{\mathcal{G}}) - D_{\mathcal{G}}\omega_{\mathcal{G}} - C_{\mathcal{G}}P)\end{aligned} \tag{14b}$$

  where $p_{\mathcal{G}}(\omega_{\mathcal{G}})$ is given by (4). It is easy to see that (1a) and (1d) are equivalent to (14a). Therefore, since by *Step 3* $\omega_{\mathcal{L}}^*(P, \omega_{\mathcal{G}})$ must satisfy (1c), the system (14) is equivalent (1a)-(1d) with $p_{\mathcal{G}}^M$ substituted by $p_{\mathcal{G}}(\omega_{\mathcal{G}})$.
- *Step 5:* Add generator dynamics to (14) by substituting $p_{\mathcal{G}}(\omega_{\mathcal{G}})$ in (14b) with $p_{\mathcal{G}}^M$ which is driven by (1e).

The derivation described above shows how to identify the network dynamics (1) as part of a variant of primal-dual algorithm for PFC (3). The same procedure can be extended to accommodate additional constraints in the optimization problem and therefore achieve more sophisticated operational constraints. The only requirement is that P and ω can only appear in constraints (3b)-(3d). This is handled by introducing *virtual flows* to implicitly impose the additional constraints.

VII. Simulations

We demonstrate the performance of the proposed framework with simulations of the IEEE 39 bus system, shown in Fig. 1. We assume that the system has two control areas that are connected by lines $(1, 2)$, $(2, 3)$ and $(26, 27)$. System parameters are obtained from the Power System Toolbox [11], with minor modifications.[4]

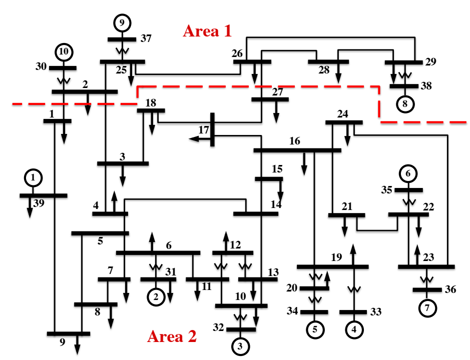

Figure 1: IEEE 39 bus system from [4].

Throughout the simulations we assume $D_i = 0.1$ pu·sec/rad for all $i \in \mathcal{N}$ where 1 pu = 100 MVA is the base power. For all generators $i \in \mathcal{G}$, we use a governor-turbine model with time constants 0.1 sec for the governor and $T_i = 2$ secs for the turbine [6]. Moreover, all generators $i \in \mathcal{G}$ are equipped with droop control $\Delta p_i = -\frac{1}{R_i}\omega_i$ where $R_i = 10$ rad/(sec · pu), in addition to any other control schemes we simulate.

[4]For numerical stability and precision of the simulations, we scale up the inertia M_i of all generators $i \in \mathcal{G}$ and all line reactances by a factor of 10.

In the simulations, we implement the unified control (UC) (6) on all generators $i \in \mathcal{G}$, whose control effort functions are $c_i(p_i) = p_i^2/(2\alpha_i)$. We assume adequate control capacities for all $i \in \mathcal{G}$ such that the constraints (5g) are not binding. Then (4) and (6f) lead to the design $p_i(\lambda_i + \omega_i) = -\alpha_i(\lambda_i + \omega_i)$ for UC. We choose $\alpha_i = 1$ pu $\cdot$ sec/rad for all $i \in \mathcal{G}$, and control gains $K_i^\lambda = K_k^\pi = 0.2$, $K_i^\phi = 0.1$, and $K_e^{\rho^+} = K_e^{\rho^-} = 1$ for all $i \in \mathcal{N}$, $k \in \mathcal{K}$ and $e \in \mathcal{E}$.[5] For comparison, we also simulate the case when all the generators run the centralized, standard automatic generation control (AGC) [6]:

$$\begin{aligned} \text{ACE}_k &= P_k^{\text{tie}} + B_k^{\text{area}} \omega_k^{\text{area}} && \forall \text{ area } k \in \mathcal{K} \\ \dot{p}_k^{\text{area}} &= -K_k^{\text{area}} \text{ACE}_k && \forall \text{ area } k \in \mathcal{K} \\ p_i &= \alpha_i p_k^{\text{area}} / (\textstyle\sum_{i \in \mathcal{G}(k)} \alpha_i) && \forall \text{ generator } i \in \mathcal{G}(k) \end{aligned}$$

where $P_k^{\text{tie}} = E_k C P$ is the deviation of net inter-area flow out of area k, ω_k^{area} is the average frequency deviation of all $i \in \mathcal{G}(k)$, i.e., generators in area k, and $B_k^{\text{area}} = \sum_{i \in \mathcal{G}(k)} \frac{1}{R_i} + \sum_{i \in \mathcal{N}(k)} D_i$. The total generation adjustments p_k^{area} for areas $k \in \mathcal{K}$ are integrals of ACE signals with gains $K_k^{\text{area}} = 0.033\ \text{s}^{-1}$.[6] The participation factors of individual generators are in the same ratio as α_i we choose for UC.

We simulate the system after a disturbance $r_{30} = -18$ pu. Fig. 2 shows the frequencies at different buses, for the two cases when all the generators run AGC, or UC, respectively.

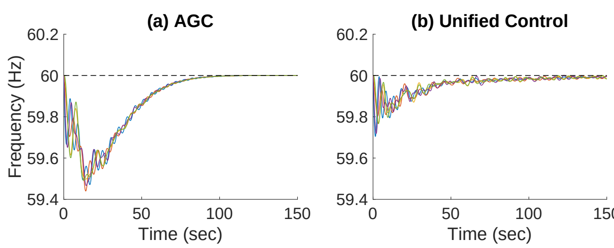

Figure 2: Frequencies at buses 3, 13, 23, 26, 33, when (a) all generators run AGC or (b) all generators run UC.

Both control schemes restore bus frequencies to 60 Hz, while UC leads to a better transient by raising the frequency nadir. *This suggests that UC has the potential to effectively address large transient frequency deviation, which is the main concern for low-inertia systems.*

Another advantage of UC is to enforce line limits at the frequency control timescale. This is illustrated in Fig. 3, which shows the flows on the three tie-lines for the two cases when all the generators run AGC, or UC, respectively. The total steady-state flows over these tie-lines are the same for both cases, since both impose inter-area flow constraints.

We run the simulations above using the simple power network model (1). We are working on a more comprehensive set of tests for UC with a much wider range of scenarios and much more realistic models including, e.g., a higher-order generator model (with flux decay, excitation, and power system stabilizer), time-varying voltage magnitudes, nonzero line resistances, and nonlinear AC power flows.

[5]Units of these gains are clear from the context and therefore omitted.

[6]We choose K_k^{area} that leads to fastest convergence without overshoot above 60 Hz. Frequency nadir is not sensitive to K_k^{area}.

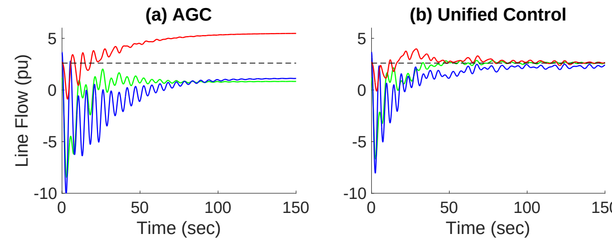

Figure 3: Line flows on $(1, 2)$, $(2, 3)$ and $(26, 27)$ which connect two control areas, when (a) all generators run AGC or (b) all generators run UC. The line limits are 2.6 pu (dashed horizontal line).

VIII. Conclusions and Discussions

We have proposed a new framework for frequency control and congestion management. Our unified control (UC) rebalances power, restores the nominal frequency and inter-area flows, and maintains line flows below their limits in a way that minimizes the disruption to the last optimal dispatch.

This can have a profound implication for power system control. The traditional congestion management is undertaken on a slow timescale, so no line overloads can be tolerated and tertiary control must solve $N-1$ SCOPF. Since UC reacts to both generator and line outages on minutes timescale, transient line overloads can be tolerated since a few minutes are not long enough to overheat a line. If UC can resolve all transmission constraints due to generator or line contingencies, then the tertiary control can solve $N-0$ OPF, instead of $N-1$ SCOPF. Otherwise SCOPF need only consider those contingencies not resolved by UC. This move from the current $N-1$ preventive dispatch towards $N-0$ corrective dispatch can result in significant savings in generation re-dispatch.

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