2015 IEEE 54th Annual Conference on Decision and Control (CDC)
December 15-18, 2015. Osaka, Japan

Distributed optimization decomposition for joint economic dispatch and frequency regulation

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Abstract— **Economic dispatch and frequency regulation are typically viewed as fundamentally different problems in power systems, and hence are typically studied separately. In this paper, we frame and study a joint problem that optimizes both slow timescale economic dispatch resources and fast timescale frequency regulation resources. We provide sufficient conditions under which the joint problem can be decomposed without loss of optimality into slow and fast timescale problems. These slow and fast timescale problems have appealing interpretations as the economic dispatch and frequency regulation problems respectively. Moreover, the fast timescale problem can be solved using a distributed algorithm that preserves the stability of the network during transients. We also apply this optimal decomposition to propose an efficient market mechanism for economic dispatch that coordinates with frequency regulation.**

I. Introduction

One of the main objectives of every power system operator is to schedule power generation to meet demand at every time instant [1]–[3]. This is a challenging task that seeks to schedule generators in a cost-efficient manner while also respecting their limitations (e.g., ramp constraints and capacity constraints) and responding rapidly to any supply-demand imbalances that may emerge (e.g., generator outages and line outages). To make matters more complex, slow timescale control is typically performed using market mechanisms while fast timescale control is done via engineered controllers.

The complexity of this global system operation problem means that it is typically broken up into two separate sub-problems – (slow timescale) economic dispatch and (fast timescale) frequency regulation – which are studied independently of each other.

Economic dispatch operates at a slow timescale (intervals of 5 minutes or longer) and focuses on efficiency with respect to costs. In particular, the economic dispatch problem seeks to optimally schedule generators so that the total generation cost is minimized subject to line limits and generation capacity and ramping constraints. Economic dispatch has a long history [1], [4]–[8]. It is currently implemented using a market mechanism known as supply function bidding. In this mechanism, generators submit supply functions to the system operator which specify (as a function of price) the amount a generator is willing to produce. The system operator uses those bids to construct the implied cost functions and solves a centralized optimization problem (over single or multiple time periods) to schedule generators in a way that minimizes system costs while meeting demand and slow timescale operating constraints (including line limits, generation capacity constraints, generation ramping constraints, minimum generation constraints, security constraints, etc.). A centralized market implementation is important both to collect bid costs from generators and also because physical laws of power flows impose coupled constraints between generators.

Frequency regulation operates at a fast timescale (from 30 seconds to a few minutes) and focuses on stability rather than efficiency. In particular, the operator seeks to compensate the remaining imbalance between generation and demand, which drives a deviation from the nominal frequency, by quickly rescheduling fast ramping generators. Frequency regulation has a long history [2], [9], [10]. It is currently implemented by a mechanism known as Automatic Generation Control (AGC), which aims to rebalance power and restore the nominal frequency within independent control areas using local measurements. Within each area, AGC uses information on frequency deviation and inter-area flows to compute the necessary change in power needed to rebalance supply and demand in the system, and allocates this change among different generators based on the market clearing allocations from the last economic dispatch execution [1].

A. Contributions of this paper

While economic dispatch and frequency regulation each, individually, have large and active literatures; these literatures are almost completely disparate. To this point, there has been no rigorous analysis of whether the combination of economic dispatch and frequency regulation solves the global system operator's goal of managing generation resources in order to minimize cost while maintaining stability.

The goal of this paper is to initiate such a study. In particular, we seek to understand when the combination of economic dispatch and frequency regulation optimally solves the global system operator's problem. More generally, we seek to understand when the global system operator's problem can be decomposed, without loss of optimality,

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ARPA-E grant DE-AR0000226, Los Alamos National Lab through an DoE grant DE-AC52-06NA25396, DTRA through grant HDTRA 1-15-1-0003, Skoltech, NSF grant 1545096 as part of the NSF/DHS/DOT/NASA/NIH Cyber-Physical Systems Program, NSF grant NETS-1518941, and NSF grant EPAS-1307794.

The authors would like to thank Steven H. Low from Caltech for insightful discussions.

978-1-4799-7886-1/15/$31.00 ©2015 IEEE

into subproblems corresponding to economic dispatch and frequency regulation.

Our main result provides an initial answer. In the context of a DC power flow model and two classes of generators (peakers and baseloads), we show that the global system operator's problem can be decomposed into two subproblems corresponding to economic dispatch and frequency regulation without loss of optimality as long as the time-average prices at each node within each slow timescale interval (used by economic dispatch) are zero (Theorem 1).

This theorem can be viewed as providing a first-principles justification for the current separation of the economic dispatch and frequency regulation problems. Furthermore, it provides a guide for the design of market mechanisms for economic dispatch and control policies for frequency regulation since it highlights a sufficient condition for such algorithms to jointly solve the global problem.

In the case of frequency regulation, the form of the problem that emerges from the optimization decomposition differs in important ways from existing frequency regulation operations (Section IV). First, the frequency regulation controller proposed in this paper (which builds on [11]), uses information on generators' costs to drive the power system to an operating point that minimizes its costs. On the other hand, existing approaches use participation factors from the latest economic dispatch run to drive the power system to a stable operating point. Since participation factors might not reflect actual generation costs, the resulting allocation might not be optimal (from the perspective of cost minimization).

In the case of economic dispatch, we illustrate that the form of the problem that emerges from the optimization decomposition can be solved using a market implementation based on supply function bidding (Section V), similar to existing operations. However, our proposed mechanism differs from existing operations in that we allocate frequency regulation resources optimally and we do so without requiring additional communication in the market. Existing operations use economic dispatch LMPs to directly compensate fast timescale frequency regulation resources, where the latter are allocated using engineered controllers without regards to generation costs. The decomposition in this paper suggests that, instead, supply functions submitted at the economic dispatch timescale should be used to allocate frequency regulation resources via the distributed algorithm in Section IV. Our main result in Section V shows that, if the conditions required for optimal decomposition hold, then the competitive equilibrium of our proposed mechanism leads to efficient (optimal) operation.

II. System Model

Our aim in this paper is to understand when the system operator's global objective can be decomposed into subproblems that correspond to economic dispatch and frequency regulation. To this end, we formulate a model of the global objective that includes the joint objectives of economic dispatch and frequency regulation and considers balancing of supply and demand at both the economic dispatch and frequency regulation timescales. We focus on a DC power flow model and consider two generation types (peakers and baseloads) which differ in the responsiveness they provide.

A. Network model

We consider a finite time horizon partitioned into K discrete intervals indexed by $k \in \mathcal{K}$ where $\mathcal{K} = \{0, 1, \dots, K-1\}$. In principle, the length of each time period k may range from as little as seconds to as long as minutes. However, in this paper, we focus on the case where the length of each time period is on the order tens of seconds.

We consider a connected network with N nodes and L links. Let $\mathcal{N} = \{1, 2, \dots, N\}$ and $\mathcal{L} = \{1, 2, \dots, L\}$ denote the set of nodes and links respectively. Index nodes by $n \in \mathcal{N}$ and links by $l \in \mathcal{L}$. Without loss of generality, we assign each link l an arbitrary orientation and let $i(l) \in \mathcal{N}$ and $j(l) \in \mathcal{N}$ denote the tail and head of the link respectively.

We assume that node n has a deterministic inelastic demand $d_{k,n} > 0$ in period k. We assume that node n has two generators which we refer to as *peaker* and *baseload* and we denote denote their production quantities in period k by $q^p_{k,n}$ and $q^b_{k,n}$ respectively. Let the vectors $d_k := (d_{k,1}, \dots, d_{k,N})$, $q^p_k := (q^p_{k,1}, \dots, q^p_{k,N})$, and $q^b_k := (q^b_{k,1}, \dots, q^p_{k,N})$. Then, $q^p_k + q^b_k - d_k$ is the vector of nodal injections in period k. Hence, the supply-demand balance constraints are given by:

$$\mathbf{1}^\top (q^b_k + q^p_k - d_k) = 0, \quad k \in \mathcal{K}, \tag{1}$$

where $\mathbf{1} \in \mathbb{R}^N$ denotes the vector of all ones.

We adopt the DC power flow model for line flows. Let $\theta_{k,n}$ denote the phase angle of node n in period k. For link l, let $p_{k,l}$ denote the power flow in period k and B_l denote the sensitivity of the flow with respect to changes in the phase difference $\theta_{k,i(l)} - \theta_{k,j(l)}$ in period k. Let the vectors $\theta_k := (\theta_{k,1}, \dots, \theta_{k,N})$ and $p_k := (p_{k,1}, \dots, p_{k,L})$ and the matrix $B := \text{diag}(B_1, \dots, B_L)$. Hence, the line flows in period k are given by:

$$p_k = BC^\top \theta_k,$$

where $C \in \mathbb{R}^{N \times L}$ is the incidence matrix of the directed graph. Then, the injections are given by:

$$q^b_k + q^p_k - d_k = Cp_k = L\theta_k, \tag{2}$$

where $L := CBC^\top$.

Note that (1) and (2) are equivalent. For any set of injections that satisfy (1), we can always find θ_k that satisfies (2). Conversely, since $\mathbf{1}^\top C = 0$, any injections that satisfy (2) also satisfy (1). Hence, the line flows can be written in terms of the power injections:

$$p_k = BC^\top L^\dagger (q^b_k + q^p_k - d_k),$$

where $L^\dagger$ denotes the pseudo-inverse of L. Let $H := BC^\top L^\dagger$. Let f_l denote the capacity of line l and let the vector $f := (f_1, \dots, f_L)$. Then the line flow constraints are given by:

$$-f \le H(q^b_k + q^p_k - d_k) \le f, \quad k \in \mathcal{K}. \tag{3}$$

B. Generation types

As mentioned, we consider two types of generation – peakers and baseloads – where peaker refers to more responsive generation and baseload refers to less responsive generation. Recall that $q^p_{k,n}$ and $q^b_{k,n}$ denote the production quantities of the peaker and baseload respectively at node n in period k.

We assume that the peaker and baseload at node n have minimum generation constraints $\underline{q}^p_n$ and $\underline{q}^b_n$ respectively, maximum generation constraints $\bar{q}^p_n$ and $\bar{q}^b_n$ respectively, and incur costs $c^p_n(q^p_{k,n})$ and $c^b_n(q^b_{k,n})$ respectively for their productions in period k, where the functions $c^p_n : [\underline{q}^p_n, \bar{q}^p_n] \to \mathbb{R}_+$ and $c^b_n : [\underline{q}^b_n, \bar{q}^b_n] \to \mathbb{R}_+$ are convex and continuously differentiable. We also assume that $c^p_n(q^p_{k,n}) \to +\infty$ as $q^p_{k,n} \to \{\underline{q}^p_n, \bar{q}^p_n\}$. Let the vectors $\underline{q}^p := (\underline{q}^p_1, \ldots, \underline{q}^p_N)$, $\underline{q}^b := (\underline{q}^b_1, \ldots, \underline{q}^b_N)$, $\bar{q}^p := (\bar{q}^p_1, \ldots, \bar{q}^p_N)$, and $\bar{q}^b := (\bar{q}^b_1, \ldots, \bar{q}^b_N)$. Then, the generation constraints are given by:

$$\underline{q}^p \le q^p_k \le \bar{q}^p, \quad k \in \mathcal{K}; \tag{4}$$

$$\underline{q}^b \le q^b_k \le \bar{q}^b, \quad k \in \mathcal{K}. \tag{5}$$

To model the fact that baseloads are less responsive than peakers, we assume that peakers may change production levels every time period while baseloads can only change production levels every S time periods where $K \bmod S = 0$. Let $\mathcal{S}$ denote the set of time periods in which baseloads may change production levels, i.e. $\mathcal{S} = \{0, S, 2S, \ldots, K-S\}$. For each $k \in \mathcal{S}$, let $\mathcal{K}_k = \{k, k+1, \ldots, k+S-1\}$ denote the set of time periods in the corresponding length-S interval during which baseload productions are constant. For each $k \in \mathcal{S}$ and $k' \in \mathcal{K}_k$, define $s(k') := S\lfloor k'/S \rfloor = k$. The constraints on baseloads' decisions can be represented by:

$$q^b_k = q^b_{s(k)}, \quad k \in \mathcal{K}. \tag{6}$$

C. System operator's objective

The global system operator's objective is to dispatch the baseload and peaker generations in order to minimize the total cost needed to satisfy demand and operating constraints. This is formalized as follows.

$$\begin{aligned} SYSTEM: \quad & \min \quad \textstyle\sum_k \sum_n \left(c^b_n(q^b_{n,k}) + c^p_n(q^p_{n,k}) \right) \\ & \text{over} \quad q^b_k,\ q^p_k, \quad k \in \mathcal{K}; \\ & \text{s.t.} \quad (1),\ (3),\ (4) - (6). \end{aligned}$$

We assume throughout that this optimization is feasible.

In addition to the constraints highlighted above there are practical issues that must be taken into account in any feasible solution. At the fast timescale, system demand must be measured precisely and controls implemented such that generators do not lose synchrony. Hence, a practical implementation must include mechanisms to restore frequency and preserve grid stability. It must also include market mechanisms to extract cost functions from generators.

The current practice in economic dispatch is to clear the market without fast timescale supply-demand constraints. Instead, these fast timescale constraints are implemented using frequency regulation controls without consideration of the costs of generation. In Section III, we provide an architectural decomposition of the global objective into slow timescale and fast timescale subproblems without loss of optimality under certain conditions. We propose a distributed frequency regulation algorithm to implement the solution to the fast timescale subproblem and a market mechanism to extract cost functions for the subproblems. We address these in Sections IV and V respectively.

D. Lagrangian relaxation

Crucial to our main result is the relaxation of the supply-demand balance constraints and line flow constraints. We associate Lagrange multipliers with the constraints as follows:

$$\begin{aligned} \lambda_k \in \mathbb{R} &: \text{supply demand constraint (1).} \\ \mu^-_k \in \mathbb{R}^L_+ &: \text{negative line flow constraint in (3).} \\ \mu^+_k \in \mathbb{R}^L_+ &: \text{positive line flow constraint in (3).} \\ \nu^-_k \in \mathbb{R}^N_+ &: \text{negative generation constraint in (4).} \\ \nu^+_k \in \mathbb{R}^N_+ &: \text{positive generation constraint in (4).} \\ \xi^-_k \in \mathbb{R}^N_+ &: \text{negative generation constraint in (5).} \\ \xi^+_k \in \mathbb{R}^N_+ &: \text{positive generation constraint in (5).} \end{aligned}$$

The Lagrangian is given by:

$$\begin{aligned} & L(x, y) \\ & = \sum_k \sum_n \left(c^b_n(q^b_{k,n}) + c^p_n(q^p_{k,n}) \right) - \sum_k \left(\mu^-_k + \mu^+_k \right)^\top f \\ & \quad + \sum_k \pi(\lambda_k, \mu^-_k, \mu^+_k)^\top \left(q^b_k + q^p_k - d_k \right) \\ & \quad - \sum_k \left(\nu^-_k - \nu^+_k \right)^\top q^p_k - \sum_k \left(\xi^-_k - \xi^+_k \right)^\top q^b_k \\ & \quad + \sum_k \left(\nu^{-\top}_k \underline{q}^p - \nu^{+\top}_k \bar{q}^p \right) + \sum_k \left(\xi^{-\top}_k \underline{q}^b - \xi^{+\top}_k \bar{q}^b \right), \end{aligned}$$

where $\pi(\lambda_k, \mu^-_k, \mu^+_k)$ denotes the vector of nodal prices in period k:

$$\pi(\lambda_k, \mu^-_k, \mu^+_k) := \lambda_k \mathbf{1} - H^\top \left(\mu^-_k - \mu^+_k \right). \tag{7}$$

Since $SYSTEM$ is convex and the constraints are linear, it can be reformulated as:

$$\max_y \quad \min_{x:(6)} \quad L(x, y).$$

We refer to the optimal y in this problem as the optimal Lagrange multipliers.

III. Architectural Decomposition

Our goal in this paper is to understand, from first principles, how the structure of the global system operator's problem can guide the architecture of power systems control. To that end, our main result is a decomposition of the global system operator's problem into fast and slow timescale problems. These subproblems can serve as guides for the design of market mechanisms for economic dispatch and control policies for frequency regulation. Importantly, our decomposition identifies a rigorous connection between economic dispatch and frequency regulation that ensures, under certain conditions, that the combination solves the global system operator's problem.

Theorem 1. *Let $(\lambda_k^*, \mu_k^{-*}, \mu_k^{+*})_{k\in\mathcal{K}}$ denote optimal Lagrange multipliers. Suppose that for each $k \in \mathcal{S}$:*

$$\sum_{k'\in\mathcal{K}_k\setminus\{k\}} \pi\left(\lambda_{k'}^*, \mu_{k'}^{-*}, \mu_{k'}^{+*}\right) = 0. \tag{8}$$

Then $(q_k^{b*}, q_k^{p*})_{k\in\mathcal{K}}$ is an optimal solution to $SYSTEM$ if and only if $(q_k^{b*}, q_k^{p*})_{k\in\mathcal{S}}$ is an optimal solution to:

$$\begin{aligned}
ED: \quad & \min && \textstyle\sum_{k\in\mathcal{S}}\sum_n \left(Sc_n^b(q_{k,n}^b) + c_n^p(q_{k,n}^p)\right)\\
& \text{over} && q_k^b,\ q_k^p, \quad k\in\mathcal{S};\\
& \text{s.t.} && \mathbf{1}^\top(q_k^b + q_k^p - d_k) = 0, \quad k\in\mathcal{S};\\
& && -f \le H\left(q_k^b + q_k^p - d_k\right) \le f, \quad k\in\mathcal{S};\\
& && \underline{q}^b \le q_k^b \le \bar{q}^b, \quad k\in\mathcal{S};\\
& && \underline{q}^p \le q_k^p \le \bar{q}^p, \quad k\in\mathcal{S},
\end{aligned}$$

and for each $k \in \mathcal{K}\setminus\mathcal{S}$, q_k^{p*} is an optimal solution to:

$$\begin{aligned}
FR_k: \quad & \min && \textstyle\sum_n c_n^p(q_{k,n}^p)\\
& \text{over} && q_k^p;\\
& \text{s.t.} && \mathbf{1}^\top(q_{s(k)}^{b*} + q_k^p - d_k) = 0; && \text{(9a)}\\
& && -f \le H\left(q_{s(k)}^{b*} + q_k^p - d_k\right) \le f; && \text{(9b)}\\
& && \underline{q}^p \le q_k^p \le \bar{q}^p; && \text{(9c)}\\
& \text{given} && q_{s(k)}^{b*}.
\end{aligned}$$

Moreover, if generation constraints are not binding at the optimal solution, i.e.

$$\begin{aligned}
&\underline{q}^b < q_k^{b*} < \bar{q}^b, \quad k\in\mathcal{K};\\
&\underline{q}^p < q_k^{p*} < \bar{q}^p, \quad k\in\mathcal{K},
\end{aligned}$$

then the converse is also true, i.e. suppose $(q_k^{b*}, q_k^{p*})_{k\in\mathcal{K}}$ is an optimal solution to $SYSTEM$ if and only if $(q_k^{b*}, q_k^{p*})_{k\in\mathcal{S}}$ is an optimal solution to ED, and for each $k\in\mathcal{K}\setminus\mathcal{S}$, q_k^{p*} is an optimal solution to FR_k, then (8) *holds.*

The proof of Theorem 1 is given in the Appendix. The result follows from a dual decomposition of the system operator's problem into separate problems that operate on two different timescales. Note that ED and FR_k can be solved in a modular fashion with causal communication. In particular, ED can be solved first once and then only the optimal baseload generations are needed as setpoints in FR_k.

We denote the two sub-optimizations by ED and FR_k because they can be interpreted as the economic dispatch and frequency regulation components, respectively, of existing operations. The correspondence is immediate in the case of ED and we discuss how the decomposition leads to improved market mechanisms for economic dispatch based on supply function bidding in Section V. However, the correspondence may not be as clear in the case of FR_k. We show in Section IV that FR_k can in fact be solved via distributed frequency regulation algorithms, although these algorithms deviate from current practice since current approaches typically do not optimize for generation costs.

The most important component of Theorem 1 is condition (8). This condition has an intuitive interpretation that time-averaged prices at every node over every slow timescale interval is zero. It is hence unsurprising that, under this condition, slow timescale quantities can be solved myopically without regard for fast timescale dispatch. However, this condition could be restrictive in practice. For instance, this condition cannot hold at every node simultaneously if all cost functions are strictly increasing. This is because:

$$\mathbf{1}^\top \sum_{k'\in\mathcal{K}_k\setminus\{k\}} \pi(\lambda_{k'}^*, \mu_{k'}^{-*}, \mu_{k'}^{+*}) = \sum_{k'\in\mathcal{K}_k\setminus\{k\}} \lambda_{k'}^* \mathbf{1}^\top\mathbf{1} > 0.$$

Here, the first equality follows from the fact that $H\mathbf{1} = 0$ and the second inequality follows from the fact that $\lambda_k^* > 0$ if all cost functions are strictly increasing. Even if cost functions could be decreasing (e.g. due to ramp-down costs), this condition is unlikely to hold often as it requires demand to have a specific profile and any perturbation in demand is likely to cause this condition to be violated. Hence, an important extension of this work is to understand the efficiency loss in cases where this condition does not hold and the decomposition is no longer optimal.

Theorem 1 is close in spirit to work in communication networks that use optimization decomposition to justify and optimize protocol layering, e.g., see [12]–[15]. Hence, Theorem 1 provides a rigorous way to think about the architectural design of power networks. Though similar in spirit, Theorem 1 highlights a crucial difference between communication networks and power networks. In communication networks, different layers in the protocol stack may coordinate by communicating primal and dual variables when solving the sub-optimizations. However, such mechanics do not apply to timescale decomposition in power networks since sub-optimizations cannot have non-causal dependencies.

IV. Distributed Frequency Regulation

The goal of this section is to illustrate that the solution of FR_k can be implemented using distributed frequency regulation controllers that respect the engineering constraints of the system. Besides achieving optimality, a practical solution should introduce changes on the power scheduling that preserve the network stability; it should be robust to unexpected system events; and it should be able to quickly aggregate distributed network information in order to guarantee constraints (9a) and (9b).

In this section we provide a distributed algorithm that not only solves FR_k, but also cleverly uses network dynamics in order to aggregate the necessary information. The algorithm can be interpreted as performing distributed frequency regulation by sending different regulation signals to each bus. Importantly, the algorithm only requires local information and can be shown to preserve the stability of the network.

A. A dynamic model

Before introducing our algorithm for distributed frequency regulation, we first need to add dynamics to our system model. In particular, we describe a model for system changes within a single time period k in the following.

Let t denote the time evolution within the time period and assume without loss of generality that $t \in (k, k+1]$. Let $q_k^p(t) := (q_{k,1}^p(t), \ldots, q_{k,N}^p(t))$ denote the quantities generated by the peakers at time t. We assume that baseloads and demand do not change within the time period. Hence, baseloads generate q_k^b and demand consumes d_k. Then, the system changes within the time period are governed by the swing equations which we assume to have the following form:

$$M\dot{\omega}_k(t) = q_k^b + q_k^p(t) - d_k - D\omega_k(t) - L\theta_k(t); \tag{10a}$$
$$\dot{\theta}_k(t) = \omega_k(t), \tag{10b}$$

where $\omega_k(t) := (\omega_{k,1}(t), \ldots, \omega_{k,N}(t))$ are the frequency deviations from the nominal value at time t, $\theta_k(t) := (\theta_{k,1}(t), \ldots, \theta_{k,N}(t))$ are the phase angles at time t, $M := \text{diag}(M_1, \ldots, M_N)$ where M_n is the aggregate inertia of baseload and peaker n, and $D := \text{diag}(D_1, \ldots, D_N)$ where D_n is the aggregate damping of baseload and peaker n. Here, the notation $\dot{x}$ denotes the time derivative, i.e. $\dot{x} = dx/dt$.

Equation (10) is a linearized version of the nonlinear network dynamics widely adopted by the power systems community, e.g., [2], [16].

B. Current practice

In today's grid, frequency regulation is implemented using a control scheme known as Automatic Generation Control (AGC) that is executed between two different executions of ED. To implement AGC, the power grid is divided into several control areas, each one of them in charge of restoring the frequency to its nominal value and compensating its own supply demand imbalance.

This is achieved for a given area A by generating a *unique* control signal known as Area Control Error (ACE_A) given by

$$ACE_A(t) = K_A\omega_A(t) + \Delta\text{Tie}_A(t),$$

where $\omega_A(t)$ represents the average frequency deviation in area A, K_A is the frequency bias setting and $\Delta\text{Tie}_A(t)$ represents the net area interchange deviation with respect to the interchange scheduled by ED. The signal $ACE_A(t)$ is then sent through a proportional-integral controller that outputs the total amount of power generation that needs to be corrected.

Finally, the total change in power needed is distributed among generators using *participating factors* that are proportional to the nodal prices $\lambda_{s(k),n} + e_n^\top H^\top(\mu_{s(k)}^- - \mu_{s(k)}^+)$ in ED where e_n denotes a unit vector with a 1 in the nth component.

There are a number of sources of inefficiency in this approach. First, AGC relies on information from the economic dispatch problem that is likely out of date due to the timescale difference. Second, AGC does not satisfy the thermal limits at fast timescales. Third, AGC requires the definition of self-balancing areas which are forced to independently rebalance supply and demand within each area [1].

These problems have recently been acknowledged by the research community [17]–[19]. The main solution strategy proposed is the use distributed algorithms that dynamically adapt to power fluctuations in order to rebalance the system while minimizing the total generation cost. While these solutions can successfully adapt to rapid changes on the network, they do not respect the ramping constraints of baseline generator and cannot be implemented together with economic dispatch.

C. Distributed frequency regulation

In contrast to AGC, we now introduce a distributed, continuous-time algorithm that provably solves FR_k, and thus (by Theorem 1) integrates with economic dispatch to optimally solve the system operator's problem while satisfying the thermal line limits.

Our solution is based on a novel reverse and forward engineering approach for distributed control design in power systems [11], [17], [18], [20]–[23]. The key step in this approach is to formulate an optimization problem whose primal-dual algorithm includes the power network dynamics as part of it and where the remaining part can be implemented using distributed communication and computation.

The algorithm operates as follows. Each peaker n updates its power generation using

$$q_{k,n}^p(t) = [c_n^{p\,\prime -1}(-\omega_{k,n}(t) - \pi_{k,n}(t))]_{\underline{q}_n^p}^{\bar{q}_n^p}, \tag{11}$$

where $c_n^{p\,\prime}(x) = \frac{\partial}{\partial x} c_n^p(x)$ and $c_n^{p\,\prime -1}$ denotes its inverse. The projection $[u_i]_{\underline{q}_n^p}^{\bar{q}_n^p}$ ensures that u_i is always within $[\underline{q}_n^p, \bar{q}_n^p]$, and $\pi_{k,n}(t)$ is a control signal generated using:

$$DFR: \ \dot{\pi}_k(t) = \zeta^\pi \left(q_k^{b*} - d_k + q_k^p(t) - L\phi_k(t)\right); \tag{12a}$$
$$\dot{\mu}_k^+(t) = \zeta^{\mu^+} \left[BC^\top \phi_k(t) - f\right]_{\mu_k^+}^+; \tag{12b}$$
$$\dot{\mu}_k^-(t) = \zeta^{\mu^-} \left[-f - BC^\top \phi_k(t)\right]_{\mu_k^-}^+; \tag{12c}$$
$$\dot{\phi}_k(t) = \chi^\phi \left(L\pi_k(t) - CB(\mu_k^+(t) - \mu_k^-(t))\right), \tag{12d}$$

where $\zeta^\pi := \text{diag}(\zeta_1^\pi, \ldots, \zeta_N^\pi)$, $\zeta^{\mu^+} := \text{diag}(\zeta_1^{\mu^+}, \ldots, \zeta_L^{\mu^+})$, $\zeta^{\mu^-} := \text{diag}(\zeta_1^{\mu^-}, \ldots, \zeta_L^{\mu^-})$ and $\chi^\phi := \text{diag}(\chi_1^\phi, \ldots, \chi_N^\phi)$ denote the respective control gains. Given vectors $x, y \in \mathbb{R}^M$ and $\mathcal{M} = \{1, \ldots, M\}$, the element-wise projection $[y]_x^+$ ensures that the dynamics $\dot{x} = [y]_x^+$ have a solution $x(t)$ that remains in the positive orthant. That is, $[y]_x^+ := ([y_m]_{x_m}^+)_{m \in \mathcal{M}}$, with $[y_m]_{x_m}^+ = 0$ if $x_m = 0$ and $y_m < 0$; $[y_m]_{x_m}^+ = y_m$, otherwise.

The proposed solution (11) – (12) can be interpreted as a frequency regulation algorithm in which each peaker receives a different regulation signal (11) depending on its location in the network.

D. Optimality and convergence

We now show how the distributed algorithm described above converges to the optimal solution of FR_k.

To this end, we first modify FR_k and define a related problem FR'_k that can be shown to be equivalent to FR_k while, at the same time, making the role of frequency in maintaining supply-demand balance explicit. This is a nontrivial modification of FR_k and is crucial to guaranteeing the stability of our distributed algorithm.

$$\begin{aligned}
FR'_k: \quad & \min && \textstyle\sum_n \left(c_n^p(q_{k,n}^p) + D_n \frac{\omega_n^2}{2}\right) \\
& \text{over} && q_k^p,\ \omega_k,\ \theta_k,\ \phi_k; \\
& \text{s.t.} && q_{s(k)}^{b*} + q_k^p - d_k - D\omega_k = L\theta_k; && \text{(13a)} \\
& && q_{s(k)}^{b*} + q_k^p - d_k = L\phi_k; && \text{(13b)} \\
& && -f \le BC^\top \phi_k \le f; && \text{(13c)} \\
& && \underline{q}^p \le q^p \le \bar{q}^p; && \text{(13d)} \\
& \text{given} && q_{s(k)}^{b*}.
\end{aligned}$$

Constraint (13a) makes explicit the fact that, whenever supply and demand do not match, the mismatch is compensated by a change in the frequency. We have also used the equivalent per node supply-demand balance constraint (2) instead of the aggregate supply-demand balance constraint (1). Constraint (13b) ensures that the optimal solution satisfies $\omega_k^* = 0$ so that supply and demand are balanced. Constraint (13c) imposes line flow limits. However, instead of using actual line flows $BC^\top\theta_k$, we impose these limits on *virtual flows* $BC^\top\phi_k$ which are identical at the optimal solution [11].

The next proposition formally relates FR_k and FR'_k and guarantees the optimality of (11) – (12).

Proposition 1 (Optimality). *Let q_k^{p**} and $(q_k^{p*}, \omega_k^*, \theta_k^*, \phi_k^*)$ be optimal solutions of FR_k and FR'_k respectively. Then, the following statements are true:*

(i) *Frequency restoration: $\omega_k^* = 0$;*
(ii) *Generation equivalence: $q_k^{p**} = q_k^{p*}$;*
(iii) *Line flow equivalence:*

$$BC^T\top\phi_k^* = BC^\top\theta_k^* = H\left(q_{s(k)}^{b*} + q_k^{p**} - d_k\right).$$

Moreover, a vector $(q_k^{p*}, \omega_k^*, \theta_k^*, \phi_k^*, \pi_k^*, \mu_k^{+*}, \mu_k^{-*})$ is an equilibrium point of (10) – (12) *if and only if it is a primal-dual optimal solution of FR'_k.*

What remains is to guarantee the convergence of the distributed frequency regulation algorithm.

Proposition 2 (Convergence). *Given the distributed frequency regulation scheme* (11) – (12) *and the network dynamics* (10). *Then, provided that $c_n^p(\cdot)$ is twice continuous differentiable with $(c_n^p)''(\cdot) \ge \alpha > 0$ (α-strictly convex), the coupled dynamics* (11) – (12) *and* (10) *converge globally to an optimal solution of FR_k.*

The proof of Proposition 2 follows from [11]. It is easy to show that by substituting the phase representation of the line flows $BC^T\theta_k$ with p_k in FR'_k and (10), the whole system (10) – (12) is a primal-dual algorithm of FR'_k (see [11, Theorem 5]). Therefore, Theorem 10 in [11] guarantees global asymptotic convergence to an equilibrium point which by Proposition 1 is an optimal solution of both FR'_k and FR_k. We remark the controllers of [11] have additional states, but the proof in this simpler case is identical.

V. A Market Mechanism for Economic Dispatch

We now move our attention to the economic dispatch component of the decomposition provided by Theorem 1. We illustrate that the solution of the economic dispatch problem formalized in ED can be implemented using a market mechanism based on supply function bidding. The mechanism we propose aligns with current practice, but differs in an important way that ensures proper coordination with frequency regulation, thus avoiding the inefficiency of approaches adopted today.

A. Current practice

The economic dispatch problem is solved in practice using complicated market mechanisms, see [5], [24]–[26] for an overview. Briefly, existing markets price slow timescale economic dispatch resources using nodal prices which are derived from Lagrange multipliers according to equation (7). As we described in Section IV-B, the nodal prices are used to compensate any frequency regulation resources dispatched within each slow timescale interval.

This implementation is adopted, in part, because of the short timescale of frequency regulation (on the order of seconds), which makes it challenging to implement separate markets for each frequency regulation interval. However, inefficiencies arise because the Lagrange multipliers may vary within each slow timescale interval so the prices computed from the economic dispatch problem may not provide the appropriate incentives for frequency regulation. Such inefficiencies are becoming more significant due to the growth of renewables [27]–[29].

B. Market Mechanism

The approach for economic dispatch suggested by the decomposition in Theorem 1 is similar to current practice, but it also provides insight on how to avoid the inefficiency highlighted above.

In particular, we propose a market mechanism that operates on the timescale of economic dispatch but includes an efficient pricing mechanism for fast timescale frequency regulation. Our proposed mechanism is efficient if the conditions for decomposability of the global problem given in Theorem 1 hold and does not require any more communication than existing market mechanisms.

Concretely, in our proposal the system operator collects supply function bids from generators at the slow timescale and solves the economic dispatch problem as in the current practice. However, instead of compensating frequency regulation resources using slow timescale nodal prices, the system operator uses the bids to allocate the peaking resources efficiently at the fast timescale.

1) Assumptions: In this section, we make the following assumptions to simplify the exposition. We assume that generators are not capacity-constrained, i.e. $\underline{q}^b = \underline{q}^p = 0$ and $\bar{q}^b = \bar{q}^p = \infty$. We also assume that peaker and baseload cost functions are given by:

$$c_n^p(\cdot) = (1/\rho_n^p)c(\cdot), \quad c_n^b(\cdot) = (1/\rho_n^b)c(\cdot),$$

for some $\rho_n^p, \rho_n^b > 0$ and some convex and continuously differentiable function $c : \mathbb{R}_+ \to \mathbb{R}_+$. This model includes the class of quadratic cost functions which is a common assumption in many studies of electricity markets [30]–[32].

2) Supply functions: We assume that supply functions are chosen from a parameterized family of functions. In particular, we represent a supply function by a parameter $\rho > 0$ and it indicates that the generator is willing to supply the quantity $s(\rho\pi)$ when the price is π. Associate with this supply function the following surrogate cost function:

$$\hat{c}(q, \rho) := (1/\rho) \int_0^q s^{-1}(\bar{q}) d\bar{q}.$$

Numerous studies have explored different functional forms of s and their impact on the efficiency of the market, e.g., [6], [26], [33]–[35]. For this paper, we assume that $s(\pi) = (c')^{-1}(\pi)$. It follows that:

$$\hat{c}(q, \rho) = (1/\rho) \int_0^q c'(\bar{q}) d\bar{q} = (1/\rho)c(q).$$

3) Mechanism: Each baseload and peaker submits supply functions to the system operator who clears the market by solving ED and FR_k using surrogate cost functions.

Formally, each baseload n submits a sequence of supply functions $(\hat{\rho}_{k,n}^b)_{k\in\mathcal{S}}$ where $\hat{\rho}_{k,n}^b$ is its supply function in period $k \in \mathcal{S}$ and each peaker n submits a sequence of supply functions $(\hat{\rho}_{k,n}^p)_{k\in\mathcal{S}}$ where $\hat{\rho}_{s(k),n}^p$ is its supply function in period $k \in \mathcal{K}$. Note that each peaker n must choose the same supply function $\hat{\rho}_{s(k),n}^p$ for all periods $k' \in \mathcal{K}_k$. Hence, the bids in the economic dispatch timescale are used as bids in the frequency regulation timescale.

We assume that baseloads and peakers are price-takers. Given a sequence of prices $(\pi_{k,n})_{k\in\mathcal{K}}$, each baseload n chooses bids $(\hat{\rho}_{k,n}^b)_{k\in\mathcal{S}}$ to maximize its profit:

$$\begin{aligned} B_n : \max \quad & \textstyle\sum_{k\in\mathcal{S}} S \Big(\pi_{k,n} s(\hat{\rho}_{k,n}^b \pi_{k,n}) \\ & \qquad -(1/\rho_n^b) c(s(\hat{\rho}_{k,n}^b \pi_{k,n}))\Big) \\ \text{over} \quad & \hat{\rho}_{k,n}^b, \quad k \in \mathcal{S}; \\ \text{given} \quad & \pi_{k,n}, \quad k \in \mathcal{S}, \end{aligned}$$

and each peaker n chooses bids $(\hat{\rho}_{k,n}^p)_{k\in\mathcal{S}}$ to maximize its profit:

$$\begin{aligned} P_n : \max \quad & \textstyle\sum_{k\in\mathcal{K}} \Big(\pi_{k,n} s(\hat{\rho}_{s(k),n}^p \pi_{k,n}) \\ & \qquad -(1/\rho_n^p) c(s(\hat{\rho}_{s(k),n}^p \pi_{k,n}))\Big) \\ \text{over} \quad & \hat{\rho}_{k,n}^p, \quad k \in \mathcal{S}; \\ \text{given} \quad & \pi_{k,n}, \quad k \in \mathcal{K}. \end{aligned}$$

Let $\hat{\rho}_k^b := (\hat{\rho}_{k,1}^b, \ldots, \hat{\rho}_{k,N}^b)$ and $\hat{\rho}_k^p := (\hat{\rho}_{k,1}^p, \ldots, \hat{\rho}_{k,N}^p)$ denote the vectors of bids in period k. Given the bids $(\hat{\rho}_k^b, \hat{\rho}_k^p)_{k\in\mathcal{S}}$, the system operator solves ED for the dispatch that minimizes the surrogate costs. The nodal prices are given by $\pi_k = \pi(\lambda_k^*, \mu_k^{-*}, \mu_k^{+*})$ where $(\lambda_k^*, \mu_k^{-*}, \mu_k^{+*})$ are optimal Lagrange multipliers in ED. Then, in each fast time scale period $k \in \mathcal{K} \setminus \mathcal{S}$, it implements the frequency regulation algorithm DFR with the surrogate costs functions to drive the system to the optimal solution of FR_k. The nodal prices are again given by $\pi_k = \pi(\lambda_k^*, \mu_k^{-*}, \mu_k^{+*})$ but now $(\lambda_k^*, \mu_k^{-*}, \mu_k^{+*})$ are optimal Lagrange multipliers in FR_k.

C. Efficiency

Given the above mechanism, our focus is on understanding the efficiency of an equilibrium. The following formalizes the notion of a competitive equilibrium we consider.

Definition 1. *We say that $(\hat{\rho}_k^b, \hat{\rho}_k^p)_{k\in\mathcal{S}}$ is a competitive equilibrium if there exists $(p_k)_{k\in\mathcal{K}}$ such that:*

(a) *For all n, $(\hat{\rho}_{k,n}^b)_{k\in\mathcal{S}}$ is an optimal solution to B_n;*
(b) *For all n, $(\hat{\rho}_{k,n}^p)_{k\in\mathcal{S}}$ is an optimal solution to P_n;*
(c) *For all k, $\pi_k = \pi(\lambda_k^*, \mu_k^{-*}, \mu_k^{+*})$, where: (i) $(\lambda_k^*, \mu_k^{-*}, \mu_k^{+*})_{k\in\mathcal{S}}$ are optimal Lagrange multipliers in ED with $c_n^p(q_{k,n}^p) = \hat{c}(q_{k,n}^p, \hat{\rho}_{k,n}^p)$ and $c_n^b(q_{k,n}^b) = \hat{c}(q_{k,n}^b, \hat{\rho}_{k,n}^b)$; and (ii) for all $k \in \mathcal{K} \setminus \mathcal{S}$, $(\lambda_k^*, \mu_k^{-*}, \mu_k^{+*})$ are optimal Lagrange multipliers in FR_k with $c_n^p(q_{k,n}^p) = \hat{c}(q_{k,n}^p, \hat{\rho}_{k,n}^p)$.*

Our main result for this section follows. It highlights that, as a consequence of Theorem 1, the mechanism we propose in this section guarantees that any efficient allocation is supported by a competitive equilibrium.

Proposition 3 (Efficiency)**.** *Suppose that* (8) *holds. Then:*

(a) *Any competitive equilibrium is efficient.*
(b) *Any efficient allocation can be sustained by a competitive equilibrium.*

Proposition 3 resembles classical welfare theorems, e.g., [34], [36]–[38]. However, it differs from typical competitive equilibria frameworks because peakers are restricted to bidding a single supply function over each economic dispatch interval even though the latter contains multiple fast timescale market instances.

This creates challenges in guaranteeing existence of equilibria and efficiency that do not arise in typical competitive equilibria frameworks. Specifically, the space of bid functions needs to be expressive enough for generators to convey their costs (in multiple fast timescale market instances) using a single bid function. This is not an issue in existing market frameworks where separate bids are collected for separate market instances. In this work, we circumvent this challenge by assuming that $s(\pi) = (c')^{-1}(\pi)$. An important extension is to understand the existence and efficiency of equilibria under other supply function bid spaces.

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APPENDIX

Proof of Theorem 1. First, substitute the baseloads' decision constraints (6) into $SYSTEM$ and eliminate the variables q_k^b for all $k \notin \mathcal{S}$. This leads to the following stationarity conditions:

$$\nabla c^b(q_k^b) - \sum_{k' \in \mathcal{K}_k} \left(\pi(\lambda_{k'}, \mu_{k'}^-, \mu_{k'}^+) + \xi_{k'}^- - \xi_{k'}^+\right) = 0,\ k \in \mathcal{S};$$
$$\nabla c^p(q_k^p) - \left(\pi(\lambda_k, \mu_k^-, \mu_k^+) + \nu_k^- - \nu_k^+\right) = 0,\ k \in \mathcal{K},$$

where the vectors $\nabla c^b(q_k^b) := (c_1^{b'}(q_{k,1}^b), \ldots, c_N^{b'}(q_{k,N}^b))$ and $\nabla c^p(q_k^p) := (c_1^{p'}(q_{k,1}^p), \ldots, c_N^{p'}(q_{k,N}^p))$. Now, the stationarity conditions for ED are given by:

$$\nabla c^b(q_k^b) - \left(\pi(\lambda_k, \mu_k^-, \mu_k^+) + \xi_k^- - \xi_k^+\right) = 0,\ k \in \mathcal{S};$$
$$\nabla c^p(q_k^p) - \left(\pi(\lambda_k, \mu_k^-, \mu_k^+) + \nu_k^- - \nu_k^+\right) = 0,\ k \in \mathcal{S},$$

and those for FR_k are given by:

$$\nabla c^p(q_k^p) - \left(\pi(\lambda_k, \mu_k^-, \mu_k^+) + \nu_k^- - \nu_k^+\right) = 0.$$

If (8) holds, then any solution to the KKT conditions of ED and $(FR_k)_{k \in \mathcal{K}\setminus\mathcal{S}}$ is also a solution to the KKT conditions of $SYSTEM$ and vice versa. Next, assume that the generation constraints are not binding which implies that $\xi_k^- = \xi_k^+ = \nu_k^- = \nu_k^+ = 0$. Suppose further that any solution to the KKT conditions of ED and $(FR_k)_{k \in \mathcal{K}\setminus\mathcal{S}}$ is also a solution to the KKT conditions of $SYSTEM$ and vice versa. Then it is straightforward to see that (8) holds. □